HANDBOOK OF
GEAR DESIGN

SECOND EDITION

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Tata McGraw-Hill Publishing Company Limited
NEW DELHI

McGraw-Hill Offices
New Delhi New York St Louis San Francisco Auckland Bogotá Guatemala
Hamburg Lisbon London Madrid Mexico Milan Montreal Panama
Paris San Juan São Paub Singapore Sydney Tokyo Toronto

Tata McGraw-Hill
A Division of The McGraw-Hill Companies

Fifth reprint 2001
RDZYCRQLRYYYB

ISBN 0-07-460237-3

Published by Tata McGraw-Hill Publishing Company Limited, 7 West Patel Nagar, New Delhi 110008, typeset at Aksharshree, Vishwas Nagar, Shahdara, Delhi 110 032, and printed at Ram Book Binding House. New Delhi 110 020.

In Memory of My Mother
and
Gratitude to My Father

Preface to the Second Edition

In the present era of sophisticated machinery, gear technology has evolved to a high degree of perfection. Several developments leading to advanced leve! design and manufacture of gear and gearing systems have taken place. These developments have prompted me to revise the first edition.

The first edition was a practical reference fulfilling the requirements of design engineers and technicians. In the new edition I have updated and broadened the scope of application with additional discussions on spiral bevel gear systems, more methods for checking gear-tooth and design of gear-tooth profile.

I have been greatly encouraged by the tremendous response from the professionals from the industry and also from academicians based in India and abroad. I take this opportunity to thank my friends, colleagues and the readers of the first edition for their appreciation and feedback. I will welcome suggestions and constructive criticism on the second edition leading to its improvement.

GITIN M MAITRA

Preface to the First Edition

Prima facie, a gear is an ordinary machine element, yet such is the modern demand and importance of this seemingly simple engineering component that the science and art of gear engineering is continuing to develop new, highly efficient and unusual forms of gear systems, gear mechanisms and arrangements to meet the exacting and ever-increasing requirements of present-day technology. Today gears represent a high level of achievement in engineering behind their seemingly simple facade.

Rapid developmentshave taken place in recent years in the design, manufacturingprocesses, material, heat-treatment and other strength-improving procedures, inspection, checking and control of gears. In the present era of sophisticated technology, mass production and high-speed machinery, gear design has evolved to a high degree of perfection. The design and manufacture of precision-cut gears, made from materials of high strength, have made it possible to produce gears which are capable of transmitting extremely large loads at extremely high circumferential speeds with very little noise, vibration and other undesirable aspects of gear drives.

It is, therefore, imperative that students and engineers alike must keep themselves abreast of the latest, rapidly changing gear-design techniques.

While books on gear design are galore in industrially developed countries, there is dearth of such books in India which take cognizance of the pragmatic approach towards this subject and which conform to the Indian Standards and use the SI units of measurements. The basic concepts of gear design and their practical use have been exhaustively dealt with in this book. Existingtreatises, foreign or indigenous, are normally restricted to sophisticated clientele or are too detailed in nature. This book is intended to present the technology of gearing in a lucid manner to help the engineering student as well as the design engineers, technicians and other skilled personnel working in the industry, so that they can 'easily appreciate the underlying principles and practice of the subject. During my tenure as a design engineer in industries at home and abroad, I have gained considerable experience in the practical side of gear design and this volume is, therefore, written with an emphasis and bias towards the practical approach. At the same time, it has been supplemented by a sufficient account of the theoretical aspects. This would be profitable for the readers wishing to probe into the justification of the design techniques involved.

Since the subject of gear technology is vast, I have felt impelled to discuss the subject in the broadest terms within the scope available. Aspects of different types of gears, gearing systems and allied subjects which are normally encountered in gear design have been discussed in this book. For those readers who wish to pursue the subject in even greater depth and detail, copious references are alluded to at relevant pages.

Acknowledgements are due mostly to the authors of the numerous books and technical papers I have consulted while preparing this book, I will be thankful if the errors that might have crept in inadvertently are pointed out. Any suggestions leading to the improvement of the book will be gratefully received.

GITIN M MAITRA

Acknowledgement

The author gratefully acknowledges the permission granted by the Bureau of Indian Standards for reproducing the following standards. These standards are available for sale from the Bureau of Indian Standards, Bahadur Shah Zafar Marg, New Delhi and from its regional and branch offices at Ahmedabad, Bangalore, Bhopal, Bhubaneshwar, Bombay, Calcutta, Chandigarh, Hyderabad, Jaipur, Madras and Trivandrum.

IS: 2535-1978
IS: 4460-1967
IS: 7443-1974
IS: 5037-1969
IS: 7403-1974
IS: 2048-1975
IS: 2293-1974

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Fundamentals of Toothed Gearing

1.1 Introduction

In engineering and technology the term "gear" is defined as a machine element used to transmit motion and power between rotating shafts by means of progressive engagement of projections called teeth.

Invention of the gear cannot be attributed to one individual as the development of the toothed gearing system evolved gradually from the primitive form when wooden pins were arranged on the periphery of simple, solid, wooden wheels to drive the opposite member of the pair. These wheels served the purpose of gears in those days. Although the operation was neither smooth nor quiet, these were not important considerations as the speeds were very low. The motive power to run these systems was generally provided by treadmills which were operated by men, animals, water wheels or wind-mills. Primitive forms of gear were known to Archimedes before the Christian era. Leonardo da Vinci also used the concept of a gear system in many of his proposed appliances and machines. In recent times, however, inventors have concentrated their efforts to devise curves for the gear teeth which would provide constant relative velocity of the gear tooth faces. The unique property applicable to all these curves is that the common normal to the curves of the two teeth in contact at their point of contact must pass through the point of contact of the two pitch circles of the mating gears. One of such curves, the epicycloid, was proposed by the famous Danish astronomer, Olaf Roemer. The involute curve, which is the most common curve used today, was presumably first suggested by the celebrated Swiss mathematician, Leonard Euler. It was, however, Prof. Robert Willis of Cambridge University who gave a practical shape to these curves as applied to present-day gear toothing. Charles Camus and Philippe de Lahire are also known as the early pioneers in the field of toothed gearing.

Gears operate in pairs, the smaller of the pair being called the "pinion" and the larger the "gear". Usually the pinion drives the gear, and the system acts as a speed reducer and a torque converter. The centre distance between the rotating shafts in a gear drive is normally not too large. When the distance is comparatively large, the other power transmitting systems, such as, belt drive, chain drive, etc. are resorted to. Now, when the centre distance is not the deciding factor, belt drives are fast becoming out-moded and are being replaced by gearing systems. Although belt drives have the inherent advantages of shock, load reaction vibration absorbing capacities, their greater space requirements, exposure to surroundings and vulnerability to slippage make the gearing systems more preferable because these systems are positively driven, can be totally enclosed, require less space and are compact driving arrangements. When two smooth cylinders are mounted on shafts with parallel axes and are pressed together lengthwise, it is possible to transmit power from one shaft to the other by friction drive. If there is no slippage during the contact, such rotating cylinders will ensure a smooth and accurate transmission of angular velocity. The angular velocities [in radians per second or revolutions per minute (rpm)] of these cylinders are inversely proportional to the diameters of the cylinders. This relation applies if the driving and the driven cylinders are perfectly accurate, and the cylinders are said to produce "uniform velocity transmission."

The above arrangement, however, is impossible to achieve in practice because it does not produce a positive drive owing to the slippage which may be caused by various factors. To overcome the problem of slippage, toothed wheels or gears are used which produce positive drive with uniform angular velocity ratio.

Although kinematically the motion of a pair of gear is analogous to that of a pair of two pitch cylinders which roll without slip, the action on the meshing teeth consists generally of a combination of rolling and sliding motions. This aspect of dual motions will be discussed in the relevant section dealing with sliding velocities of gear teeth.

For mechanical power transmission, gears are generally categorised into three distinct types: (*a*) those transmitting power and motion between parallel shafts, namely, spur and ordinary helical gears; (*b*) those for shafts with intersecting axes, the angle between the shafts being generally 90°, e.g. bevel gears; and (*c*) those where the shafts are neither parallel nor intersecting, the axes generally making 90° (or some other angle) to each other but in different planes, e.g. worm and worm-wheel, crossed-helical gears, and hypoid gears.

In recent times, the gear design has become a highly complicated and comprehensive subject. A designer of a modern gear drive system must remember that the main objective of a gear drive is to transmit higher power with comparatively smaller overall dimensions of the driving system which can be constructed with minimum possible manufacturing cost, runs reasonably free of noise and vibration, and which requires little maintenance. He has to satisfy, among others, the above conditions and design accordingly, so that the design is sound as well as economically viable.

Although the types and the modalities of gear design vary widely, the following data can be used as guidelines.

Spur and helical gear drives The usual reduction ratios in spur and helical gear drives are: 1:8 (max. 1:20) for single stage, 1:45 (max. up to 1:60) for double stage, and 1:200 (max. up to 1:300) for triple stage.

Depending on the number of stages, type of design and size, this type of gearing can have output up to 18000 kW, speed up to 100000 rpm and circumferential velocity up to 200 metre/second, overall efficiency generally up to 96-99%. Planetary gear drives using spur and helical gears usually have reduction ratio up to 8 with a max. value of around 13. Efficiency is about 98-99%, power output up to 7500 kW and speed up to 40000 rpm.

Worm and worm-wheel drives The reduction ratio in these drives is usually up to 60, but it can be increased up to and over 100 depending on the numbers of stages. Efficiency ranges from 97% decreasing to 45% with increasing reduction ratios and slower sliding velocity. Output is usually up to 750 kW, output torque up to 250,000 Nm, speed up to 30000 rpm, and circumferential velocity up to 70 m/s. The worm and worm-wheel drive produce lesser noise and vibration than any other kind of gear drive. It is also comparatively cheaper and is recommended for those cases where a self-actuated reversal of power flow must not occur, i.e. it can ensure irreversibility when desired.

Bevel gear drives The reduction ratio is usually up to 6, in bevel gear drive, but still higher values are possible in some cases. For higher service requirements and operational conditions, hardened spiral bevel gears are usually specified.

Selection of the right kind of gear for the right kind of application is an open issue and there is no ready method which can be specified for the purpose. Generalisations can, of course, be made which can lead to the selection of process of manufacture to be adopted and the type of gear to be specified for specific purposes. Final decision, however, will depend entirely on the discretion and technical skill of the designer who will weigh the merits and demerits of the several choices available before making the final selection. As regards applications, the following guidelines are of relevance.

Gears used in machine tools must be accurate and rugged. The gear teeth are finish-machined by one of the precision-producing methods employed. Alloy steels with good machinability are used for hardness of Brinell Hardness Number 2500 to 3500 N/mm². With the introduction of carbide-tipped tools, higher cutting speeds are involved which in turn necessitates harder and more precise gears. The hardness can go up to 60 Rockwell Hardness C or even higher in certain specific applications.

In case of automobiles, which use spur, helical as well as bevel gears for transmission gear boxes and differentials, gears are generally cut from low-alloy steel forgings which after teeth cutting are heat-treated to the desired hardness. The gear teeth should be very accurate in the initial stage itself as no post-hardening tooth-correcting processes are employed. Case hardened automobile gears usually have a surface hardness of around 60 HRC and core hardness of around 30 HRC. This imparts the gears the properties of wear-resistance, strength, plus the shock absorbing capabilities.

Gears used in marine applications are very large, powerful and run at high speeds. Herring-bone gears are usually employed. As high circumferential velocities and load carrying capacities are required, extreme accuracy in tooth-spacing is essential.

A class of gears, called control gears are used as timing gears in machines and as setting gears for guns in ships and aeroplanes, etc. Here, the main objective is transmitting precise angular motion, transmission of power being secondary. Backlash is extremely small, being practically zero in certain applications.

For household gadgets medium-carbon steel gears, sintered metal, non-metallic, laminated gears are commonly used. Die-cast gears of non-ferrous metals are often used for their low cost. Punched gears and moulded-plastic gears are also widely used in domestic applications and appliances.

Terminology, symbols and notation for toothed gearing It has been the practice with different authors to use various symbols to represent gear parameters according to their own choice. To avoid confusion and to promote international usage, the International Organisation for Standardisation (ISO) has issued the following recommendations in this regard.

The ISO recommendation No. 888 entitled "International Vocabulary of Gears" lays down the nomenclatures, and the ISO Recommendation R 701 entitled "International Gear Notation, Symbols for Geometrical Data" lays down the relevant notations.

The terminology and the notations for toothed gearings are also covered by the Indian Standard Specifications IS: 2458 and IS: 2467 which broadly tally with the ISO Recommendations.

In this book, the same symbols, notations and subscripts have been used with a few minor exceptions.

1.2 General Classification of Gears

Depending upon the relation between the axes, shape of the solid on which the teeth are developed, curvature of the tooth-trace and any other special features, gears are categorised into the following types.

Spur gears In a pair of mating spur gears, the axes of the component gears are parallel, that is, they are mounted on shafts which are parallel to each other. The reference or the pitch solid is a cylinder. The gear teeth are straight along the length and are parallel to the axis. A rack is a straight-sided gear and can be thought of as a spur gear of infinite diameter.

Parallel helical gears In these gears also the axes are parallel and the pitch solid is cylindrical. The tooth-traces or the elements of teeth are helices and these helices may be left-handed or right-handed. (In this connection, it may be mentioned that some authors designate the spur gears as straight spur gears and the helical gears as helical spur gears. This practice will not be followed in this book.)

Herringbone gears Also known as double-helical gears, these gears are actually two helical gears of opposite hands, placed side by side and cut on the same blank to obtain a composite unit.

Straight bevel gears In this type of gearing, the axes are intersecting. The angle between the two axes, known as the shaft angle, is usually 90°, but it can be of other value also. The gear blank is a cone on which teeth are generated. The teeth are straight, but the height of teeth gradually decreases and the sides of teeth are tapered so that all lines, when extended, meet at a common point called the pitch cone apex. In case where the bevel gears are required to have uniform clearance throughout the length of the teeth, only the pitch cones of the two gears intersect at the apex point. Bevel gears having straight teeth but mounted on non-interesecting axes are known as skew gears. After the advent of hypoid gears, these gears are seldom used.

Spiral bevel gears In this type of bevel gears, the tooth elements are curved in the shape of a spiral so that the contact between the inter-meshing teeth begins gradually and continues smoothly from one end to the other. The contour of the spiral depends on the particular make, e.g. circular arc for Gleason system, involute for Klingelnberg system, etc. Also, the lengthwise tooth height may be tapered towards the apex or it may be uniform throughout its length.

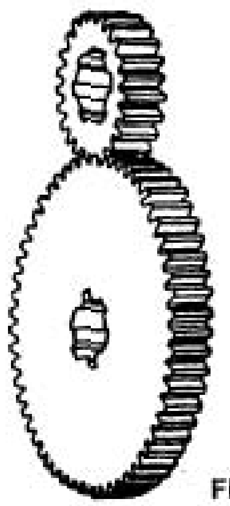
Zero bevel gears These are spiral bevel gears where the spiral angle is zero. It is a patented item of the Gleason Works of USA.

Hypoid gears These are similar to spiral bevel gears, but have non-intersecting axes, i.e. the axis of the pinion is off-set relative to the gear axis. However, the planes containing the two axes are usually at right angles to each other. If the off-set is sufficient so that the two shafts can pass one another with adequate clearance, the straddle mounting on bearings for both the pinion and gear is possible. In such cases, obviously the component gears need not be overhung. The blanks of hypoid gears are hyperboloids of revolution. Hence the name.

Crossed helical gears These are cylindrical helical gears, but their axes are at an angle when in mesh and do not intersect. Crossed helical gears are also sometimes termed as "spiral gears" and "screw gears" but such names are discouraged as they are rather confusing.

Worm and worm-wheel In this system of gearing, the axes are non-intersecting and the planes containing the axes are normally at right angles to each other. The tooth elements of both the components are helices. The system can be single-enveloping or double-enveloping types.

The above mentioned types are the major classes of gears commonly in use. There are other special types such as coniflex bevel gears, crown or face gears, spiroid gears, beveloid gears, helicon gears, planoid gears, and revacycle bevel gears. Most of these special types are patented items bearing registered trade names of different manufacturing companies, such as Gleason Works, Vinco Corporation, Michigan Tool Co., Illinios Tool Works. De Laval Holroyd Co.



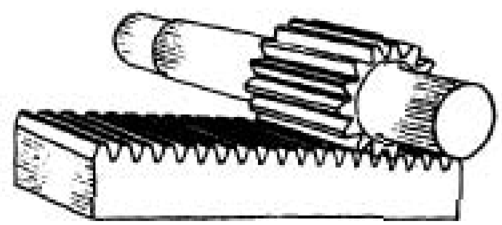


Fig 1.2 Rack and pinion

Fig. 1.1 Spur gear

The characteristics and uses of each of the types of gears which are commonly in use will be discussed in detail in the relevant chapters and sections later. The different types of gears described above have been illustrated in Figs 1.1-1.10.

1.3 Principles of Transmission and Conjugate Action

When a pair of mating gear teeth act against each other, rotary motion is produced which is transmitted from the driver to the driven gear. If such a pair of gears have tooth profiles which are so designed that a constant angular velocity ratio is produced and maintained during meshing, the two gears are said to have conjugate action and the tooth profiles are said to have conjugate curves. In other words, conjugate action is assured if

$$\frac{\omega_1}{\omega_2} = \text{constant}$$

where ω_1 = Angular velocity of the driver component of the mating pair, generally called pinion, in radians per second

ω_2 = Angular velocity of the driven component of the mating pair, generally called gear, in radians per second.

If the tooth profile of one member of the pair is given, it is possible to construct the tooth profile of the other member in order to have conjugate action when the gears mesh. However, we will

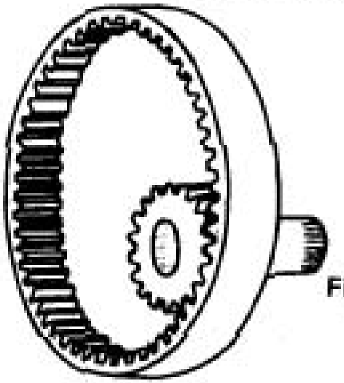


Fig. 1.3 Internal gear

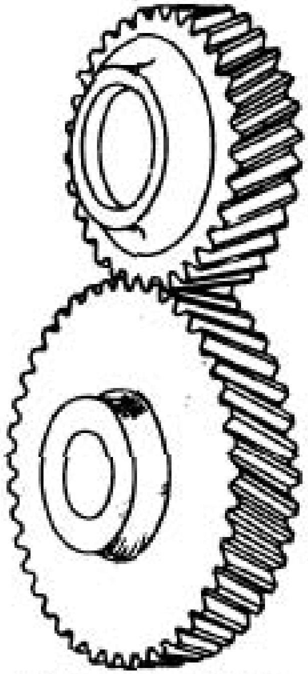


Fig. 1.4 Helical gear

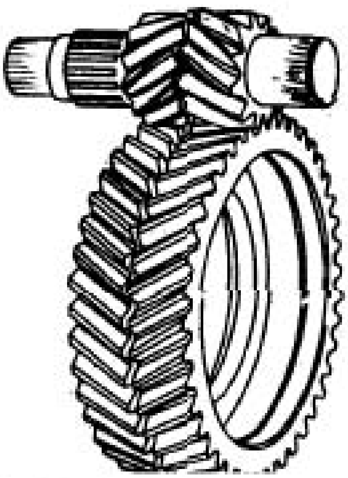
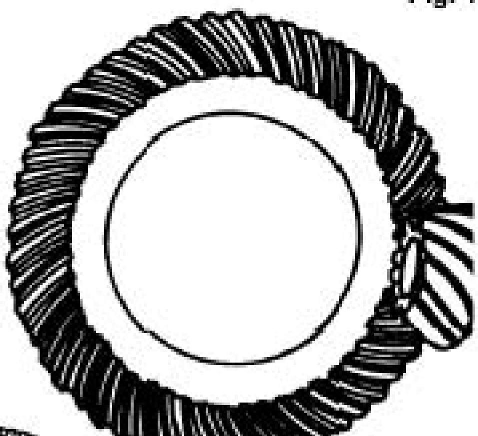


Fig 1.5 Herringbone gear



ig. 1.7 Spiral bevel gear

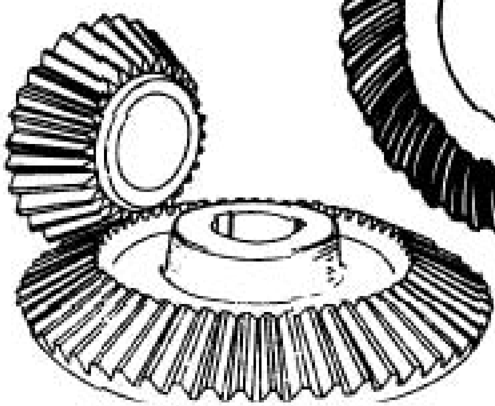


Fig. 1.6 Straight bevel gear

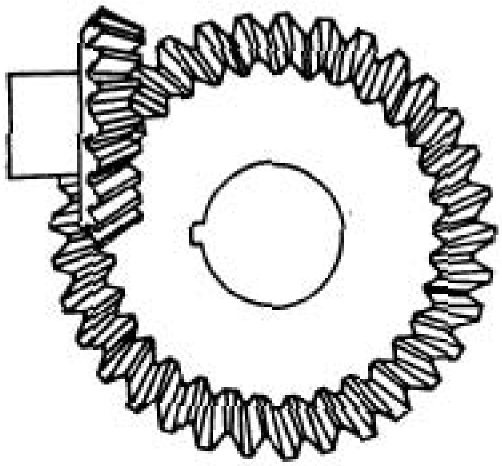


Fig. 1.8 Hypoid gear

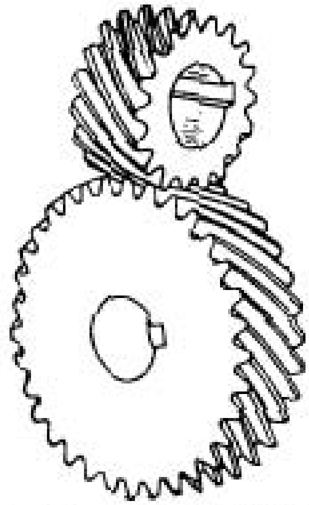


Fig. 1.9 Crossed helical gear

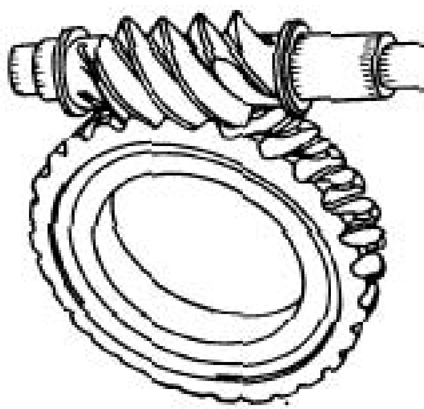


Fig. 1.10 Worm and worm-wheel

see later that the most commonly used curves which serve this purpose are the family of involute and cycloidal curves.

Figure 1.11 shows two curved body-surfaces which are in contact with each other. Body 1, with centre at O_1 and having angular velocity of ω_1, is pushing body 2 of which the centre is at O_2. This produces rotary motion and body 2 rotates with an angular velocity of ω_2. The point of contact at this instant is at Q where the two surfaces are tangent to each other. The common tangent to the curves is T-T and the transmission of forces takes place along the common normal N-N which is also called the line of action. The line of action N-N intersects the line of centres O_1O_2 at P which is called the pitch point. Circles drawn through P, having centres at O_1 and O_2, are termed as pitch circles. The diameters of these circles, called pitch circles diameters (PCD), are the representative parameters of the two gears.

For producing a constant velocity ratio, the curved profiles of the mating teeth must be such that the law of gearing is satisfied. This law states that:

> In order to have a constant angular velocity ratio, the tooth curves must be so shaped that the common normal to the tooth profiles at the point of contact will always pass through the pitch point, irrespective of the position of the point of contact during the course of action.

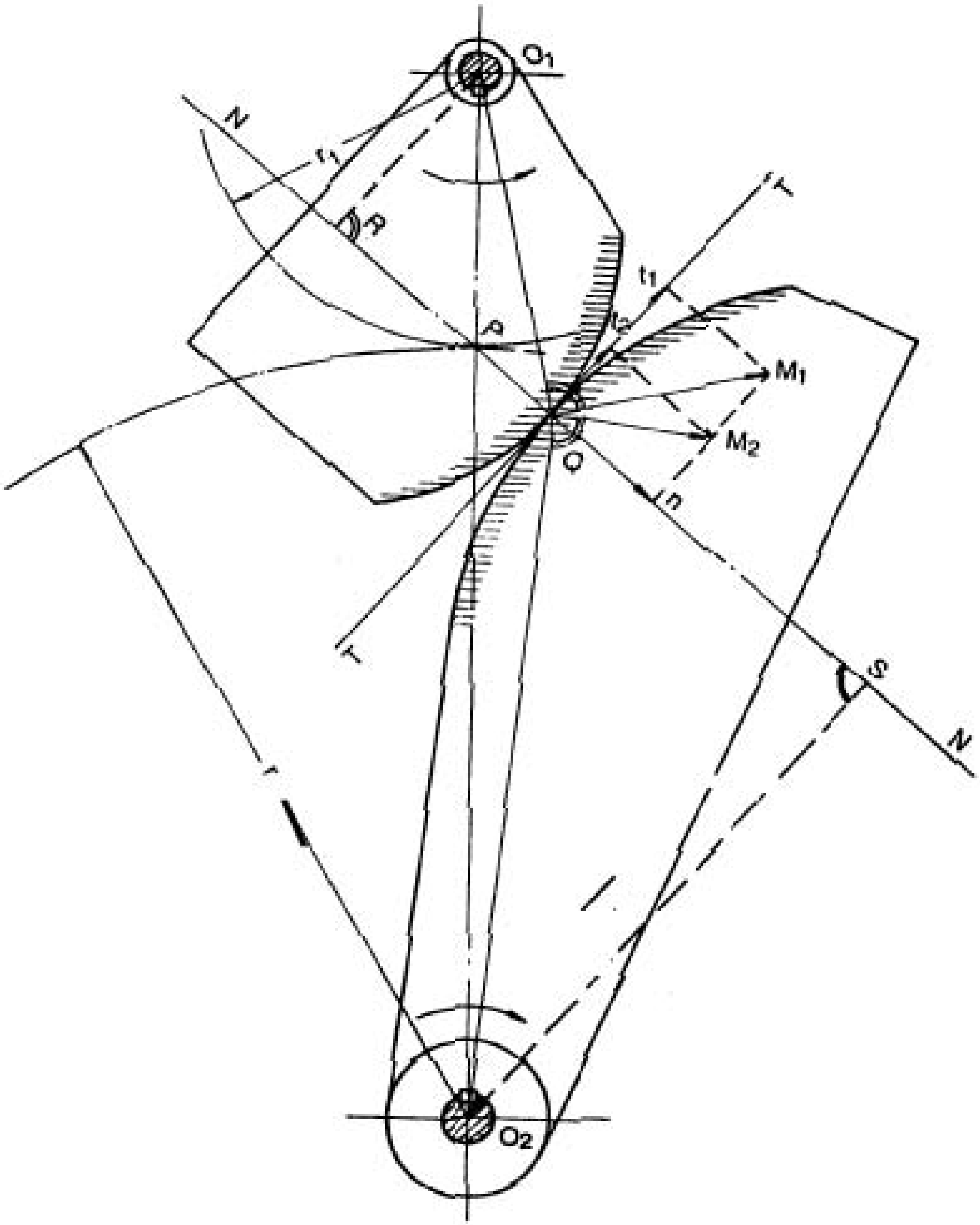


Fig. 1.11 Transmission of motion and conjugate action
Based on Grundzuege der Verzahnung, Thomas, 1957 Edition, Fig. No. 5.2, p. 28. Carl Hanser Verlag, Munich, Germany.

For a constant angular velocity ratio, the pitch point must remain stationary at a fixed point. Every line of action for every instantaneous point of contact must pass through the pitch point P.

If the teeth have cycloidal profiles, the lines of action vary in position at different points of contact during the course of action, although each line passes through the pitch point, thus observing the law of gearing. In other words, the path of contact formed by joining the different points of contact at different positions of the inter-meshing teeth is curvilinear. This is a disadvantage of the cycloidal curve as will be seen in the section dealing with cycloidal toothing.

In case of involute profile, however, all points of contact take place on the same straight line (line N-N in Fig. 1.11)—which means that the point Q, which is the instantaneous point of contact at any position, moves up and down along the line N-N. And since all the normals to the tooth profiles at any point of contact during action coincide with the line N-N, the law of gearing is obviously maintained. This aspect will be discussed in detail in the section dealing with the properties of involute toothing.

The condition that for a constant angular velocity ratio, the common normal must intersect the line of centres at a fixed point can be proved as given below.

Gear teeth move with a combination of rolling and sliding motion during the period of engagement. The sliding does not affect the transmission of motion at a constant velocity ratio because the motion at the pitch point P is one of pure rolling. This will be discussed in detail in the section on the sliding phenomenon of gear teeth. In this connection, it must be remembered that the pitch circles are the representative circles of the gears, and the pitch cylinders are analogous to two smooth cylindrical bodies, one of which drives the other through friction-contact, thus producing pure rolling action.

In Fig. 1.11, the lines QM_1 and QM_2 represent the linear velocity vectors of the two gears at the instantaneous point Q. Since $v = \omega r$, v being the circumferential velocity at radius r, these velocity vectors can be easily laid, knowing ω_1, ω_2, and the instantaneous radii O_1Q and O_2Q. And since the two bodies are rigid, the common point Q can have only one velocity component on the line of action N-N and this is represented by the line Qn. When resolved, QM_1 and QM_2 have components Qt_1 and Qt_2 respectively on the common tangent T-T. The vector difference of Qt_1 and Qt_2 gives the magnitude of the relative sliding velocity which will be treated in Sec. 2.6.

When the contact takes place at the pitch point P, the vectors QM_1 and QM_2 will be equal and in the same direction. Since the tangential components must also be equal and in the same direction, the vector difference $t_1 t_2$ vanishes. Consequently the relative velocity of sliding becomes zero and the two gears have pure rolling motion. Therefore, we come to the important conclusion that for pure rolling motion, the point of contact must lie on the line of centres.

Now, the following relations can be established from Fig. 1.11.

$$\omega_1 = \frac{QM_1}{O_1Q}, \quad \omega_2 = \frac{QM_2}{O_2Q} \text{ and } \frac{\omega_2}{\omega_1} = \frac{QM_2}{O_2Q} \times \frac{O_1Q}{QM_1}$$

From similar triangles QM_1n and O_1QR, we have

$$\qquad\quad Qn$$
$$O_1Q \qquad O_1R$$

From similar triangles QM_2n and O_2QS, we have

$$\frac{QM_2}{O_2Q} = \frac{Qn}{O_2S}$$

Therefore

$$\frac{\omega_2}{\omega_1} = \frac{Qn}{O_2S} \times \frac{O_1R}{Qn} = \frac{O_1R}{O_2S}$$

Again, O_1PR and O_2PS are similar triangles, from which we have

$$\frac{\omega_2}{\omega_1} = \frac{O_1R}{O_2S} = \frac{O_1P}{O_2P} = \frac{r_1}{r_2} \tag{1.1}$$

From the above discussions, we come, therefore, to the following important inferences.

1. When a pair of curved surfaces are in direct contact transmitting conjugate motion, the angular velocities of the two bodies are inversely proportional to the segments into which the line of centres is cut by the common normal.
2. To produce constant angular velocity ratio, the common normal must intersect the line of centres at a particular, immovable point.

Thus the law of gearing and conjugate action are satisfied.

1.4 Characteristics of the Involute Curve

It has been stated in Sec. **1.3** that to satisfy the law of gearing, that is, to maintain a constant velocity ratio in a pair of inter-meshing gears, the tooth curves are to be so designed that the common normal to the tooth profiles at the point of contact will always pass through the pitch point. The curves satisfying such a condition are termed as conjugate curves.

We shall see that one such curve is the involute curve and its relation *vis-a-vis* the maintenance of conjugate action and the law of gearing will be treated in Sec. **1.5.** The characteristics of the cycloid curve will be discussed later.

The involute of a circle is defined as the curve which is generated by the end point of a cord which is kept taut while being unwound from a circle. Any other point on the cord will also generate a similar involute curve as the cord is progressively unwrapped from the circle. The geometrical construction of the involute curve is shown in the Appendix A.

The involute is a spiral beginning from the periphery of a circle called the "base circle" which is the heart of the involute. A family of involute curves which are generated from points at equal distances on the same base circle is equidistant.

Some important relations of the involute curve which are relevant to gearing are given below.

From Fig. **1.12** it can be seen that the base circle radius r_b is given by

$$r_b = r \cos \alpha \quad \text{or} \quad r = \frac{r_b}{\cos \alpha} = r_b \sec \alpha \tag{1.2}$$

Using polar coordinates and the theorem from calculus, we have

$$\tan \alpha = r \frac{d\phi}{dr} = \frac{d\phi}{d\alpha}\frac{d\alpha}{dr} r \tag{1.3}$$

where ϕ is the angle subtended at the centre by the radius vector r, and the reference axis from where the involute is generated (in this case, the y-axis).

From Eq. 1.2, by differentiating we get

$$\frac{dr}{d\alpha} = r_b \sec \alpha \tan \alpha = r_b \frac{1}{\cos \alpha} \frac{\sin \alpha}{\cos \alpha} = \frac{r_b \sin \alpha}{\cos^2 \alpha} \tag{1.4}$$

From **Eq. 1.3**,

$$\frac{d\phi}{d\alpha} = \tan\alpha \frac{1}{r}\frac{dr}{d\alpha} = \tan\alpha \frac{1}{r} r_b \frac{\sin\alpha}{\cos^2\alpha} \text{ (from Eq. 1.4)}$$

$$= \tan\alpha \frac{1}{r} r\cos\alpha \frac{\sin\alpha}{\cos^2\alpha} = \tan\alpha \frac{\sin\alpha}{\cos\alpha} = \tan^2\alpha$$

By integrating and solving, we get

$$\int d\phi = \int \tan^2\alpha \, d\alpha$$

or

$$\phi = \tan\alpha - \alpha \quad (\alpha \text{ in radian})$$

The quantity ϕ has been termed as involute α or inv α.

$$\therefore \quad \text{inv}\,\alpha = \tan\alpha - \alpha \tag{1.5}$$

The length of the involute curve for any portion can be determined as follows. This length is sometimes required in shops for laying out templates or for copying machines. Using the usual parametric equations of the involute curve, we have.

$$x = r_b(\sin\theta - \theta\cos\theta) \tag{1.6}$$

$$y = r_b(\cos\theta + \theta\sin\theta) \tag{1.7}$$

Using the general formula for finding out the length of a curve, we have, length L, of involute from point **A** to point P is given by:

$$dL^2 = dx^2 + dy^2 = r_b^2\,\theta^2 d\theta^2$$

or

$$dL = r_b\,\theta d\theta$$

whence

$$L = r_b \int_0^\theta \theta\, d\theta = r_b \frac{\theta^2}{2} = r_b \frac{\tan^2\alpha}{2} \tag{1.8}$$

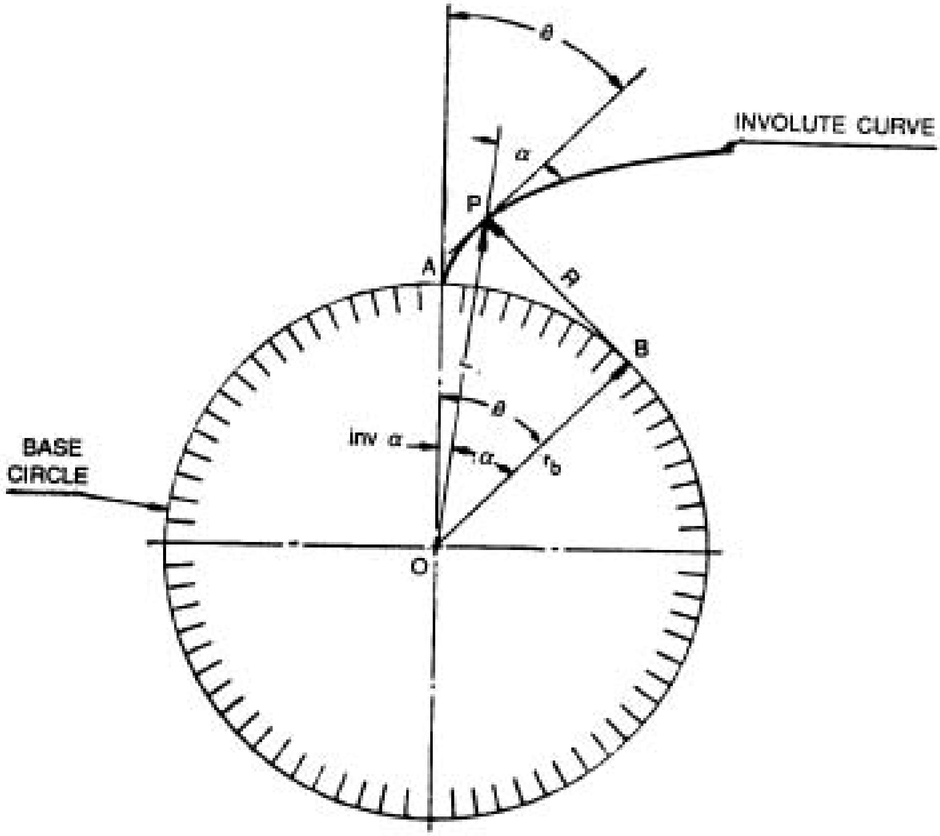
Note that $\theta = \tan\alpha$ and the radius of curvature R at the point P is given by

$$R = r_b\,\theta = r_b \tan\alpha \tag{1.9}$$

Since by definition, an involute curve is generated by the point of a cord which is unwound from the periphery of a circle and which is always held stiff, it naturally follows that the instantaneous radius of curvature of a point on the involute thus generated is the same as the stiff length of the cord which spans that point to the point on the circle from where the cord has just been unwrapped. Equation 1.9, however, can be established mathematically:

From calculus we know that the radius of curvature of any point on a curve is given by

$$R = \frac{\left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right]^{3/2}}{\frac{dy\,d^2x}{d\theta\,d\theta^2} - \frac{dx\,d^2y}{d\theta\,d\theta^2}}$$



α = pressure angle at P

θ = roll angle at P

$$\text{inv } \alpha = \theta - \alpha = \frac{\text{Arc } AB}{r_b} - \alpha = \frac{BP}{OB} - \alpha$$

$$= R/r_b - \alpha = \tan \alpha - \alpha$$

(all angles in radians)

Fig. 1.12 Involute curve

By differentiating we get from Eqs 1.6 and 1.7

$$\frac{dx}{d\theta} = r_b \cos\theta - r_b \cos\theta + r_b \theta \sin\theta = r_b\theta \sin\theta$$

$$\frac{d^2x}{d\theta^2} = r_b \theta \cos\theta + r_b \sin\theta$$

$$\frac{dy}{d\theta} = -r_b \sin\theta + r_b \sin\theta + r_b \theta \cos\theta = r_b \theta \cos\theta$$

$$\frac{d^2y}{d\theta^2} = r_b \cos\theta - r_b \theta \sin\theta$$

$$\therefore R = \frac{\left[r_b^2\,\theta^2\sin^2\theta + r_b^2\,\theta^2\cos^2\theta\right]^{3/2}}{r_b\,\theta\cos\theta\,(r_b\,\theta\,\cos\theta + r_b\,\sin\theta) - r_b\theta\sin\theta\,(r_b\cos\theta - r_b\,\theta\sin\,\theta)}$$

$$= \frac{r_b^3\,\theta^3}{r_b^2\theta^2} = r_b\,\theta$$

Thus, the radius of curvature of the involute varies continuously as θ varies. The radius is zero at the base circle where the tracing point of the cord just leaves the base circle.

Referring to Eq. 1.8 since L is directly proportional to the square of tan α, the length of the involute increases rapidly with equal increment of α. Another important conclusion to be drawn from the above mathematical treatment is that the radius of curvature R of the involute curve increases as the generating point P proceeds farther away from the base circle. In other words, the curvature of the curve (which is the reciprocal of R) decreases, resulting in increased flattened shape of curve. This fact is of relevance in case of (positively) corrected gear teeth as they are made of flatter portions of the involute curve as compared to the portions used by the uncorrected, normal gear teeth. This aspect with its implication has been dealt with in sections on corrected gears.

1.5 Involute Curve and Gear Tooth Profile

We have seen in Sec. 1.3 that to satisfy the conditions of conjugate action and the law of gearing, the tooth profiles of a pair of mating gears must be made of curves shaped to meet the requirements of the above conditions. It was also stated that two curves are generally used—the involute and the cycloidal curves. Of the two, the involute curve is the basis of nearly all tooth profiles in general use now. How the involute curve meets the requirements for use as the gear tooth profile is discussed below:

In Fig. 1.13 (a) two pulleys of radii r_{P_1} and r_{P_2} have been shown of which pulley 1 drives pulley 2 by means of a crossed cord. From the above arrangement, we can easily see that:

1. the pulleys rotate in opposite direction;
2. the ratio of angular velocities ω_1 and ω_2, that is $\omega_1 : \omega_2$, will be constant if no slippage is assured;
3. the ratio of the angular velocities is inversely proportional to the ratio of diameters of the pulleys, remembering that the linear velocity v of the cord is equal to the circumferential velocities of the two pulleys, so that

$$v = \omega_1 r_{P_1} = \omega_2 r_{P_2}$$

whence

$$\omega_1/\omega_2 = r_{P2}/r_{P1}$$

4. the ratio of angular velocities is independent of the centre distance O_1O_2 and consequently it will not change when the centre distance is altered and it remains unaffected.

Now refer to Fig. 1.13 (b). One side of the cord has been removed (to facilitate clarity of explanation) and a piece of card-board has been attached rigidly underneath pulley 1. Fix a pencil at any point Q on the cord and rotate pulley 2 counter-clockwise, always keeping the cord in taut condition. We see the following two developments:

1. As the pulleys rotate during the course of action from the beginning at pulley 1 to the end

at pulley 2, point Q will trace a straight line on the plane of paper (disregarding the card-board for the moment) on which the drawing has been made. This straight line is the common tangent to the circles representing the two pulleys.

2. Since during the process pulley 1 turns clockwise, it progressively unwraps the cord which is always kept taut. And as the pencil point moves towards pulley 2, it traces an involute on the rotating card-board, remembering the fact that from the very definition of the involute curve, the same effect would have been produced had the cord been cut at Q and the portion of the cord, initially wound on pulley 1 from point a' to a, unwrapped keeping the unwrapped portion of the cord always rigid as the unwrapping process progressively proceeded. Obviously, arc a'u ≅ straight line Qa.

The above process is repeated on pulley 2 and we have an involute on the card-board of pulley 2 as shown in Fig. 1.13(c). Imagine now that the above two individual processes are merged together. The result will be that the pencil point will now trace both the involutes simultaneously on the two rotating card-boards as point Q moves up and down the length of the cord shown.

If now the two card-boards are cut along the two curves thus generated and discarding the cord, the curves are made to come in contact, then the involute curve on pulley 1 can now be used to push the involute on pulley 2 to have a positively driven arrangement to transmit motion and force. This has been shown in Fig. 1.13(d). We can further come to the following important conclusions:

1. The line of action is the same as the cord on which the point of contact Q of the two involutes moves.
2. This line is always normal to both the involutes at the point of contact Q wherever its instantaneous position may be along the line T_1T_2.
3. This line cuts the line of centres O_1O_2 at a fixed point P, dividing the line of centres into segments O_1P and O_2P.
4. As in the case of the pulleys with crossed cord, the angular velocity ratio remains inversely proportional to the diameters of the pulleys.
5. If circles (known as the pitch circles) are now drawn through P with O_1 and O_2 as centres, their diameters $O_1P(=d_1)$ and $O_2P(=d_2)$ will also bear the same relation with the angular velocity ratio as d_1 and d_2 are proportional to the diameters of pulleys 1 and 2 repectively.

In other words, ω_1/ω_2 = Diameter of pulley 2/Diameter of pulley 1

$$= d_2/d_1 = n_1/n_2$$

where n_1 and n_2 are the speeds in rpm of the two pulleys respectively.

From the above discussion it follows that the stipulations of the conjugate action and the law of gearing are satisfied and therefore, the involute curve is suitable for the purpose. Another very important conclusion we can draw from the above discussion, a characteristic which is typical of the involute curve, is that if the centre distance is changed, involute 1 will still drive involute 2. Only, this time the different portions of the two involutes will be in contact from the portions which were in action originally. It is obvious that the angular velocity ratio remains unchanged, so long as the diameters of the pulleys (from which the involute curves originate) remain unchanged. This property, i.e. the maintenance of the constant velocity ratio remaining unaffected in spite of the alteration in the centre distance, is a great advantage of the involute curve over the cycloidal curves and is one of the reasons why this curve has replaced the cycloidal curves in most of the gearing systems. The mathematical treatment of the above aspect is given in Sec. 2.13.

In Fig. 1.13(d) T_1 and T_2 are the points of tangency of the line of action to the two base circles, that is, the circles from which the involute curves have originated. The portions T_1Q

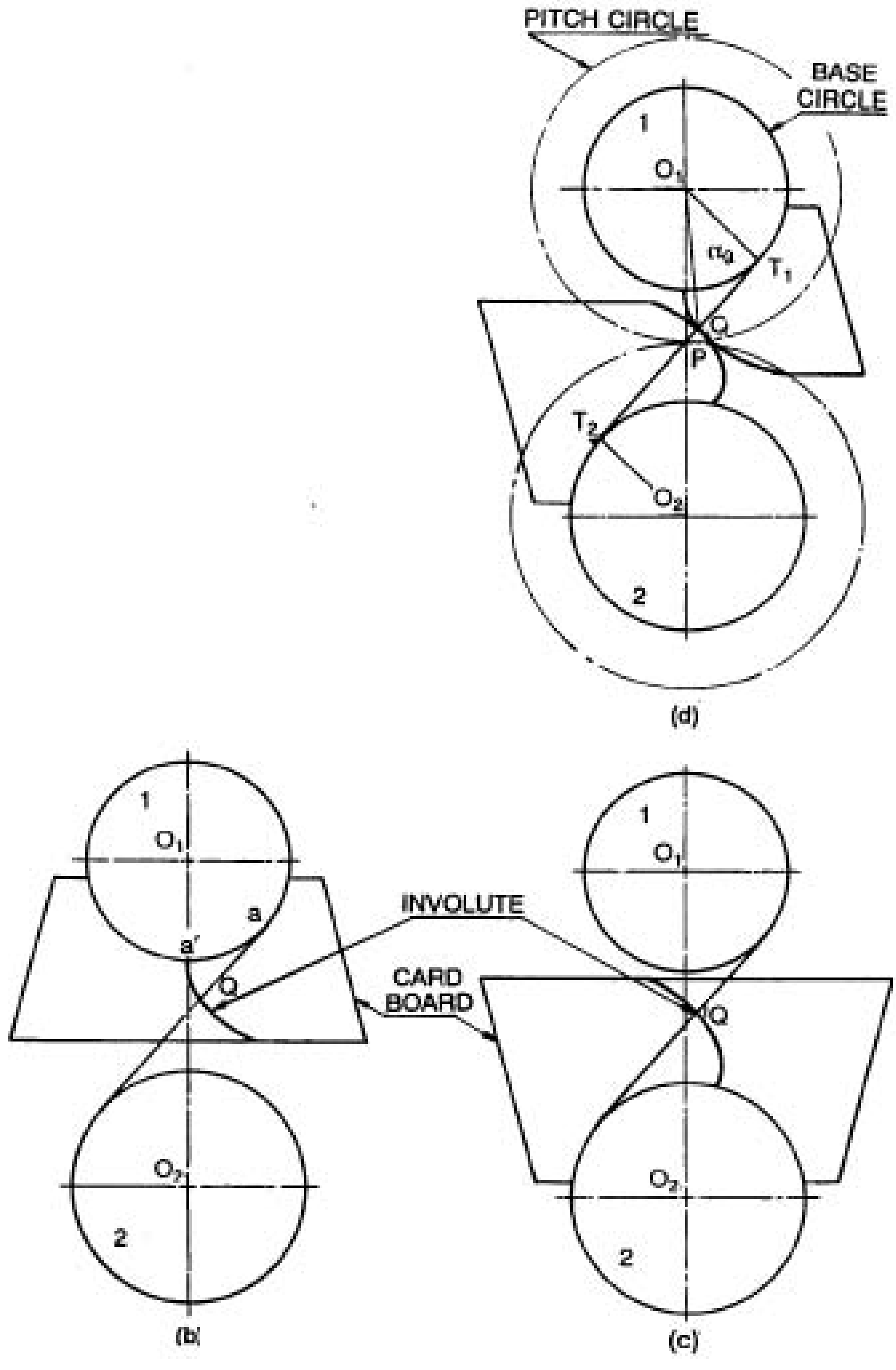


Fig. 1.13 Involute curve and law of gearing

subtends an angle α_Q at the centre O_1. This is known as the "pressure angle" and is a parameter of the point Q on the involute at the point of contact. Angle α_Q is peculiar to the point Q only and varies as Q moves on the involute. This angle must not be confused with what is commonly known as the "pressure angle" in gear terminology. This latter angle is the pressure angle only of the pitch point *P* when the two involutes of the contacting gears are in mesh at that point, its value being normally 20°. A hob or any other generating type of gear cutter of "built-in" pressure angle or "cutting" pressure angle of 20° will generate conjugate gears which will normally mate with each other and have a pressure angle of *20°*. Later on, we will see that the pressure angle of an individual point (say, *Q*) will vary along the involute. Also, in case of two normal 20° pressure angle gears, the "working pressure angle" α_w is a function of the actual centre distance when the two mating gears are mounted on the gear box and also of the addendum modification or "correction" which may be imparted to the gears.

All these aspects will be explained in detail in the sections on "correction" of gears in Chap. **2.**

1.6 Characteristics of the Cycloidal Curves

We have seen in the previous sections that to maintain a constant velocity ratio for a pair of mating gears the profiles of the meshing teeth must be so chosen that the common normals to the profiles at all the points of contact must pass through a fixed point on the line joining the centres of the mating gears. This fixed point, called the "pitch point", is the common centre of the system.

The above condition defines the law of gearing and any two curves which satisfy this law are known as "conjugate curves". This has already been discussed in Sec. **1.3.** Theoretically, if one curve of the two contacting surfaces is chosen, it is possible to construct the shape of the mating curve so as to conform to the above-stated law of gearing. A method of constructing the conjugate profile, when one profile is given, is shown in the Appendix **C.**

In practice, however, this method is not followed for gearing system and is studied for academic interest only. Mathematically, it has been established that the two families of curves, the involutes and the cycloids, satisfy the law kinematically and these curves are adopted for all practical purposes. Of these two curves, the involute has been universally adopted for practically all types of applications, except a few cases. The properties of involute have been dealt with in the previous sections. For power transmission, involute gears have completely replaced cycloidal gears. In some cases, however, cast cycloidal gears are still in use for load transmission.

The cycloidal family of curves was the first correct form to be adopted for gearing. The advantages of the involute curve, however, are so numerous that apart from the applications where the unique characteristics of the cycloidal curves are of particular relevance, the gear teeth having involute profiles are almost exclusively used.

We know from mathematics that the family of cycloidal curves is produced by rolling a circle called the "generating" or "rolling" circle. When the circle rolls on a straight line, a point on the circumference of the circle generates a cycloid. The same circle while rolling on the outside of another circle or inside of another circle gives rise to an epicycloid or a hypocycloid respectively in the same manner. The profile of a basic rack in the cycloidal system is formed by parts of a cycloid curve. But on a gear of finite diameter, the face of the gear tooth has the outline of an epicycloid while the flank has that of a hypocycloid as will be shown in Sec. 1.7. The profile of a cycloidal tooth, therefore, is of "double curvature", which is unlike the profile of an involute tooth, It has been stated before that, except for certain special cases, the involute curve is almost exclusively used in modern gearing systems. The advantages of this curve over the cycloidal system has been summarised below.

1. Since the rack in an involute system has straight sides and since the generating cutters usually have rack profiles, these cutters can be easily manufactured. A hob cutter for the cycloidal gear is not as easily made. Consequently, involute gears can be produced more accurately and at a lesser cost.
2. While the cycloidal tooth profile has double curvature, an involute tooth has single curvature which facilitates ease of manufacture.
3. For effective conjugate action, i.e. for maintaining a constant velocity ratio, it is imperative that the pitch circles of cycloidal gears must be exactly tangent. In other words, for a mating pair of these gears there is only one theoretically correct centre distance for which these will transmit motion maintaining a constant angular velocity ratio. In case of involute gearing system, the centre distance can be changed without affecting the angular velocity ratio (see Sec. 1.5). This advantage of the involute system is of prime importance as most of the modern gears are corrected ones having extended centres (see sections on corrected gears). Also, even in case of a gearing system having standard centre distance, it is practically not possible to accurately achieve or maintain that distance due to mounting inaccuracies, misalignment and a number of other diverse factors involved.
4. In involute gearing as the path of contact is a straight line and the pressure angle is constant, there is a constant force acting on the axes. In cycloidal gears, the pressure angle continuously changes (see Sec. 1.7 on cycloidal gears). This results in separating force of variable magnitude which in turn gives rise to unquiet operation, jerky running and consequently shorter life.
5. The number of cutters required is less in case of involute gearing system to produce complete sets of interchangeable gears of any particular pitch.

The cycloidal system, however, has the following advantages:

1. Cycloidal gears do not have interference and problems thereof.
2. A cycloidal tooth is stronger than a standard involute tooth. This is because it has spreading flanks whereas an involute tooth has radial flanks. Consequently, there is more material at the root portion of a cycloidal tooth as compared to an involute tooth (see Fig. 2.7).
3. Cycloidal teeth have less sliding action and hence wear less due to rubbing.
4. Since there is no problem of interference, pinions having number of teeth as low as 6 or 7 are possible in the cycloidal system. It can even be 3 or 4 in certain special cases. This is the reason why these gears are extensively used in clocks, watches and similar instruments where a low number of teeth combined with adequate strength is necessary. It is also possible to have a low number of teeth in case of involute toothing system, but this calls for a large amount of profile correction which may lead to "peaking", and if it is not corrected, the teeth may become heavily "undercut". This has been elaborated in sections on profile correction of gears.

1.7 Cycloidal Gears

In Sec. 1.6 properties of the cycloidal curves have been discussed. Appendix B gives the method of construction of cycloidal curves. In this section we will see how these curves can be adopted in gearing systems.

It has been pointed out that among the many advantages of the involute gears over the cycloidal gears, the one which is of vital importance from the practical point of view is that an involute gearing system remains unaffected by changes in the centre distance between the mating gears. In case of a cycloidal gearing system, however, the theoretical centre distance must be strictly maintained in the mounted condition. In other words, the pitch circle must be exactly tangent to each other in order to satisfy the law of gearing.

The cycloidal family of curves was the first to be used in gearing since the cycloidal curves can be made to satisfy the law of gearing. In Sec. 1.6 it was stated that when a circle, called the generating or the rolling circle, rolls over a straight line or on the outside or the inside of another circle, the curves thus generated are called cycloid, epicycloid or hypocycloid, respectively. The profile of a rack tooth in the cycloidal system consists of two separate portions — the face and the flank — both being portions of a cycloid. This is clearly illustrated in Fig. 1.14.

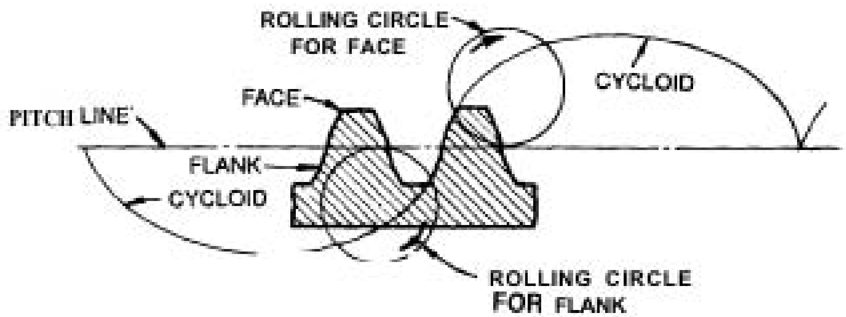


Fig. 1.14 Formation of cycloid rack

It can be seen that the profile is of double curvature and there is no smooth point of transition. It may be recalled that the profile of an involute rack is a straight line.

The following points should be noted carefully in case of two cycloidal gears of finite diameters mating with each other:

1. The face of a cycloidal gear consists of a portion of an epicycloid and the flank consists of a portion of a hypocycloid.
2. The epicycloid and the hypocycloid are generated by a rolling circle which rolls on the outside or inside of a circle called the pitch or directing circle. The form of a cycloidal gear tooth depends upon the ratio: d_G/d, where d_r is the rolling circle diameter and d is the pitch or directing circle diameter. It is desirable to construct the tooth-form with the biggest possible rolling circle. Normally, the generating or rolling circles, used for producing gear tooth profiles, bear the following relation with the pitch circles:

$$d_{G1} = 0.33 \text{ to } 0.4 d,$$

$$d_{G2} = 0.33 \text{ to } 0.4 d,$$

where d_{G1} and d_{G2} are the generating circle diameters of the pitch circle diameters d_1 and d_2, respectively.

3. To satisfy the law of gearing, the flank and face of two gears in contact must be generated by the same rolling circles. In other words, (a) the generating circle G_1 generates the flank (hypocycloid) of gear 1 and the face (epicycloid) of gear 2 and (b) the generating circle G_2 generates the flank (hypocycloid) of gear 2 and the face (epicycloid) of gear 1.

Since the flank of gear 1 mates with face of gear 2, both the curves in contact are produced by the same rolling circles, viz $G1$ and G2. The construction has been represented in Fig. 1.15.

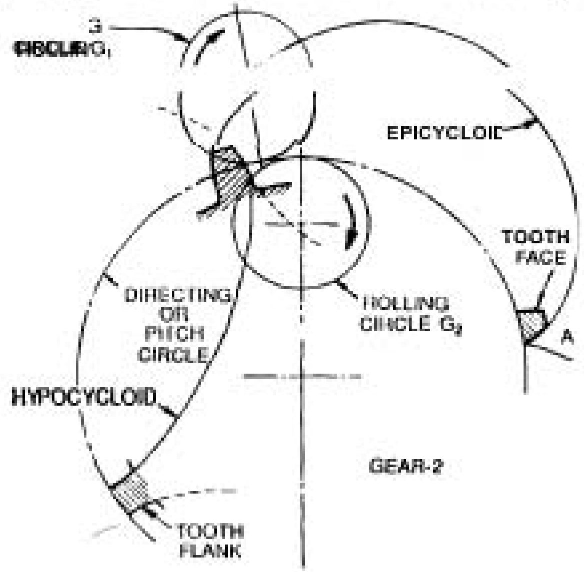


Fig. 1.15 Formation of cycloidal tooth

Action of Cycloidal Teeth

Referring to Fig. 1.16 the hypocycloid P-m has been generated by rolling $G1$ inside the pitch circle of gear 1. The flank of a tooth of gear 1 is formed by a portion of P-m, suitably placed. The epicycloid P-s is obtained by rolling $G1$ on the outside of the pitch circle of **G2**. Part of P-s forms the face of the tooth of gear 2. Thus the flank of the tooth of gear 1 and the face of the tooth of gear 2 are both generated by the same rolling circle, $G1$. These two curves mate with each other. In a similar manner, the rolling circle G2 forms P-n for the flank of the tooth of gear 2 and P-r for the face of the tooth of gear 1, respectively. These two portions also mate in a similar way during the course of action.

The two pitch circle d_1 and d_2 meet at the pitch point P. The beginning of tooth contact takes place at point A which is the point of intersection of the generating circle $G1$ and the tip circle or the addendum circle of gear 2. From A the flank A-d of gear 1 and the face A-c of gear 2 have been generated as described earlier.

We shall now examine whether the law of gearing is satisfied. From books on mechanism, we know that the point P is the instantaneous centre of rotation (or centro) of the circle $G1$. This is true irrespective of whether $G1$ is rolling inside the pitch circle of gear 1 or on the outside of the pitch circle of gear 2. Under such circumstances each point on $G1$ is moving in a direction perpendicular to the line joining that point with P. Obviously point A, which lies on $G1$, also moves in a direction perpendicular to its straight line distance from P. In other words, the velocity vector of point A at this particular instant, is normal to the line A-P. And since A is the point of contact at this particular instant and as such it is a common point to both the curves, and since the common normal to both the profile-curves, i.e. A-d and **A-c**, at the point of contact A passes through the pitch point P, the law of gearing is satisfied. It can be seen that the same reasoning

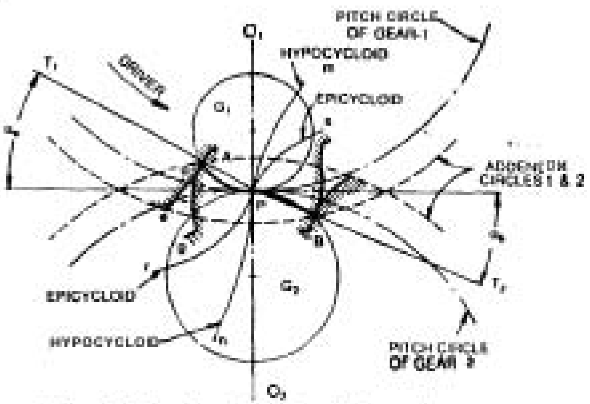


Fig. 1.16 Cycloidal tooth in action

is applicable for other points of contact also. In this connection, the following points should be clearly understood:

1. The points of contact always lie on the generating circles during the course of action.
2. The line of action or the path of contact is not rectilinear as in the case of involute toothing. It is curvilinear and is denoted by the curve *A-P-B*. Here ***B*** is the end point of contact where the tip circle of driver (gear 1) cuts the line of action.
3. The pressure angle varies at each point of contact, unlike that in the case of involute toothing where it is the same, no matter where the contact takes place. The pressure angle has the maximum values at ***A*** and ***B***. It is zero at *P*. The consequences of this varying pressure angle have been discussed in Sec. 1.6. Note that the pressure angle α_a and α_b are different. Consequently, the directions of the normal tooth forces along T_1P and PT_2 are also different.
4. It has been pointed out that the faces and the flanks of teeth which come in contact during tooth action must be generated by the rolling circle of the same size. One consequence of this condition is that if any particular cycloidal gearing set is meant to be interchangeable, then all the faces and flanks of teeth are to be generated by the rolling circle of the same size.
5. In a system of interchangeable cycloidal gears, the common rolling circle should be the one which corresponds to the smallest gear of the set, that is, the teeth of all the gears belonging to that set must be generated by the rolling circle of the same size which is relevant in case of the smallest gear. Normally, in such a system, the smallest number of teeth taken in practice is 11 and the diameter of this rolling circle is taken to be half of that of the pitch circle. That is

$$2\,r_G = r = \frac{m \cdot z}{2} = \frac{11}{2}\,m \quad \text{or } r_G = 2.75\,m,$$

where r_G = radius of the rolling circle

r = radius of the pitch circle,

m = module, and

z = number of teeth.

A cycloidal tooth has been shown in Fig. *B.2*, Appendix *B*.

1.8 Gear Materials, their Properties and Treatment

In recent years selection of the proper gear materials and their treatment commensurate with the service conditions they are subjected to has become a highly sophisticated procedure. Besides other factors, the satisfactory performance of a gear is a direct function of the material it is made of as well as the treatment thereof. Ultimately, it is the designer's responsibility to specify the material and its appropriate treatment. Hence, the designer has to delve deep into the subject and acquire a thorough knowledge of the different materials available, their treatment, types of uses, probable causes of material failure, machineability, strength, operational characteristics and all the other allied subjects. Proper selection of material and other data is a pre-requisite for an efficient gear design. In Appendix E a table showing the strength values, other data and use of common gear materials has been given. This table will be made use of in the appropriate section of this book. Gear materials and their properties are also given in tables contained in Sec. 2.25 dealing with the strength calculations and power ratings of gears. In this section the relevant properties of the different types of gear materials are discussed.

Before an elaborate description is given on the individual materials, it will be useful to remember the following guidelines for material selection:

1. Cast iron as a gear material is used where the stress conditions are light in nature.
2. Structural steels and steel castings are meant for light to medium duty gears.
3. When the stress demands are high and exacting, hardened and tempered steels as well as case-hardened steels should be used.
4. For different special properties, such as resistance to corrosion, heat or wear, etc. various alloy steels, including stainless steels, are available. Some of these combine maximum ductility with high tensile strengths — a property not found in common carbon steels.
5. Bronzes as well as some aluminium and zinc alloys display high strength combined with good sliding properties. They are normally intended for worm-wheels and similar toothed gearing.
6. Non-metallic materials offer good operational properties and noiseless running.

Normally, the teeth of a pinion is more frequently stressed than the teeth of the mating gear during the course of service. While selecting the material, unhardened pinion and gear having the same hardness values should not be paired. In general, in such cases the material for the pinion should be so chosen that its strength is around 50 N/mm² higher than that of the material for the gear. In other words, since the ultimate tensile strength for carbon steels is about 0.36 times the Brinell Hardness, we can write:

$$HB_{Pinion} = HB_{Gear} + 150 \text{N/mm}^2$$

The above rule, however, is not applicable to gears of cast iron or to hardened gears. Guidelines for typical pinion/gear hardness combinations are given in Table 1.1. The pinion, having lesser number of teeth than the gear, is more prone to be used up as it has to do more work than gear during any specified time. It follows, therefore, that it is prudent to make the pinion harder than the gear. The difference in hardness serves to equalise the rate of wear. In the case of case-hardened steels, the depth of hardness of the case is of prime importance. A very rough rule is:

$$\text{Depth of the case} = \frac{1}{6} \times \text{root thickness of the tooth}$$

A relation regarding this aspect is given in Table 1.2. The maximum load carrying capacity is attained by the hardened gears. Such gears find application, among others, in heavy vehicles, aircrafts, machine tools. For big gears used in rolling mill drives and turbine drives ring gears made of heat-treatable for hardened steel and shrink-fitted on suitable hubs are quite common.

Table 1.1 Pinion and gear hardness combinations for heat-treated steel gears

	Hardness Range in HB (N/mm²)									
Pinion	2100 2500	2450 2850	2650 3050	2950 3350	3100 3500	3250 3650	3400 3800	3750 4150	3900 4300	4150 4550
Gear	1800 2200	2100 2500	2250 2650	2550 2950	2700 3100	2850 3250	3000 3400	3350 3750	3500 3900	3750 4150
	Hardness Range in HRC									
Pinion	55 60	58 63								
Gear	55 60	58 63								

Table 1.2 Relation between depth of case, case-hardening process and module, m

Case-hardening process	Depth of hardness (mm)
Case carburising	$0.25m$ for $m = 1.5$ to 4, $0.5\sqrt{m}$ for $m = 4$ to 30.
Induction and flame hardening	$0.3m$
Nitriding	0.1 to 0.6
Soft nitriding	0.0 15
Cyaniding	up to 0.4

Based on Maschinenelemente, Niemann, Vol. II, 1965 edition, p. 75. Springer Verlag, Heidelberg.

Guidelines for selection of materials, their properties, areas of probable uses, heat-treatment and other relevant data will now be discussed.

Steel

The different kinds of steels are the most commonly used gear material because of their versatility to meet a whole gamut of a variety of divergent specifications along with their easy availability and their ability to combine greater strength per unit volume coupled with low cost per kilogram. A wide variety of steels are in use, ranging from carbon steels to high-alloy steels. Again, carbon steels used also vary in carbon content — from low to high carbon. The ultimate choice, of course, will depend upon such factors as strength values, required size of gear, service conditions and other design criteria.

The single most important factor for designing a gear-set is the hardness of the component gears comprising the set. By changing from gears with low hardness values to full-hardened ones alone, it is possible to reduce the main dimensions of a power-transmitting gear-box by half, which means an eight-fold reduction in weight, resulting in a compact design and thus effecting considerable saving in money, material and space requirements. For example, a gear-set having normally hardened gears of Brinell Hardness Number (BHN of 2000N/mm² can be replaced by a set having full-hardened gears of BHN 6000 N/mm², both sets delivering the same power, but the second set weighing only one-eighth of the first one.

As far as the hardness is concerned, gear steels can be divided into two broad categories: those meant for surface-hardening and those for through-hardening. Surface hardening produces a hard case on the tooth surface, leaving the core comparatively soft. Surface hardening can be achieved by the common case-hardening processes, such as, case carburising, nitriding, induction and flame-hardening. We will discuss later in Ch. **2** that one of the most important factors for such calculation is the contact stress or Hertz stress. The power rating of a gearing depends mainly on what is known as the "surface durability". Surface durability, also called surface endurance or wear strength is a measure of the ability of the gear surface to resist fatigue type of tooth-surface failure known as "pitting" caused by contact stress. This aspect will be discussed in detail in Ch. 2.

Surface durability is a function of the compressive strength which in turn is almost directly proportional to hardness. The compressive stress developed between two contacting tooth surfaces is proportional to the square root of the tooth-load. It follows, therefore, that higher compressive strength greatly enhances the load capacity because the allowable load on the teeth varies as the square of the compressive strength. Hence, we can conclude that as the compressive strength is proportional to hardness and as greater surface durability is ensured by having higher compressive strength, the designer should strive for higher hardness. Certain limitations, however, impose restrictions on this rule-carbon content of the steel sets a practical top limit to the achievable hardness, material becomes brittle and notch sensitive at very high hardness levels, thereby rendering the teeth weaker in beam strength. The designer should, therefore, endeavour to strike a balance between these two aspects, i.e. greater hardness combined with adequate beam strength. (See also Fig. 1.17.)

In general, case-hardened gears can withstand higher loads than through hardened ones, but the through hardened gears are quieter in operation in normal cases, have high endurance limit and cost less. However, since through-hardened gears are vulnerable to distortion due to heat-treatment, they are not recommended for high speed drives. Moreover, unless grinding of gear teeth is practicable, through-hardened gears should not be used in applications where accuracy is of utmost importance. These gears have higher core strength because of higher carbon content, but are less ductile and less resistant to wear. Hardness normally varies from HRC **30** to 55. These gears are generally suitable for moderate strength and impact resistance.

As a post-treatment, hardened gear steels are sometimes tempered to permit machining of the teeth. Hardness is somewhat sacrificed thereby, but other properties are marginally altered.

Due to its comparative softness at the core, the case-hardened gears possess interior toughness. This in turn imparts shock or impact resisting capability to these gears. There are different case-hardening processes in practice which will now be briefly discussed.

Case *Carhurising* Case carburising is by far the most widely used process for hardening gear teeth surfaces. Heavy duty transmission gears are generally all case carburised. In these process,

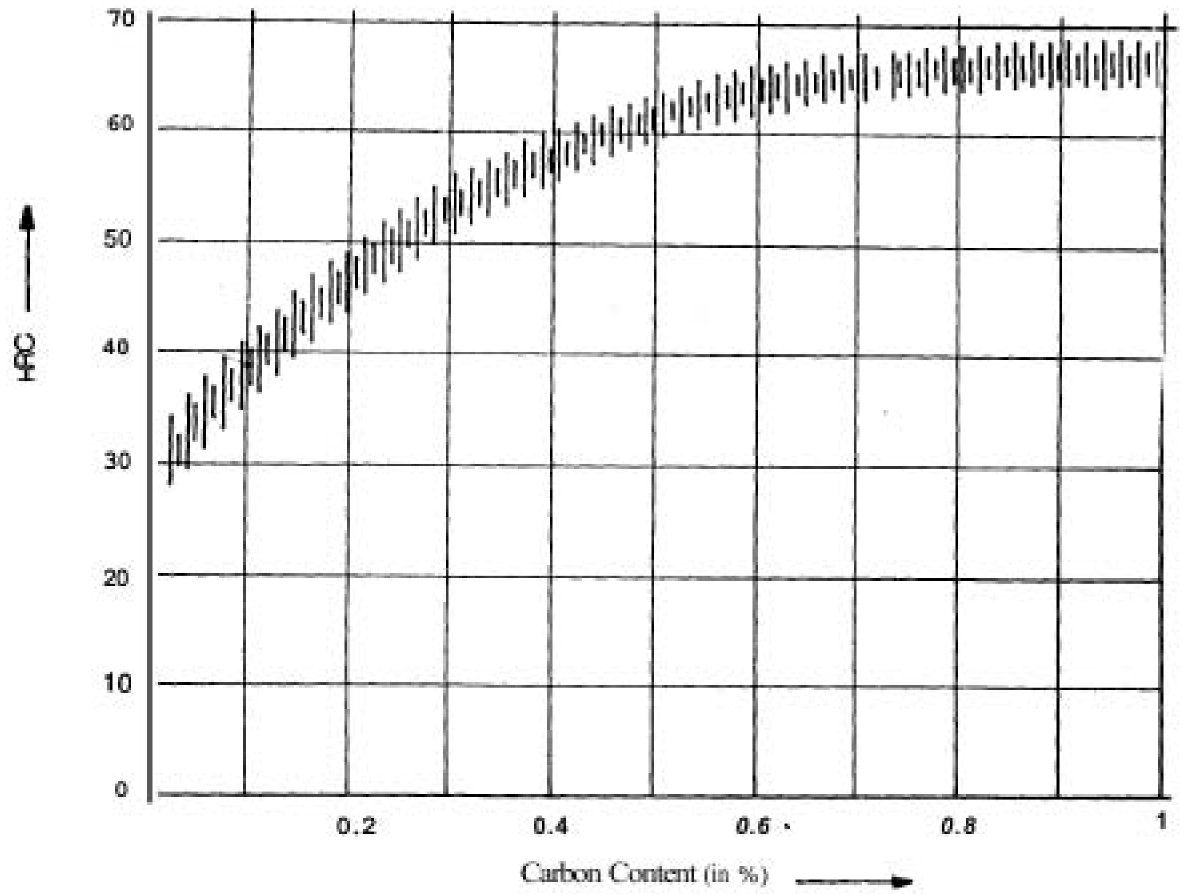


Fig. 1.17 Hardness of carbon steels after hardening in relation to carbon content

Based on MAAG Taschenbuch of M/s. MAAG Zahnraeder AG. Zurich, Switzerland, Fig. No. 13.01, page 649.

the common practice is to begin with a low carbon steel. Steel having a carbon content of 0.10 to 0.25% is heated in a medium which is enriched in carbon. The surface soaks up carbon and becomes hard after quenching. The carbon content of the carburised surface should be carefully controlled to obtain the appropriate case. Best results are achieved when it lies around 0.80 to 0.90%. A case which is too thin results in unsatisfactory tooth strength and wear resistance. On the other hand, an unnecessary deeper depth renders the tooth to be too brittle. Too much carbon in the outer case may lead to "spalling" which is a kind of gear tooth failure described later in Ch. 8. Again, too little carbon in the carburising medium will not develop the desirable results of hardness and wear resistivity. For best load carrying capacity, hardness of the case should be around HRC **55** to 60 and that of the core around 30 to **40**.

Nitriding The case properties of nitriding steels are practically the same as in the case of carburising steels, except that these steels are more suitable for high temperature services. In this process, the hardening agents are nitrides formed on the surface of gear teeth due to absorption of nitrogen when the gears are treated in a nitrogenous medium like ammonia gas. Like carburising, nitriding is a diffusion process, but it takes longer time to form a nitrided case. Nitrided gears can resist "scoring" and abrasion better than other types due to their high hardness which can go as high as HRC 65 to 70, though the case obtained may be rather shallow.

Induction Hardening By this process, special steels can develop hardness around HRC 60 but the process involves considerable technical skill. High frequency alternating current is used to heat up the gear surfaces locally. Hardness is obtained after subsequent quenching.

Flame Hardening This process is similar to induction hardening. Only, the heat required in this case is provided by an oxy-acetylene flame. Flame hardening can be used to harden the tooth surface layers as well as the whole tooth, if necessary.

Steel Castings for Gears When gears are to be made of cast steel, the steel used should conform to the requirements and chemical compositions as laid down in the relevant codes for steels meant for cast gears. Usually, such steel is made in open hearth or electric furnaces. Converter steel is not considered suitable. Special care should be taken to thoroughly normalise or anneal all steel castings meant for gears. The temperature and the time for heat-treatment should be properly chosen so that the characteristic grain structures of untreated castings are totally eliminated.

So far we have discussed carbon steels only. Among alloy steels, high manganese-nickel steels offer high resistance to heat, wear and bending stresses. Martensitic stainless steels are used for gears when requirements include high degree of resistance to corrosion combined with excellent mechanical properties. When a gear of large section needs hardening, alloy steels are sometimes recommended as the alloy content provides the necessary hardenability due to the fact that alloy steels are more amenable to proper heat-treatment procedures than carbon steels without detrimental result in case of gears with certain shapes and sections.

For industrial use in general, heat-treated plain carbon steels are far more common than alloy steels, the reason being that although the heat-treated alloy steels have superior properties, most of the design considerations and service conditions do not warrant their use because of the additional cost involved. Plain carbon steels are quite satisfactory and economical for most of the industrial applications. If, however, the advantages gained by using these steels over the plain carbon steels off-set the additional cost, then it is justified. When compared to the heat-treated plain carbon steels, the heat-treated alloy steels offer the following advantages.

1. Carbon content and quench being the same, alloy steels can produce greater surface hardness and depth.
2. Alloy steels have higher yield point, elongation and reduction of area. Toughness is, therefore, greater.
3. A lower quenching temperature may produce the same surface hardness. Hence, parts of alloy steels are less liable to distortion.
4. Grain size is finer, resistance to impact and to wear is more.
5. Better machinability at higher hardness than that offered by plain carbon steels.

Various alloying elements impart different properties on steels meant for gears. The characteristics of these elements are indicated below in brief. The properties of these alloy steels *vis-a-vis* those of the plain carbon steels are understood to be applicable in the cases where both the kinds of steels have the same carbon content. In other words, carbon percentage being the same, the properties are functions of the alloying elements only.

Nickel—Nickel increases hardness and strength, the reduction of ductility is marginal.

Chromium—Hardness and strength are increased more than those obtained by alloying with nickel, but the reduction in ductility is more.

Vanadium — It increases hardness, strength and toughness. Impact strength of the alloy is high but machining is difficult.

Manganese — Its effects are similar to vanadium.

Molybdenum — It increases strength but does not affect ductility. It imparts excellent impact properties to the alloy.

Chrome-Nickel—This combination produces properties of high strength, greater ductility and wear-resistance. However, machinability is not good and heat-treatment is difficult.

Chrome-Vanadium — This has the same tensile properties as Cr-Ni steels. Hardenability, impact strength and wear-resistance properties are better.

Chrome-Molybdenum — Properties are practically the same as plain molybdenum steel but the depth of hardness can be greater and wear-resistance is increased. Good machinability and heat-treatment are assured.

Nickel-Molybdenum — Properties of Ni-Mo steels are similar to Cr-Mo steels. Toughness is greater but machinability is not very good.

Cast Iron

Cast iron gears are cheap and have good damping capacity. When gears are to be of complicated shape, cast iron may well be the only choice in certain cases. The material is used in applications where strength is not the main criterion of material selection, as cast iron is relatively weak and brittle compared to steel. A large amount of graphite is present in cast iron parts and this acts as a lubricant, hindering wear of teeth. If increased load capacity and resistance to contact stresses are desired, then spheroidal graphite or pearlitic malleable cast iron may be used. In recent years, sintered iron gears made by the powder metallurgical processes from iron powder are much used in low cost machineries where strength requirements are small. These gears have wear-resistance properties and are easy to lubricate.

Non-Ferrous Metals

Among the various non-ferrous metals used in gear manufacturing, bronze is the most common alloy used. Various types of bronzes are used as gear materials, mainly because of their ability to withstand heavy sliding loads which are encountered in applications such as worm-gear sets. Also, like cast iron, bronzes are easy to cast into complicated shapes when necessary.

Phosphor bronze is generally recommended for worm-wheels which mesh with worms of high hardness and accuracy, and is normally meant for medium loads and medium to high speeds.

Tin bronzes offer strength, resilience and hardness, and can be used for worm-wheels for general purposes. Silicon bronzes have similar properties as phosphor bronzes. Besides, leaded bronze, manganese bronze, aluminium bronze and nickel bronze also find wide application as gear materials.

Best operational results from bronzes are obtained when the blanks are centrifugally cast.

Non-metals

Various types of non-metals have been widely used as gear materials since earliest times. These materials are chiefly selected because of their quietness during service at high speed, resilience, vibration damping ability and low cost in bulk manufacture. Non-metallic gears are also used as timing gears and also various other classes of gearing. Such materials used for gear making are generally reinforced phenolic moulding materials, moulded plastics like nylon, reinforced thermosetting laminates, raw hide, resin-bonded pressed materials, hard fabrics, etc. For rough calculation as to their strength properties and power-transmitting capabilities, these materials can be considered to have the same properties as those of cast-iron. However, for accurate calculation such data for each material are available (samples of such calculations shown in Ch. **2**.). Meanwhile, it may be pointed out that though the tensile strength of non-metallic gear materials may be less than cast iron, their resilience is high which enables them to withstand impact and abrasive wear better than cast iron.

Initially, raw hide was used as the non-metallic gear material. Later on, materials with improved properties came to be marketed under various trade names, such as, Formica, Textolite, Phenolite, Micarta, etc. These materials are chiefly made of layers of canvas with bakelite impregnation. These are then fused under high hydraulic pressure. **As** a consequence of pressure and heat, a dense rigid mass results. Impregnated canvas is more durable than cast iron or raw hide.

For gears made of non-metallic substances, phenolic laminated materials are most commonly used for their various advantageous properties. Such gears have entirely different characteristics as compared to metallic gears. Since the values of the moduli of elasticity of these materials are low, the resulting elastic deformation under load takes care of the usual detrimental effects of errors, such as tooth shape error, spacing error, etc. These errors, therefore, have little effect on the overall strength and performance of the gears, which is not so in case of metallic gears as we shall see in the section dealing with the strength calculations of usual gears.

Experimental results have established the fact that the best tooth form for these materials is the 20° stub-tooth system. It has also been found that the load-carrying capacity of a driving pinion made of these materials is considerably affected by the cutting action of the corner of its metallic mating gear as the pair come into mesh. The remedy lies in reducing the approach action which ensures a greater and safe load transmission.

Phenolic laminated materials are most suitable for high speed duty. Their performance is not satisfactory in general when the speed is low, starting torque high, load is of fluctuating nature and when really high shock loads are involved. Good results are obtained when the pitch line velocity is about **3** m/s or more.

As regards the materials for mating gear, hardened steel with Brinell Hardness Number over **4000** N/mm² leads to best operational performances and greater durability under load. Cast iron can also be used for the mating gear, but softer steel or non-ferrous metals (like brass and bronze) tend to produce excessive abrasive wear.

2
Spur Gears

2.1 The Basic Rack

The basic rack represents the normal section of a tooth in any gear-tooth system and determines the form or shape of tooth as well as the various relations of tooth form dimensions thereof, namely, the module, the whole depth of tooth, circular pitch and the fillet radius. Within the involute system, many variations of tooth forms are possible, viz. 20° full-depth tooth system, 20° stub-tooth system, and similarly for other pressure angles, such as $14\frac{1}{2}°$, 15°, 25°, 30°, etc. To standardise any gear-tooth system, it is only necessary to give the relevant proportions of the rack tooth belonging to that system, because of the fact that the rack is the basis or foundation of a standard system of interchangeable gears.

In IS: 2535-1978, the basic rack for involute cylindrical gears for general engineering purposes has been specified. The salient features of this tooth profile are represented in Fig. **2.1** which is based on the figure of the basic rack. Each side of the profile of the basic rack is a straight line which is a special case of the involute curve when the base circle diameter is infinite. Sometimes tip relief is provided in which case the end profile lines are slightly curved as shown in the figure. The pressure angle is **20°** and the height of the tooth (or whole depth) is 2.25 times the module m. Normally, the maximum value of the radius at the root of tooth r is **0.38** m, but in certain cases the maximum value may be exceeded up to 0.45 m. In any case, this radius must be kept as large as possible because the root fillet plays an important role in notch effect and stress concentration aspects as will be seen later in this book. All the tooth relations are given in the figure.

The profile reference line, MM cuts the basic rack in such a manner that on this line the following relation exists:

The nominal tooth thickness s = the nominal tooth space width e = half pitch $p/2$

That is
$$s = e = \frac{p}{2} \tag{2.1}$$

Here, p is the circular pitch in mm and m is the module in mm The circular pitch of a gear is one of the important specification criteria of a gear. It is defined as the length of the arc of the pitch circle between two adjacent teeth, and is given by

$$p = s + e = \frac{\pi d}{z} = \frac{\pi m z}{z} = \pi m$$

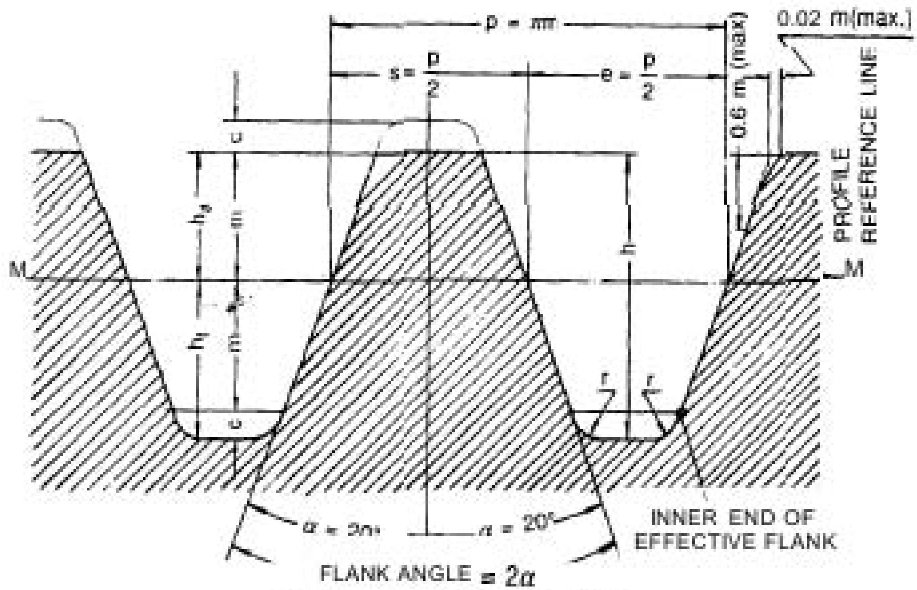


Fig. 2.1 **Basic rack** geometry

(2.2)

where z = Number of teeth of gear, and d = Pitch circle diameter = mz

Similarly, the base pitch of a gear is analogous to the circular pitch, except that it is measured on the base circle, and is given by

$$p_b = \frac{\text{Circumference of the base circle}}{z} = \frac{\pi d \cos a}{z} = \frac{\pi m z \cos a}{z}$$

$$= \pi m \cos a = p \cos a$$

$$\therefore \quad p_b = p \cos a \tag{2.3}$$

The fundamental dimensions bear the following relations

Addendum = $h_a = m$, Dedendum = $h_f = m + c$, where c = Top clearance = 0.25 m, as per IS: 2535
$\therefore h_f = m + 0.25\,m = 1.25$ m. Whole depth of tooth

$$= h = h_a + h_f = (1 + 1.25)\,m = 2.25\,m \tag{2.4}$$

2.2 Basic Nomenclatures and Gear Tooth Terminology

The basic terms associated with gears and gearing systems are explained in Figs 2.2 and 2.3 and defined below. In Fig. 2.3 parameters relating to helical gears have also been included for comparison. These will be taken up again in Chap. 3. However, in this section we will discuss the terms relating to spur gears.

Pitch circle: This is the circumference of an imaginary cylinder which rolls without slipping when in contact with another such cylinder as in friction drive. The two rolling cylinders are

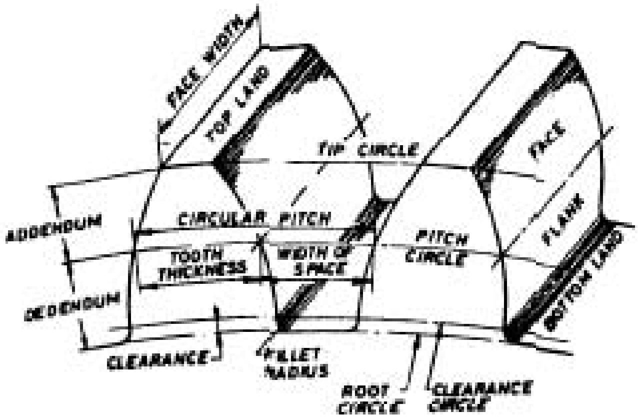


Fig. 2.2 Gear tooth parameters

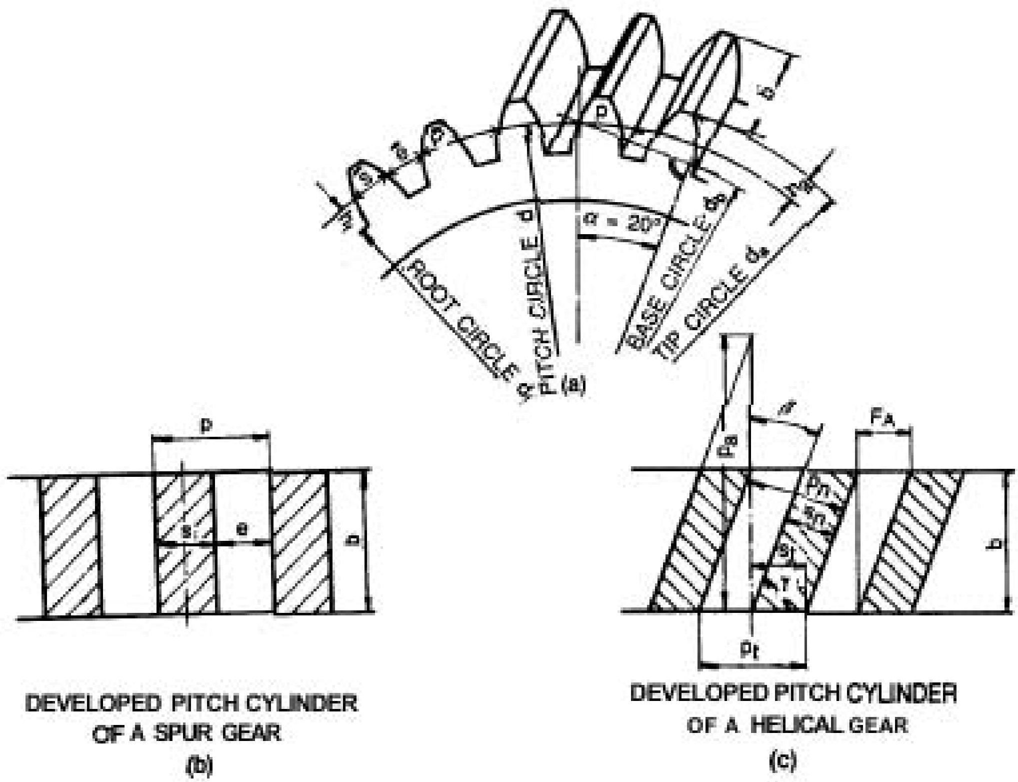


Fig. 2.3 Spur and helical gear parameters

called pitch cylinders. According to the law of gearing explained in Chap. 1 the angular velocity ratio must remain unchanged. Since this is not practicable in friction drive, the cylinders are replaced by toothed wheels called gears. The pitch circles of two mating gears are the same as the circumferences or the end projections of the two rolling pitch cylinders having the same angular velocity ratio. In any gear, the relevant pitch circle is the reference circle of that gear and though imaginary, it is the basis of measurement of other parameters of the gear. The diameter of this circle is called the pitch circle diameter (pcd for short) or simply pitch diameter. It is denoted by the letter "*d*" with proper subscripts, e.g. d_1 for pinion and d_2 for the mating gear.

Tip circle: This is also known as the addendum circle or the outside circle. This is a circle which bounds the outer edges of the teeth of a gear and its diameter is denoted by d_a.

Root circle: Also known as the dedendum circle, it is the circle which bounds the bottoms or the roots of the teeth and its diameter is denoted by d_f .

Base circle: This is the circle from which the involute tooth profile is developed. Its diameter is denoted by d_b.

Addendum: It is the radial distance between the pitch circle and the tip circle and is denoted by h_a.

Dedendum: It is the radial distance between the pitch circle and the root circle, and is denoted by h_f.

Land: The top land and the bottom land are the surfaces at the top of the tooth and the bottom of the tooth space respectively.

Working depth: This is the distance of engagement of two mating teeth and is equal to the sum of the addendums of the mating teeth of the two gears in case of standard system.

Whole depth: This is the height of a tooth and is equal to the addendum plus dedendum.

Clearance: This is the radial distance between the top land of a tooth and the bottom land of the mating tooth space.

Face width: This is the width of the gear and is the distance from one end of a tooth to the other end.

Face of tooth: This is the surface of the tooth between the pitch cylinder and the outside cylinder.

Flank of tooth: This is the surface of the tooth between the pitch cylinder and the root cylinder.

Module: It is defined as the ratio of the pitch diameter to the number of teeth of a gear. The value of module is expressed in millimeters. The module is one of the major and determining parameters of a gear.

Pinion: It is smaller of the two gears in mesh and is usually the driving component of a gearing system. The other component, which is larger, is usually referred to as "gear".

Chordal addendum: This is the height bounded by the top of the tooth and the chord corresponding to the arc of the pitch circle representing the circular tooth thickness.

Chordal tooth thickness: This is the chord referred to above. Both chordal addendum (or chordal height, as it is sometimes called) as well as the chordal tooth thickness are of importance in checking of gears as will be explained in Sec. 2.28.

Diametral pitch: This is a term used in gear technology in the **FPS** system. It is defined as the ratio of the number of teeth to the pitch diameter in inch. It is usually denoted as "DP". It is equal to the number of gear teeth per inch of pitch diameter. The unit of DP is the inverse of inch. The following relation exists between DP and module

$$DP \text{ (inch}^{-1}\text{)} = \frac{25.4}{\text{Module (mm)}}$$

Approximate values of DP versus module are given in Appendix D for comparison.

Besides the above terms, there are many other terms associated with gears and gearing systems. These terms will be defined and discussed in the relevant sections in the text.

2.3 Relations between Gear Parameters

The basic terms which have been defined in Sec. 2.2 have the values as given in Table 2.1 in a normal, standard gearing system. In a normal, standard gear, the profile reference line *M-M*

Table 2.1 Dimensions for standard gearing

Description	*Pinion*	Gear
Number of teeth	z_1	z_2
Pitch circle diameter	$d_1 = z_1 m$	$d_2 = z_2 m$
Tip circle diameter	$d_{a1} = d_1 + 2m$	$d_{a2} = d_2 + 2m$
Root circle diameter	$d_{f1} = d_1 - 2 \times 1.25\, m$	$d_{f2} = d_2 - 2 \times 1.25\, m$
Base circle diameter	$d_{b1} = d_1 \cos a$	$d_{b2} = d_2 \cos a$
Tooth thickness on pitch circle	$s = e = \frac{p}{2} = \frac{\pi m}{2}$	
Centre distance	$a = \frac{d_1 + d_2}{2} = m \frac{z_1 + z_2}{2}$	

of the basic rack is tangent to the pitch circle of the gear at the pitch point. When two such gears mesh, we have a standard gear set, as distinct from the ('corrected' gearing which will be taken up later. The principal parameters of an uncorrected gear set are summarised in Table 2.1.

In Fig. **2.4** two standard gears have been shown in mesh. The terms described below are defined in relation to two gears in mating condition.

Arc of action: This is the arc on the pitch circle through which a tooth travels from the beginning of contact with the mating gear tooth to the point where the contact ends. Since the two pitch cylinders are in rolling contact (without slippage as per the theory of gearing), the lengths of the arcs of action of the two gears are the same. That is

$$\text{Arc of action} = r_1\theta_1 = r_2\theta_2 \tag{2.5}$$

where r_1 and r_2 are the pitch circle radii, and θ_1 and θ_2 are the angles (in radians) subtended by the two arcs at their respective centres, The arc of action, in each case, is divided into:

Arc of approach, which *is* the arc through which the tooth moves from the initial contact up to the pitch point ***P***, and

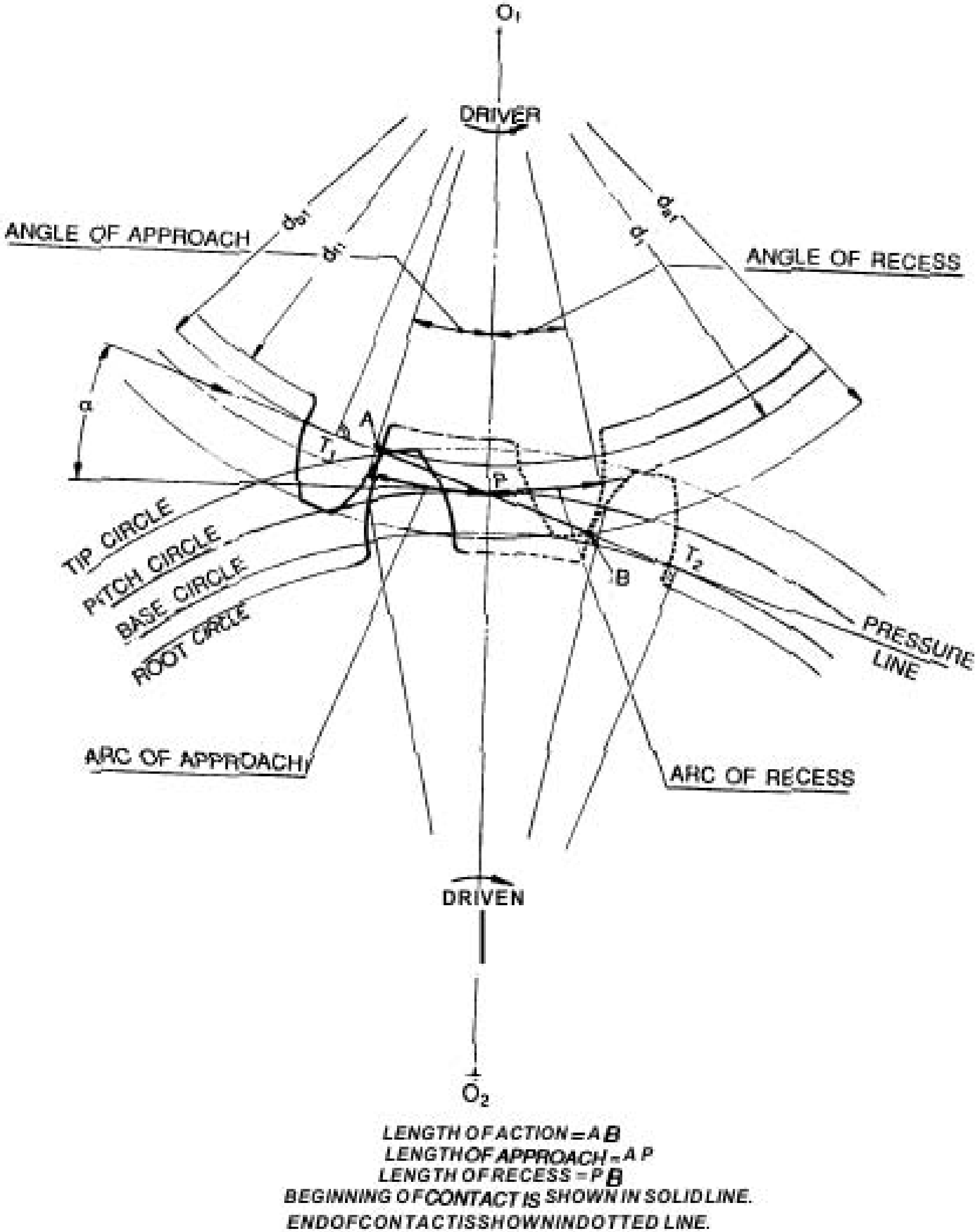


Fig. 2.4 Characteristics of tooth action

Arc of recess: which is the arc through which the tooth travels from the pitch point to the end of contact.
The arcs of approach and recess have been illustrated in Fig. **2.4.**

Angle of approach and recess: These are the angles subtended at the centre by the arc of approach and the arc of recess respectively.

It should be noted here that though the lengths of arcs are the same for both the gears, the corresponding angles are different for the two gears. This is apparent from **Eq.** 2.5 as r_1 and r_2 have different values.

Points of tangency: T_1 and T_2 are the points of tangency where the common tangents of the two base circles through the pitch points P meets the base circles.

Line of action: This is the line along which the point of contact of the two mating tooth profiles moves. This is also known as the path of contact and is the same common tangent referred to above.

Length of action: That portion of the line of action on which the point of contact moves during the course of action is known as the length of action. This length is bounded by the points A and B on the line of action, as shown in Fig. 2.4. The length of action AB, which in effect represents the beginning and the end of contact of the two mating gear teeth, is sub-divided into length of approach AP, and the length of recess PB, as shown in the figure.

Pressure Angle

If a tangent is drawn to the involute profile of a tooth at any point on the curve and if a radial line is drawn through this point of tangency, connecting this point with the centre of the gear, then the acute angle included between this tangent and the radial is defined as the pressure angle at that point.

Referring to Fig. 2.5(a), A is the point at which the tangent has been drawn and α_A is the pressure angle for point **A.** Referring to Fig. 2.5 (b), P is the pitch point where the two standard pitch circles meet on the line of centres, O_1O_2. If a common tangent to the two tooth-profiles is drawn through P, then the pressure angle for point P is obviously the angle subtended by this tangent to the line of centres at P. The segments O_1P and O_2P of the line of centres also happen to be the radials for the point P. This pressure angle (at the pitch point P) is one of the most important specification factors of a gear. This pressure angle is denoted commonly by the Greek letter α. A pressure angle at any other point is designated by a subscript, e.g. α_A for the point **A** and so on. The symbol α for the pressure angle at the pitch point carries no subscript. The value of the pressure angle α for the standard tooth as per the basic rack, IS: 2535, is $20°$, but the pressure angle can have other values also, viz. $14\frac{1}{2}°$, $15°$, $25°$, $30°$, etc. depending on the tooth standard, We will see in sections dealing with "corrected" gears that the working pressure angle α_w can be quite different from the standard pressure angle, depending upon the correction factors involved and the mounting dimensions. The cutters which generates the gears, however, are called "20° — pressure angle cutters", irrespective of whether they produce standard or corrected gears. Pressure angle of the cutter is also referred to as the "built-in" pressure angle.

Since the involute is generated from the base circle, the pressure angle at the starting point is zero. Referring to Fig. 2.4, a tangent drawn on the base circle and passing through the pitch point P also subtends an angle equal to α at the centre. That is

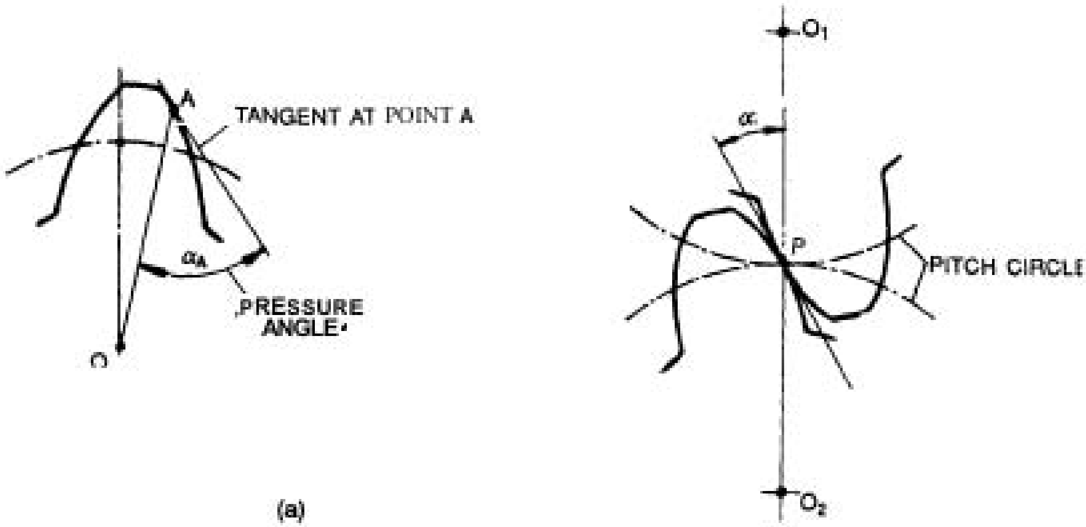


Fig. 2.5 Pressure angle

$$\text{angle } T_1O_1P = T_2O_2P = \alpha$$

Hence, we get the relation

$$d_b = d \cos \alpha \tag{2.6}$$

The relation between the pressure angle of a point on the involute tooth profile and the pressure angle of some other point on the same involute will be given later in this section.

From Fig. 2.4 it can also be seen that the common tangent line T_1T_2 to the base circles also makes an angle α to the perpendicular to the line of centres through P. Line T_1T_2, which is variously known as the pressure line, line of action or path of action,,is the line along which the contact of mating teeth takes place at various points during the course of action. This is also the direction in which the driving force acts.

In automotive industries a 25° pressure angle is quite common. For involute splines, this angle is usually 30°. With increasing pressure angle, the load carrying capacity becomes more. Pressure angles in case of helical gears will be discussed in Chap. 3.

The effects of increasing the pressure angle are summarized as below:

1. The limiting number of teeth to avoid undercutting is lowered. That is to say if the pressure angle is increased, pinions with comparatively lesser number of teeth can be generated without undercutting (see section on interference and undercutting).
2. The shape of the tooth becomes more pointed or peaked.
3. Tooth flank becomes more curved.
4. The relative sliding velocity is reduced.
5. The contact ratio and overlap are reduced.
6. The tooth pressure and axial pressure increase.
7. Tooth load-carrying capacity increases.

The Base Circle and Root Circle

The module and the pressure angle remaining same, the mutual position of the base circle and the root circle will depend upon the number of teeth for any particular basic rack. It is wrong to presume that the root circle is the smallest circle in a gear. Taking a standard basic rack

Base circle diameter, $d_b = d \cos \alpha = m z \cos \alpha$

Root circle diameter, $d_f = d - 2 \times 1.25m = m z - 2.5\, m = m\,(z - 2.5)$

If $d_b = d_f$, then $m z \cos \alpha = m\,(z - 2.5)$ or $z \cos \alpha = z - 2.5$ or $z(1 - \cos \alpha) = 2.5$

$$\therefore\ z = \frac{2.5}{1 - \cos \alpha} = \frac{2.5}{1 - \cos 20^\circ} = \frac{2.5}{1 - 0.93969} = 41 \text{ teeth}$$

This is a borderline case. If the number of teeth exceeds 41, the root circle becomes greater than the base circle.

For example, if $z = 50$, $m = 2$, $\alpha = 20^\circ$, then

$$d_b = m z \cos \alpha = 2 \times 50 \times 0.93969 = 93.969 \text{ mm}$$

$$d_f = m\,(z - 2.5) = 2(50 - 2.5) = 2 \times 47.5 = 95.000 \text{ mm}$$

Hence, in this case, root circle is greater than base circle. Therefore, theoretically the involute has already started before the dedendum circle or root circle. However, in actual practice, fillets with suitable radii are provided at the roots of the teeth to nullify the detrimental effects of stress concentration and notch effect, irrespective of whether the base circle or the root circle is the bigger of the two.

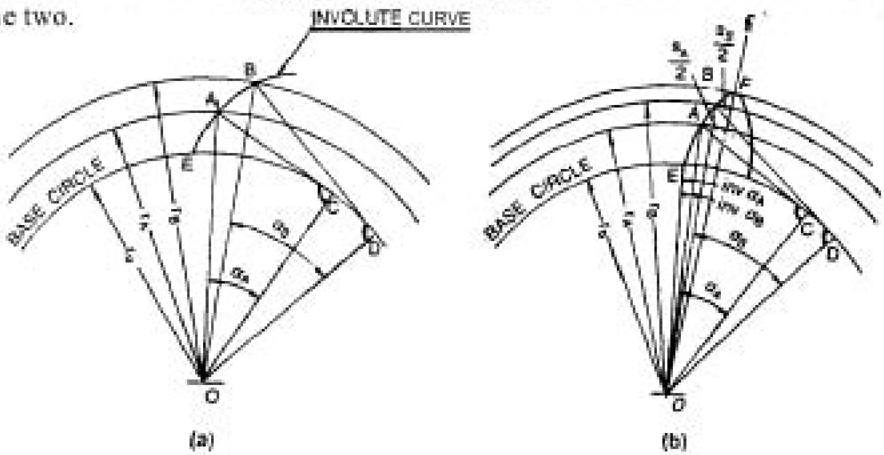


Fig. 2.6 Involute curve and gear tooth thickness

Tooth Thickness

Referring to Fig. 2.6 (a), an involute curve has been generated from the base circle of radius r_b. A and B are two points on this involute at distances of r_A and r_B respectively from the base circle centre O. From geometry, we get the following relations

$$r_b = r_A \cos \alpha_A = r_B \cos \alpha_B$$

whence we get the important equation

$$\cos \alpha_B = \frac{r_A}{r_B} \cos \alpha_A \tag{2.7}$$

From Eq. 2.7 we can determine the pressure angle at any point on the involute in relation to the parameters of another known point. For example, if the circle passing through A happens to be the pitch circle the diameter of which is known, then for a standard system

$$\cos \alpha_B = \frac{d/2}{r_B} \cos 20° = \frac{r}{r_B} \cos 20°$$

Figure 2.6 (b) shows a gear tooth enclosed between the tip circle and the base circle. From the properties of involute discussed in earlier section, we know that

arc DE = straight length DB

arc CE = straight length CA

Also, angle EOD (in radian) $= \dfrac{\text{arc DE}}{r_b} = \dfrac{\text{DB}}{r_b} = \tan \alpha_B$

Similarly, angle EOC = $\tan \alpha_A$.

Now, angle EOB = angle EOD $- \alpha_B$. Or, angle EOB = $\tan \alpha_B - \alpha_B$

Similarly, angle EOA = angle EOC $- \alpha_A = \tan \alpha_A - \alpha_A$

From Ch. 1, we know that the expression: $\tan \alpha - \alpha$ has been termed as: inv α. Therefore,

angle EOB = inv $\alpha_B = \tan \alpha_B - \alpha_B$

and angle EOA = inv $\alpha_A = \tan \alpha_A - \alpha_A$

Values of involute function for different angles are given in the Appendix. We shall now see how these relations and values can be used to find expressions for tooth thicknesses at different positions along the tooth profile.

Referring to Fig. 2.6 (b), we can easily see that

angle EOF = angle EOB + angle BOF

The centre line through point F divides the tooth in two equal and symmetrical halves. Therefore, the circular tooth thickness, s_B, at the circle passing through B is given by

$\dfrac{s_B}{2} = r_B \times$ angle BOF (in radians), $\quad\therefore$ angle EOF = inv $\alpha_B + \dfrac{s_B}{2r_B}$

Similarly, angle EOF = inv $\alpha_A + \dfrac{s_A}{2r_A}$

We can now establish the following relation after equating

$$\frac{s_B}{2r_B} = \frac{s_A}{2r_A} + \text{inv } \alpha_A - \text{inv } \alpha_B$$

or

$$s_B = 2r_B \left[\frac{s_A}{2r_A} + \text{inv } \alpha_A - \text{inv } \alpha_B \right] \tag{2.8}$$

Equation 2.8 is important for gear calculations because it enables the designer to calculate the tooth thickness at any cylinder if the parameters of a particular cylinder are known.

For example, it is often necessary, especially in case of positively corrected gears, to calculate and check the tooth thickness at the top land. For an uncorrected, standard tooth, the tooth thickness at the pitch circle is given by:

$$s = e = \frac{p}{2} = \frac{\pi m}{2}$$

The top land thickness s , or s_a is given by

$$s_a = 2r_a\left[\frac{s}{2r} + \text{inv}\,\alpha - \text{inv}\,\alpha_a\right]$$

where r and r_a, are the radius at the pitch circle and tip circle respectively, and α_a is the pressure angle at the tip circle. The value of α_a can be ascertained from **Eq.** 2.7.

$$s = d\left|\frac{\pi m/2}{} + \text{inv}\,20° - \text{inv}\,\alpha\right| \qquad (2.9)$$

Sometimes, for positively corrected gears, it may be necessary to calculate the diameter at which the tip becomes pointed or "peaked". Obviously, at this point $s_a = 0$. Incidentally, recalling Eq. 2.7, we have

$$\cos\alpha_b = \frac{r\cos\alpha}{r_b} = \frac{r_b}{r_b} \text{ (as per Eq. 2.6)} = 1 \quad \therefore \quad \alpha_b = 0°$$

Hence, the pressure angle at the base circle is 0°, as mentioned earlier.

The tooth thickness at the pitch circle of a positively corrected gear is not equal to $p/2$, but somewhat greater, depending upon the correction factor chosen. An expression for this tooth thickness will be given in Sec. **2.12.**

2.4 Types of Gear Tooth

Besides the 20° full-depth involute tooth discussed in Sec. 2.1 which shows the basic rack as per IS: 2535, there are other types of tooth forms which are commonly used. These will be described now. It is to be noted that while establishing any gear tooth standard, only the proportions of the relevant rack tooth need be given because the rack is the basis or the foundation of any type of standard tooth system of interchangeable gears. To ensure interchangeability, the mating gears must be produced by cutters having the same pressure angle. This particular pressure angle can be termed as the "built-in" pressure angle, as mentioned in earlier sections. In sections dealing with "corrected" gears, we will be using the term "working" pressure angle which is quite different from the built-in pressure angle. Besides pressure angle, the interchangeable gears must also be of the same module (or diametral pitch) and of the same magnitude of pressure angle and helix angle at the pitch circles in case of helical gears mounted on parallel shafts. The following different types of tooth proportions are normally used.

1. Full-depth, **20°** involute system: This is by far the most widely used tooth system and is fully discussed in Sec. 2.1. Due to the increase of pressure angle to 20°, such teeth alleviate the interference and undercutting problems, and are also of broader and stronger root section as compared to the systems having smaller pressure angles.

2. Full-depth, $14\frac{1}{2}°$ involute system: This was one of the earliest systems used, and was preferred because the sine of $14\frac{1}{2}°$ is about $\frac{1}{4}$. This somewhat simplified the machine-setting problems in early gear-cutting machines. Here the pressure angle is $14\frac{1}{2}°$, the whole depth is 2.157 times the module and the working depth is twice the module which characterises it as a full-depth system as in the case of 20° full-depth system. Other proportions are: addendum = 1x module, dedendum = 1.157x module and clearance = 0.157x module. This system is quite satisfactory so long as the number of teeth of the gear is large enough to avoid undercutting.

3. Full-depth, 15° involute system: Here the pressure angle is 15°. This system is much used in Continental Europe. In both $14\frac{1}{2}°$ and 15° systems, the undercutting begins when the number of gear teeth is below 32.

4. Stub-tooth system: This class of tooth systems reduces the interference problem by having shorter addendum and large pressure angle, usually 20°. For standard stub-tooth system, the tooth proportions are: whole depth = 1.8 x module, addendum = 0.8 x module, dedendum = 1.0 × module, working depth = 1.6x module, clearance = 0.2 x module. One defect of the stub-tooth system is its lesser contact ratio as compared to the full-depth system, which results in adverse effect on the wear of teeth and also leads to greater noise in running gears unless this aspect is specially taken care of by accurate machining and mounting. The running of gears is not generally smooth. The system gives better results when the pinion has less than 25 teeth.

In automotive transmissions, the 20° stub-tooth finds wide applications because it affords the possibility to have relatively small gears and the maximum power-transmitting capacity for a given module or material. In such application, of course, extremely accurate gears and mountings are imperative. Helical stub-teeth are also used in such cases for smoother operation and lesser noise.

It will be seen in Sec. 2.9 on interference and undercutting that the possibility of interference sets a limit to the value of the addendum. It follows, therefore, that if the interferring portion of the tooth is cut off, it cannot dig into the mating flank. This is the principle on which the stub-tooth system is based. It is to be noted, however, that this system does not eliminate interference entirely.

The advantages of the stub-teeth are:

1. Greater strength because there is more material at the root as compared to a normal, 20° pressure angle, full-depth tooth; also, because of shorter moment arm, the bending moment is less;
2. Reduced chances of interference, as indicated before;
3. Quicker production as the material to be removed during cutting is comparatively of smaller quantity, and
4. Sliding is reduced and therefore wear caused by sliding only is also reduced.

Other stub-tooth systems include Fellows stub-tooth and Nuttall stub-tooth systems.

***Fellows** Stub-tooth:* This system was introduced by the Fellows Gear Shaper Co. of the **USA.** The peculiarity of this system is that the different basic dimensions of the tooth are derived by using two diametral pitches. One of them is used to obtain the dimensions of the addendum and dedendum while the other is used for determining pitch circle and tooth thickness values.

***The Nuttall** Stub-tooth:* In this system, developed by the R.D. Nuttall Co. of **USA,** the tooth dimensions are based directly on the circular pitch. In both Fellows and Nuttall Stub-tooth systems, the pressure angle is 20°.

Besides the above tooth systems, in some applications, 25° pressure angle full-depth teeth are used, specially in automotive industries, to impart greater strength to the teeth. **In USA,** there is a fine-pitch system which is similar to the **20°** full-depth, American basic rack system, except that for gears with diametral pitches (DP) of **20** and finer, a slight increase in whole depth is provided to allow for the greater proportional clearance. In involute spline systems, pressure angle of **30°** and **45°** are also used besides the usual pressure angles. Involute spline with **30°** pressure angle is most common.

Different types of gear tooth forms, including cycloidal tooth form, have been illustrated in **Fig. 2.7** for comparison purposes.

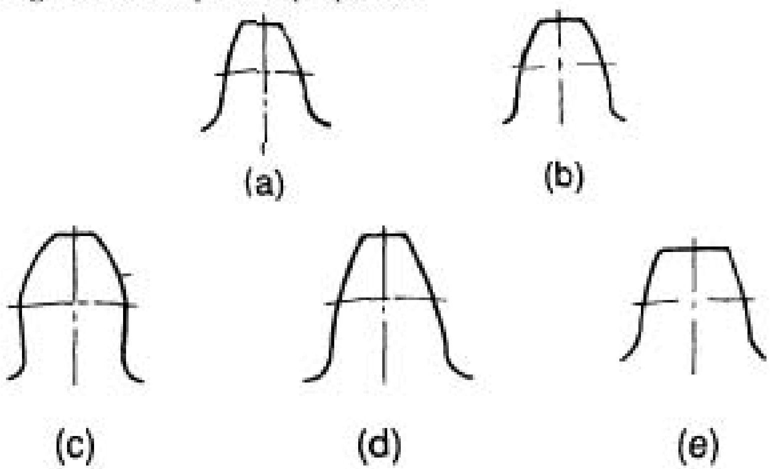


Fig. 2.7 Comparison of gear tooth profiles
(a) Cycloidal, (b) Involute, (c) $14\frac{1}{2}°$ full-depth involute,
(d) 20° full-depth involute, (e) 20° stub involute

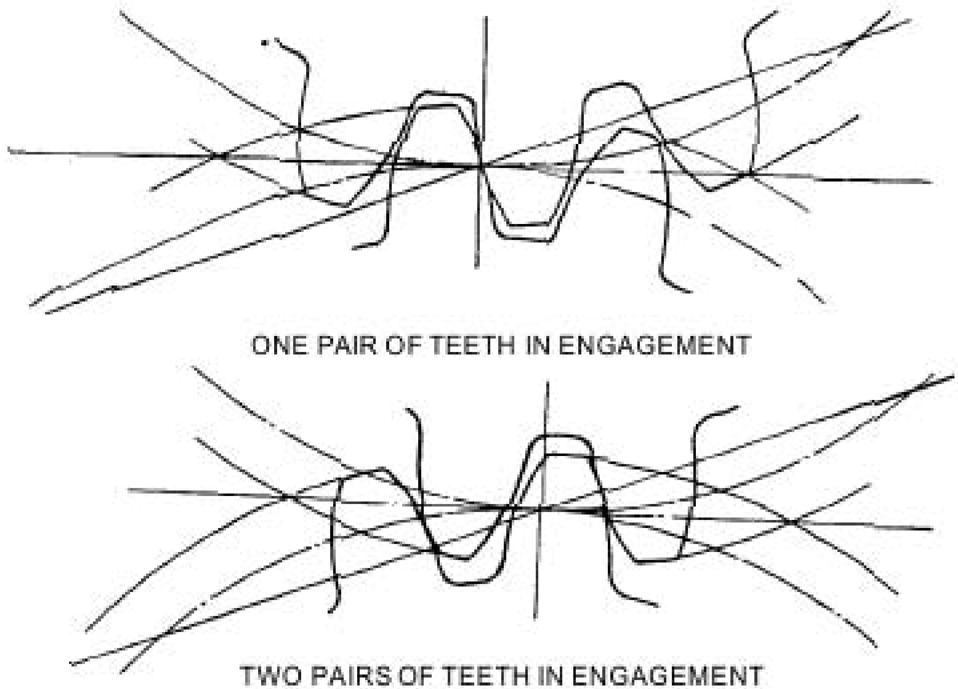


Fig. 2.8 Different stages of gear teeth engagement during meshing

2.5 Nature of Tooth Engagement in Spur Gear Drive

In a pair of meshing spur gears, the line of contact along the width of the gears is parallel to the gear axes and shifts its position along the tooth profile curve from top to bottom region of tooth height or vice versa as the engagement proceeds during the course of action.

In Sec. 2.7, the significance of the contact ratio has been explained. The value of the contact ratio can be taken as a measure of the number of the pairs of teeth in mesh during the course of action. Depending upon the mesh position at a particular moment, there can be one-pair or two-pair engagement as shown in Fig. 2.8. Since the contact ratio is normally greater than 1, two pairs of teeth share the load part of the time. This is elaborated below.

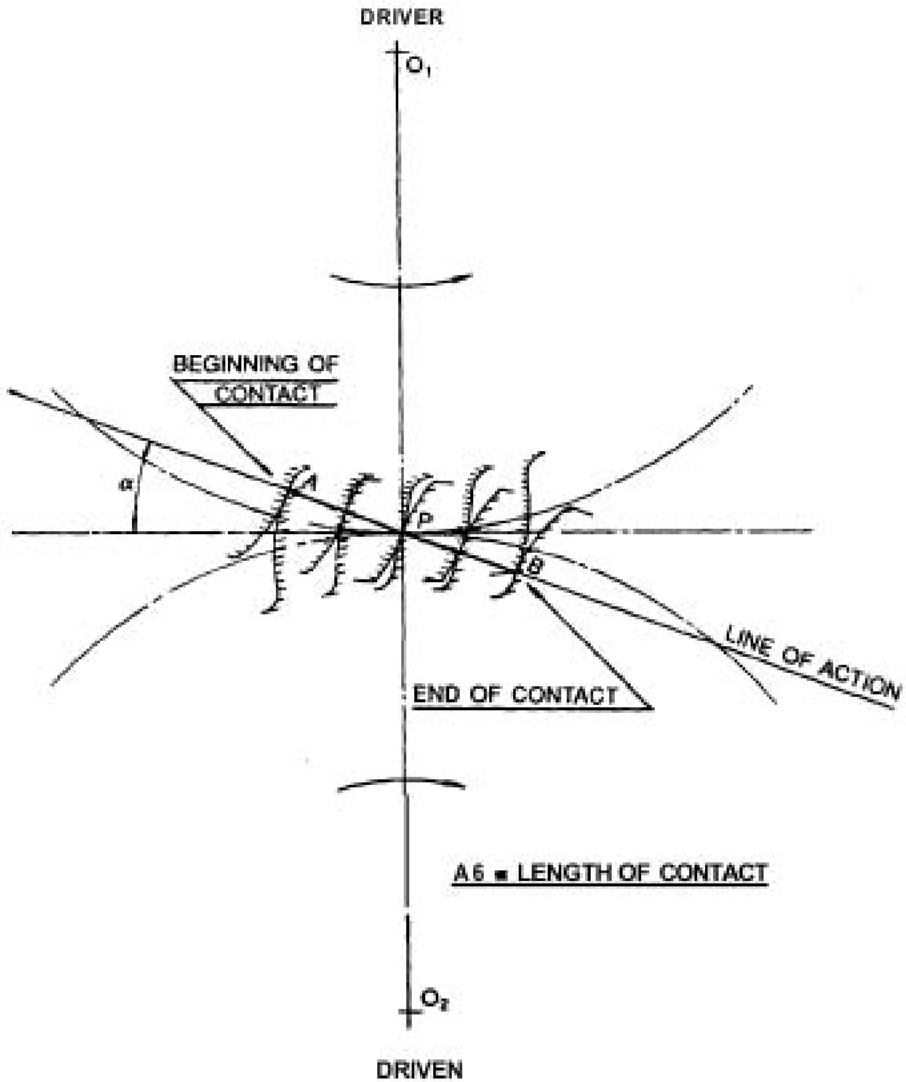


Fig. 2.9 Course of tooth contact

Referring to Fig. 2.9, the tooth contact begins at A and terminates at B during the course of tooth action. In Fig. 2.10 (a) the tooth z_2 of the driving pinion comes in contact with the top of the tooth z_2' of the driven gear, as the course of action begins. The previous pair $z_1 z_1'$ is already in mesh so that the load F_N is shared by these two pairs. This condition continues for a short time till the pair $z_1 z_1'$ goes out of mesh as shown in Fig. 2.10 (b). From this point onwards, the pair $z_2 z_2'$ takes the full load and continues to do so till a new pair comes in mating position. Thereafter, the load is again shared by the pair $z_2 z_2'$ and the new pair for a short while till $z_2 z_2'$ goes out of mesh. A pictorial view of two pairs of teeth in mesh has been shown in Fig. 2.10 (c). In all these figures the load zones have been demarcated by extra thick lines. In Fig. 2.10 (d), the sequence of tooth contact has been illustrated on the tooth profile during the course of action.

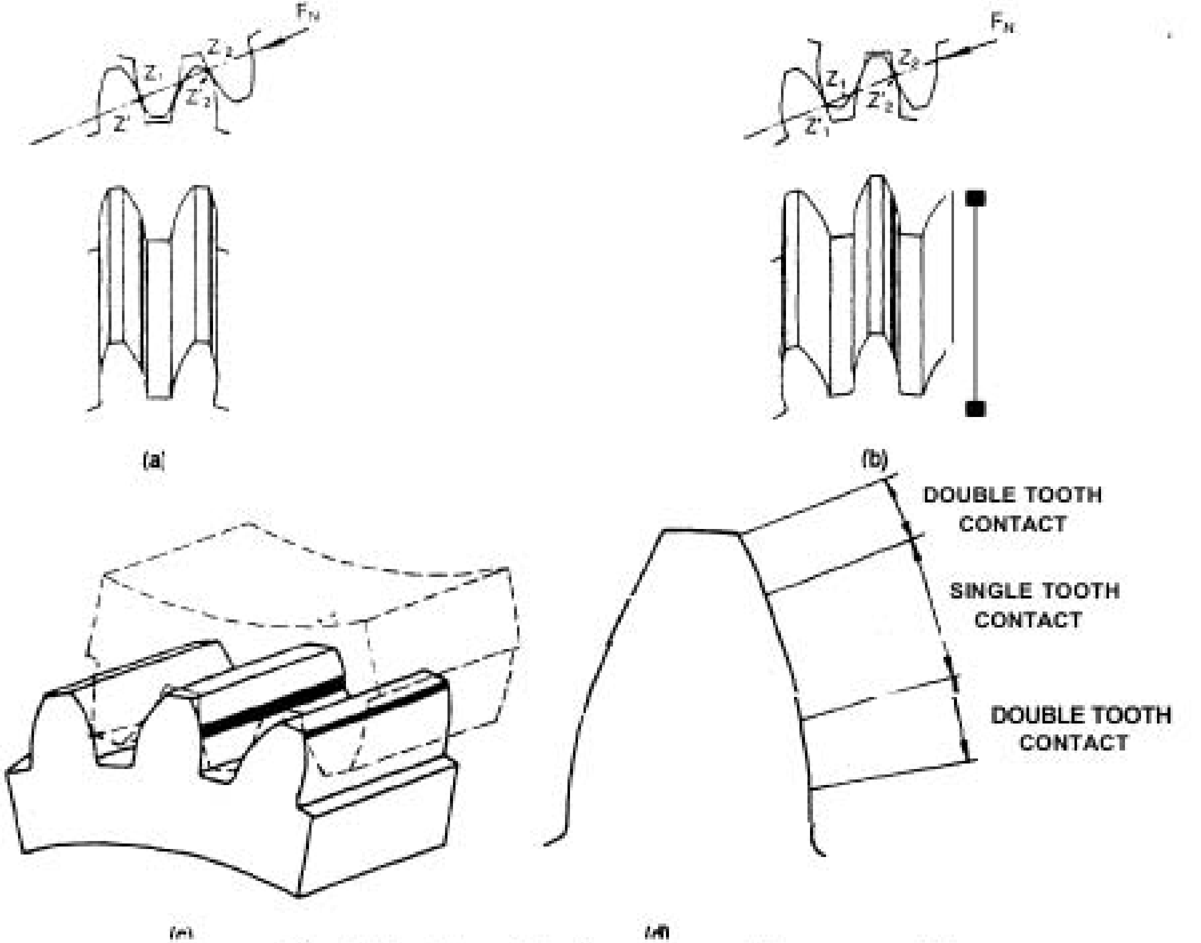


Fig. 2.10 Nature of tooth engagement in spur gear drive

In Fig. 2.11, the progression of tooth contact and load distribution has been diagramatically represented to clarify further what has been discussed so far. At point A, the contact begins. Each of the pairs $z_2 z_2'$ and $z_1 z_1'$ carries a load of $F_N/2$. At point G, the whole load is on $z_2 z_2'$ as $z_1 z_1'$ goes out of mesh at point B. This condition has been shown by thin, dotted contours. This single pair bears the load till point H is reached. Double-pair engagement begins from here onwards till $z_2 z_2'$ goes out of mesh, this time the two pairs being $z_2 z_2'$ and the new, in-coming pair.

In Fig. 2.11, P is the pitch point, $T_1 T_2$ is the line of action, AB is the length of contact as before and P_b is the base pitch. (For contact ratios CR,, CR, and CR, see Sec. 2.7.)

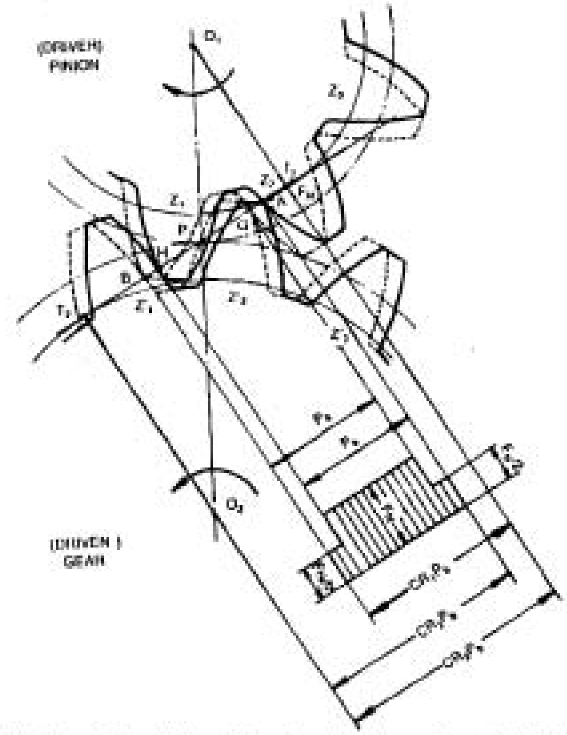


Fig 2.11 Distribution of tooth load during single-pair and double-pair engagement

2.6 Sliding Phenomenon of Gear Teeth

In Sec. 1.3 we have seen that when the profiles of two meshing teeth contact at the pitch point P, the motion is one of pure rolling without slippage. As the contact point moves up or down the line of contact (action), the motion is a combination of rolling and sliding. Farther the contact point goes away from the pitch point, higher is the rate of sliding. Thus, in Fig. 2.12 if the length of contact (along the path of contact) is AB, the maximum values of the relative sliding velocities are at A and B which are the initial and final points of contact respectively.

In mechanisms which are positively driven by direct contact, such as a pair of gears, cam and follower, it is often required to find the amount of this sliding velocity. This is a parameter which has got a direct bearing on the amount of abrasion-wear which may ensue and also on the type of proper lubricant to be selected, among other aspects. Besides purely kinematic reasons, a knowledge of sliding velocity is necessary because researchers of gear technology have found that in case of very high-speed gears, the product of Hertz (contact) stress at the area of tooth contact and the maximum sliding velocity is a very useful design criterion for such gears and is a limiting factor of the power-transmitting capacity of the gear teeth.

By means of the various velocity-analysis methods, one can find the relative sliding velocity at any point of contact during the course of tooth engagement, but the following method is by far the simplest one:

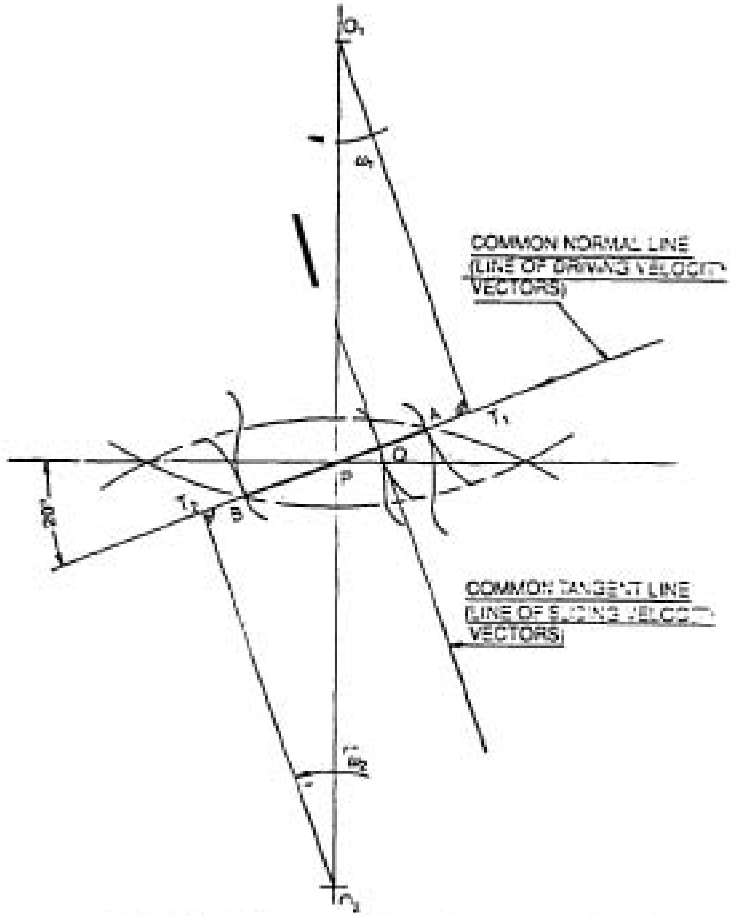


Fig. 2.12 Sliding velocities of two **gears in mesh**

In Fig. 2.12 let Q be the momentary point of contact between the mating teeth. The sliding velocity vectors will be at right angles to the path of contact and will be along the common tangent to the tooth profiles as shown. Now, referring to Fig. 1.11 we can derive at the following relations:

When two gears are in mesh, their angular velocities are in opposite senses. Since the rotations are co-planar, an arithmatic sum of their angular velocities will represent the vector addition of these quantities. The relative angular velocities of the two gears will be given by

$$\text{Relative angular velocity} = \omega_1 + \omega_2$$

where ω_1 and ω_2 are the angular velocities of the pinion and the gear respectively. In Fig. 1.11 let the hatched curves represent tooth profiles in conjugate action. The velocity vector of the point Q is represented by the line QM, for the driving pinion rotating at an angular velocity of ω_1, NN is the common normal to the two curves in mesh and TT is the common tangent. Line NN is also the line of action or the path of contact of the two curves in motion. Vector QM_1 is resolved into two components: Qn along the common normal and Qt, along the common tangent. Since we are dealing with rigid bodies in contact, the normal component of the velocity of Q, when considered as a point on body 1, is equal to the normal component of the velocity of Q, when considered as a point on body 2. The instantaneous directions of velocities of Q in body 1 and 2 are as shown by the arrows of QM_1 and QM_2 which make right angles to the radial lines O_1Q and O_2Q respectively. Since Q_n is the common component, the magnitude of the velocity vector QM_2 of body 2 can be found as shown in the figure. The components of QM_1 and QM_2 on the common tangent TT are Qt_1 and Qt_2 respectively. These are individual sliding components of QM_1 and QM_2. When Q coincides with the pitch point P, the sliding components become equal in magnitude and being in the same direction, the relative sliding velocity is zero. The motion is then one of pure rolling.

From Fig.1.11 it can be easily seen that the relative sliding velocity is the difference between the sliding components since, in this case, the components happen to be in the same direction. Had they been in opposite directions, the components would have to be added to get the relative sliding velocity. An expression for this relative sliding velocity will now be derived, so that it will not be necessary to draw the velocity diagrams to arrive at the value. This is explained below

$$\text{Relative sliding velocity} = Qt_1 - Qt_2.$$

Drop perpendiculars O_1R and O_2S on the common normal NN.

Since linear velocity (v) = Angular velocity (ω) x Radius (r), we have

$$\omega_1 = \frac{QM_1}{O_1Q} \quad \text{and} \quad \omega_2 = \frac{QM_2}{O_2Q}$$

From geometrical relation, we see that the following triangles are similar

$$QM_1t_1 \text{ and } O_1QR$$
$$QM_2t_2 \text{ and } O_2QS$$
$$O_1PR \text{ and } O_2PS$$

Now

$$PQ = QR - RP = \frac{O_1Q}{QM_1} \times Qt_1 - RP = \frac{1}{\omega_1} \times Qt_1 - RP \text{ or } RP = \frac{Qt_1}{\omega_1} - PQ$$

Again

$$PQ = PS - QS = \frac{O_2P}{O_1P} \times RP - QS = \frac{\omega_1}{\omega_2} \times RP - QS \text{ (Eq. 1.1)}$$

and

$$QS = \frac{O_2Q}{QM_2} \times Qt_2 = \frac{1}{\omega_2} \times Qt_2$$

$$\therefore PQ = \frac{\omega_1}{\omega_2} \times RP - \frac{1}{\omega_2} \times Qt_2 = \frac{\omega_1}{\omega_2} \times \left(\frac{Qt_1}{\omega_1} - PQ\right) - \frac{1}{\omega_2} Qt_2 = \frac{Qt_1}{\omega_2} - \frac{\omega_1}{\omega_2} PQ - \frac{Qt_2}{\omega_2}$$

or

$$\omega_2 PQ = Qt_1 - Qt_2 - \omega_1 PQ$$

whence

$$Qt_1 - Qt_2 = PQ(\omega_1 + \omega_2)$$

Thus we arrive at the following important relation:

The relative sliding velocity of a point of contact = The distance of that point from the pitch point x The sum of the angular velocities of the two gears.

Hence, to find the relative sliding velocity of a particular point of contact, we need only to measure its distance from the pitch point and then multiply it by the sum of the gear angular velocities which are already known from their speed in rpm.

EXAMPLE: Given: speed of pinion $n_1 = 1200$ rpm, reduction ratio $i = 2$, $PQ = 53.1$ mm. To find the relative sliding velocity.

Solution: Speed of gear = 1200/2 = 600 rpm, $\omega_1 = 2\pi n_1 = 2\pi\,1200$, $\omega_2 = 2\pi\ 600$.

$$\therefore \quad \text{Relative sliding velocity} = \frac{53.1(1200 + 600)2\pi}{60 \times 1000} = 10 \text{ m/s}$$

It is to be noted that the maximum values of the relative sliding velocities are attained at the beginning and at the end of contact. These values are respectively given by

$$v_b = \text{Length of approach} \times (\omega_1 + \omega_2)$$

and

$$v_e = \text{Length of recess} \times (\omega_1 + \omega_2)$$

To find the power loss due to sliding, we proceed as follows:

Linear velocity at pitch point $P = v = \omega_1 r_1 = \omega_2 r_2$. Reduction ratio $= i = \dfrac{\omega_1}{\omega_2} = \dfrac{r_2}{r_1}$.

$$\therefore \ \omega_1 = \frac{v}{r_1} \text{ and } \omega_2 = \frac{v}{r_2} = \frac{v}{ir_1} \text{ or } \quad \omega_1 + \omega_2 = \frac{v}{r_1} + \frac{v}{r_2} = v\left(\frac{1}{r_1} + \frac{1}{ir_1}\right)$$

$$= \frac{v}{i \cdot r_1}(i + 1)$$

$$\text{Relative sliding velocity} = PQ(\omega_1 + \omega_2) = PQ\frac{v}{ir_1}(i + 1)$$

Average relative sliding velocity, $v_a = \dfrac{1}{2}\left[PQ_{max}\dfrac{v}{ir_1}(i + 1)\right]$, where PQ_{max} is the maximum distance of Q from P.

Power loss due to sliding $P_s = v_a F_N \mu$, where F_N = Normal gear force (see Sec. 2.18)

The coefficient of friction μ depends on the material, finish of tooth surface, and the state of lubrication. It is not of a constant value, but varies with load changes, velocity changes, etc. besides the above determining factors. Average value of μ can be taken as around 0.07.

Nominal power $P = F_t v$, where F_t = Transmitted load (see Sec. 2.18).

$F_N = \dfrac{F_t}{\cos \alpha}$ (for spur gear) $= \dfrac{F_t}{\cos \alpha \cos \beta}$ (for helical gear), α and β being the pressure angle and the helix angle (in case of a helical gear) respectively.

Expressed as a percentage of nominal power,

$$\frac{P_s}{P} \times 100 \approx \frac{PQ_{max}}{d_1}\,\frac{i + 1}{i}\,\frac{\mu}{\cos \alpha \cos \beta}\,100\% \tag{2.10}$$

Pressure angle $\alpha = 20°$ normally, and for spur gear $\beta = 0°$.

Neglecting power losses at the bearings and lubrication losses, and considering the kinematics of the teeth only, a simplified version of the efficiency (η) can be given as:

(2.11)

where, f = Loss factor due to tooth friction. It has an average value of **2.6** for 20° standard toothing, and has been calculated on the basis of an average value for contact ratio of **1.65**. Coefficient of friction μ is taken as 0.07. In the above equation positive sign is meant for external gearing and negative for internal gearing.

2.7 Contact Ratio

It can be seen from Fig. **2.11** that during the course of action of teeth engagement along the path of action in a meshing pair of gear teeth, i.e. from the beginning of contact to the end of contact comprising the length of contact, the load is transmitted by a single tooth of the driving gear for part of the time and by two teeth during rest of the time. That is, a new pair of teeth comes into action before the preceding pair goes out of action. For continuous contact, the angle of action must be greater than the angle subtended at the centre by the arc representing the circular pitch (called "pitch angle"). The relation between these two angles is termed as the "contact ratio".

The physical significance of the contact ratio lies in the fact that it is a measure of the average number of teeth in contact during the period in which a tooth comes and goes out of contact with the mating gear. A contact ratio of **1** means that only one pair (one tooth from each gear) is engaged at all times during the course of action. This is the case when the angle of action is just equal to the pitch angle. Contact ratio **1.6** means that during the period of engagement one tooth each from the mating gears is in contact **100%** of that period, while during the same period, two teeth each from the mating gears are also in mesh, but **60%** of the time only.

To ensure smooth and continuous operation, the contact ratio must be as high as possible, which the limiting factors permit. Definite values are difficult to specify, but for satisfactory performance of power transmitting gears, a value of 1.4 is used as a practical minimum. The value, if the situation warrants it, may be permitted to have a lowest value of up to 1.2 sometimes in extreme cases. A lower contact ratio also necessitates a higher degree of accuracy in machining to ensure quiet running of the gear set.

We have seen in Ch. **1** on involumetry that the common normal to the two involutes in contact is tangent to the two base circles. This common normal is also the line of action TT. Contact begins when the line of action intersects the tip circle of the driven gear. At this point the flank of driver touches the tip of the driven gear. Contact ends when the line of action intersects the tip circle of the driver. At this point the tip of the driver just leaves the flank of the driven gear. These two points are shown as **A** and B in **Fig. 2.13** and the portion of the line of contact AB of T_1T_2 is called the length of contact or the length of action.

An expression for the contact ratio can be found in the following manner: Referring to Fig. **2.13** which illustrates a simplified version of the teeth positions during the course of action, the contact begins at A and ends at B as stated before. At the base circles, T_1 and T_2 are the points of tangency.

The angle of action consists of the angle of approach and the angle of recess in each case, both these angles being separated by the central line O_1PO_2. The angles of action in this case are: fO_1f' and hO_2h'. Since the radii of the two gears are different, the angles also are different. But since both the gears are supposed to roll on their pitch cylinders without slippage, the two arcs on the two pitch circles are equal in length during the movement which takes place for a particular time. In other words

$$\text{arc}\, ff' = \text{arc}\, hh' = \text{arc of contact or action}$$

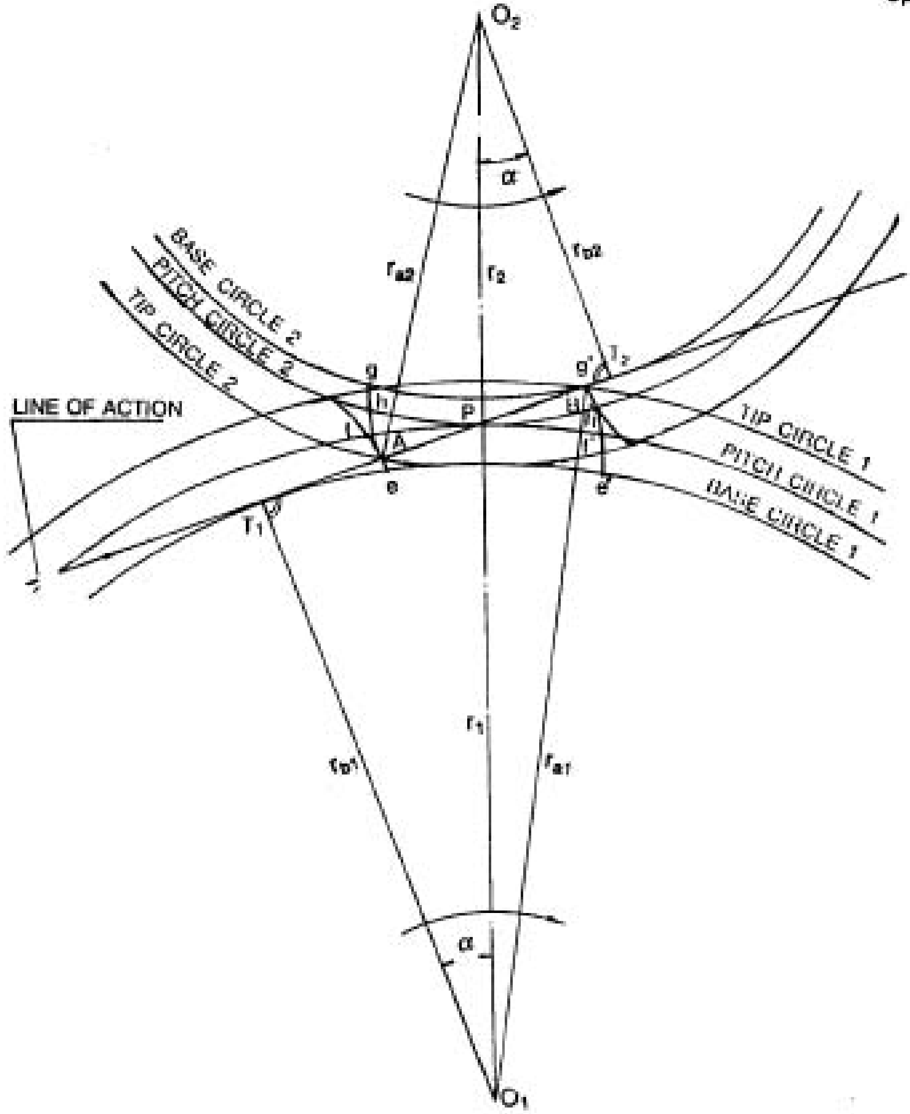


Fig. 2.13 Derivation of contact ratio

Now, the involute starts at e and traces to A on the line of action. Similarly, e' traces to B. From properties of involute curve, we know that

$$\text{arc } T_1e = \text{length } T_1A, \text{ and arc } T_1e' = \text{length } T_1B$$

Hence,

$$\text{arc } ee' = \text{length } AB = \text{length of action}$$

We can now establish the following relations from geometry

$$AB = AP + PB = (AT_2 - PT_2) + (BT - PT_1)$$

$$= \left[\sqrt{r_{a_2}^2 - r_{b_2}^2} - r_2 \sin a\right] + \left[\sqrt{r_{a_1}^2 - r_{b1}^2} - r_1 \sin a\right]$$

Here the length of action AB is divided into AP = length of approach, and PB = length of recess. The respective IS symbols are: AB = g_a $AP = g_f$ and PB = g_a.

Simplifying the above equation, we get

$$AB = \left[\sqrt{r_{a_1}^2 - r_{b_1}^2} + \sqrt{r_{a_2}^2 - r_{b_2}^2}\right] - [r_1 + r_2] \sin a$$

$$= \left[\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a_2}^2 - r_{b_2}^2}\right] - a \sin a$$

Contact ratio (CR) is given by

$$CR = \frac{\text{Angle of action}}{\text{Pitch angle}} = \varepsilon \text{ (IS symbol)}$$

Now, the angle subtended by the arc ee' at the centre O is equal to the angle subtended by the arc ff' at the centre O_1. Therefore, calling this angle as θ, we have

$$\text{arc } ee' = r_{b1}\theta, \text{ and arc } ff' = r_1\theta$$

But, $r_{b1} = r_1 \cos a$, hence, arc ff', $= \dfrac{\text{arc } ee'}{\cos a}$

Calling the pitch angle as ϕ, we have CR $= \dfrac{\theta}{\phi}$

Transposing, we get $\theta = \dfrac{\text{arc } ff'}{r_1} = \dfrac{\text{arc } ee'}{r_1 \cos a} = \dfrac{AB}{r_1 \cos a}$, and $\phi = \dfrac{\pi m}{r_1}$

(since circular pitch, $p = \pi m$).

$$\therefore CR = \frac{\theta}{\phi} = \frac{AB}{r_1 \cos a} \times \frac{r_1}{a m} = \frac{AB}{a m \cos a} = \frac{\text{arc } ee'}{\cos a} \times \frac{1}{a m}$$

$$\equiv \frac{\text{arc } ff'}{\pi m} = \frac{\text{arc of action}}{\text{circular pitch}}$$

Now, $a m \cos a = p \cos a = p_b$ = Base pitch.

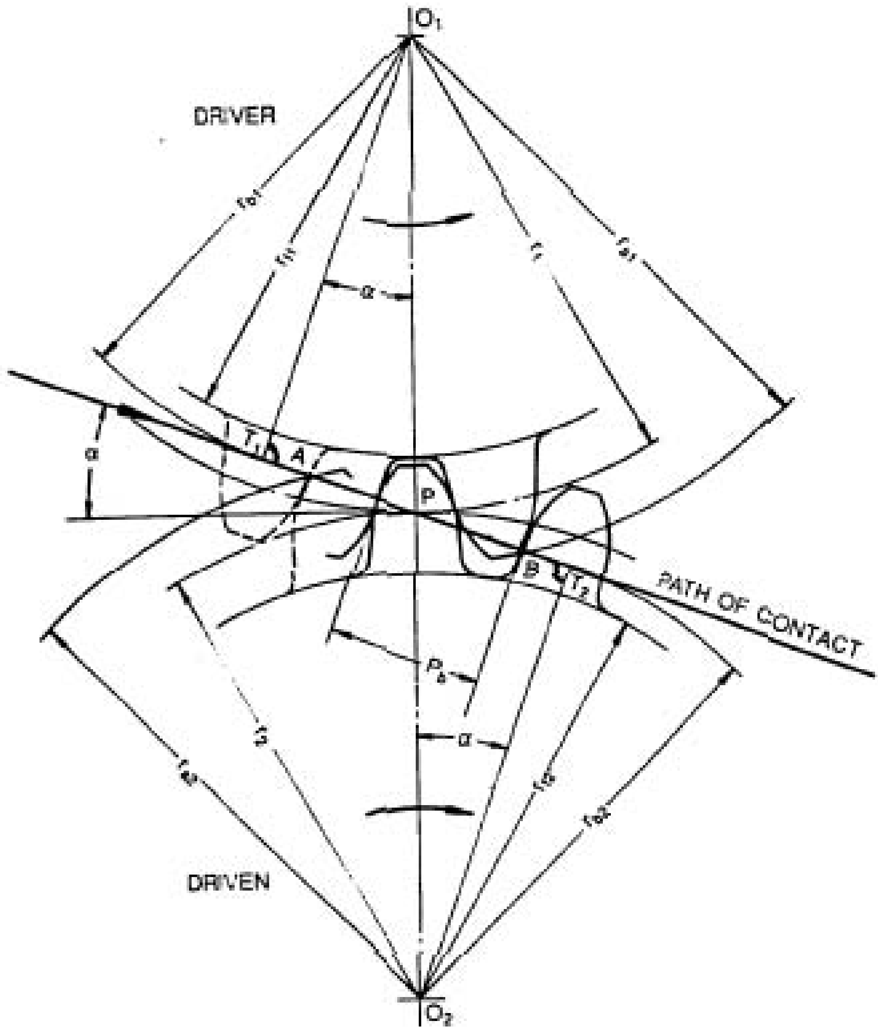
We finally come to the following relation

$$\text{Contact ratio (CR)} = \frac{\text{Angle of action}}{\text{Pitch angle}} = \frac{\text{Arc of action}}{\text{Circular pitch}} = \frac{\text{Length of action}}{\text{Base pitch}}$$

$$\text{Therefore, CR} = \frac{\sqrt{r_{a_1}^2 - r_{b_1}^2} + \sqrt{r_{a_2}^2 - r_{b_2}^2} - a \sin a}{p \cos a} \tag{2.12}$$

$$= \frac{\sqrt{r_{a_1}^2 - r_{b_1}^2} + \sqrt{r_{a_2}^2 - r_{b_2}^2} - a \sin a}{p_b} \tag{2.13}$$

Figure **2.14** shows the outlines of the teeth in contact in a more elaborate manner to illustrate the contact ratio relations.



$$\text{CONTACT RATIO} = \frac{\text{LENGTH OF CONTACT}}{\text{BASE PITCH}} = \frac{AB}{p_b}$$

BEGINNING OF CONTACT AT "A" (SHOWN IN DOTTED LINES)

END OF CONTACT AT "B" (SHOWN IN SOLID LINES)

Fig. 2.14 Contact **ratio**

For a gear mating with a rack, the contact ratio is given by

$$CR = \frac{\sqrt{r_a^2 - r_b^2} - r \sin\alpha + \dfrac{h_r}{\sin\alpha}}{p \cos\alpha},$$

where, r_a, r_b and r relate to the gear and h_r is the addendum of the rack and is usually equal to the module m.

For corrected gears, described later in this chapter, the corrected values of r_{a1}, r_{a2} and the centre distance a are to be inserted. Also, α will be replaced by the working pressure angle α_w.

In connection with the discussion on the contact ratio relations, it is interesting to note that this is a ratio of the length of action AB, which is a straight line, and the base pitch p_b, which is the arc of a circle. This may seem odd to a reader. In Fig. 2.15 two adjacent teeth are shown in the figure where the base pitch has been laid off as per its definition. This is arc ac in this figure. From point a, involute ab is generated with reference to point T on the base circle. Involute cd is similarly generated. From Ch. 1 on the characteristics of the involute curve (Sec. 1.5), we know that

$$\text{arc } Tc = \text{straight line } Td$$

Similarly

$$\text{arc } Ta = \text{straight line } Tb$$

or,

$$\text{arc } Tc - \text{arc } Ta = \text{line } Td - \text{line } Tb$$

$$\therefore \text{arc } ac = \text{Base pitch} = \text{line } bd$$

Since bd lies on the line of action, the ratio: length of contact AB to the base pitch ($ac = bd$) is a measure of the contact ratio.

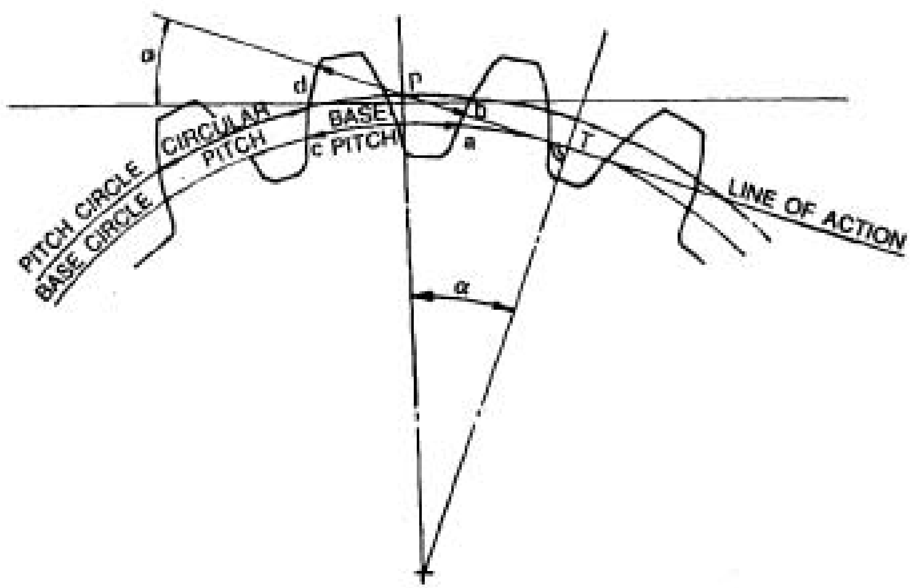


Fig. 2.15 Base pitch and circular pitch

In Appendix F tables of contact ratio of spur gears in mesh are given for different z_1/z_2 combinations. The tooth profile is standard with addendum = m, and the pressure angle = 20°.

2.8 Backlash in Spur Gears

Backlash can be generally defined as the play between a mating pair of gear teeth in assembled condition. It is the amount by which the width of a tooth space exceeds the thickness of the meshing tooth measured on the pitch circle. This is called the circumferential or torsional, or angular backlash, and is designated as j_t. If the backlash is measured on the line of action, it is termed as the normal or linear backlash j_n. These backlashes have been shown in Fig. 2.16. The relation between these two types of backlashes is given later. Unless otherwise specified, the values of backlash are given with reference to the pitch circles. Proper amount of backlash

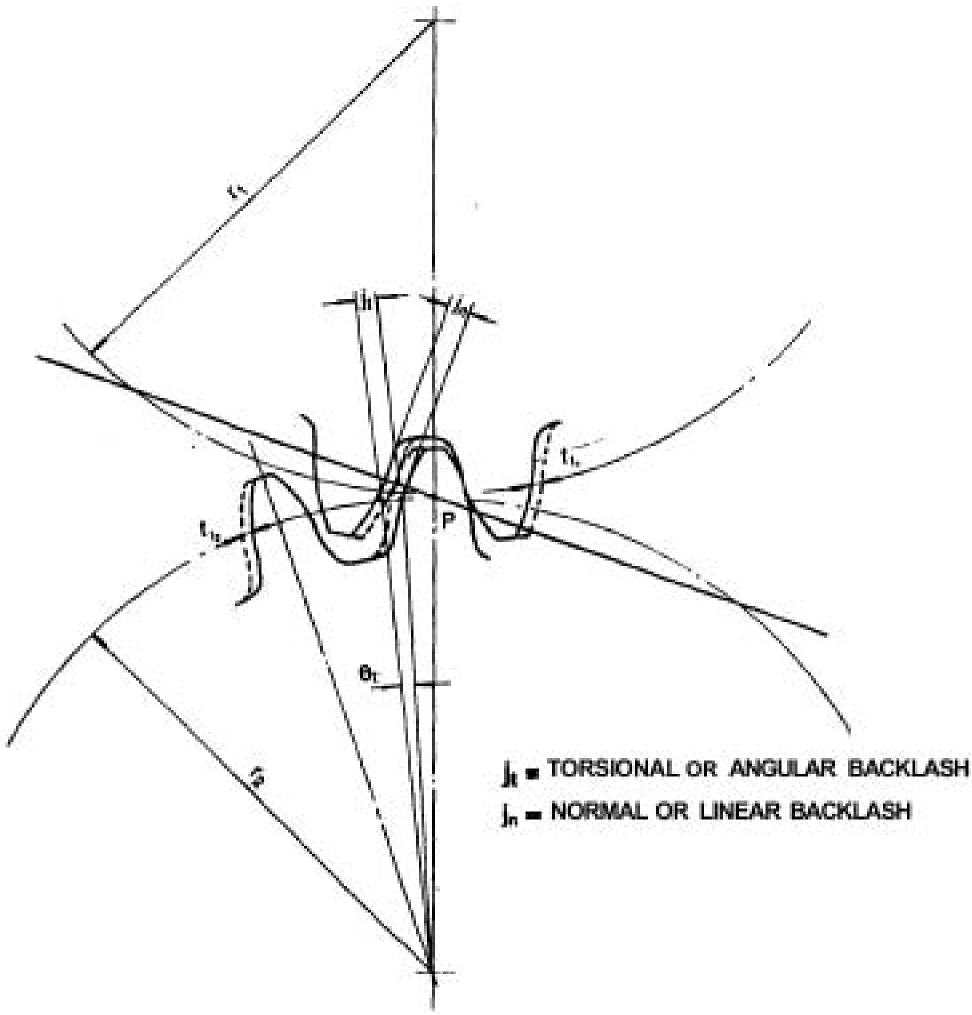


Fig. 2.16 Backlash in gears

ensures smooth running of the gear set. Except in case of timing gears, indexing and some other special purpose applications where the meshing gears have practically zero backlash, the main purpose of providing backlash is to prevent jamming and to ensure that no contact is made on both sides of the teeth simultaneously. Too little backlash may lead to overloading, overheating, jamming and ultimately seizure and eventual failure of the system. Moreover, a tight mesh, may result in objectionable noise during running. On the other hand, excessive backlash may cause non-uniform transmission of motion especially if the amount of backlash varies from tooth to tooth due to machining and other errors. Excessive backlash may also cause noise and impact loads in case of reversible drives.

Specification of proper amount of backlash, therefore, is of prime importance. Moreover, an unnecessarily small amount of backlash allowance should be avoided because this will increase the cost of gears as allowances for run-out, pitch error, profile and mounting errors are also to be kept correspondingly smaller.

In selecting the proper amount of backlash, many factors are to be taken into consideration. They are the run-out and errors in tooth thickness, pitch tooth spacing, profile, helix angle, etc. It should be noted that backlash in no way affects the involute action.

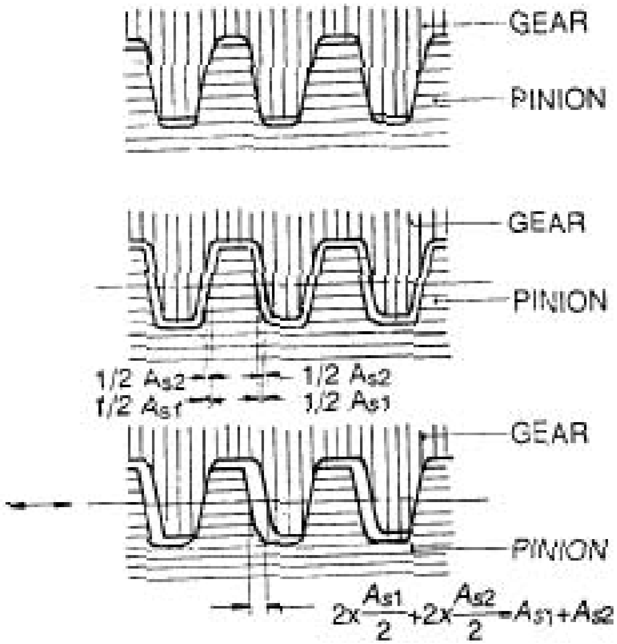
In the mounted condition, the backlash will consist of the amount by which the thickness of the teeth has been reduced as well as the tolerance on the centre distance. The tolerances specified on the teeth (which are always negative), either on individual teeth or on block measurement as described later in Sec. 2.28, determine the amount by which the tooth thicknesses are reduced. This will again depend on the quality and the zone of tolerance selected (see Secs 2.27 and 2.28). The reduction in tooth thickness is usually obtained by sinking the cutter deeper into the blank to correspond to the tolerance pre-selected. In some cases the cutter itself is so dimensioned that the tolerance is taken care of when the tool cuts up to the relevant standard tooth depth. Determination of the final backlash in the mounted condition has been illustrated diagramatically in Fig. 2.17.

For measurement of circumferential or torsional backlash, one gear of the pair is held stationary and the other one is rotated till its tooth touches the corresponding tooth face of the other component. The movement is then registered by a dial indicator suitably mounted. For measuring normal backlash, suitable feeler-gauges or similar measuring instruments may be employed.

Gears, when meshed with a rack, have no backlash. When base circles are shifted away from each other, as in the case of non-standard centre-distance gear systems, the mating involutes will have zero backlash with the theoretical rack as shown in Fig. 2.18(b), but they will have an actual backlash between themselves which is quite apparent from the figure. When the centre distance is non-standard, the working, i.e. the actual pressure angle α_w is different from the nominal pressure angle of the basic rack. Under such conditions, the individual mating gear tooth profiles have zero backlash with respect to the common reference profile of the rack, but they will contact the rack profile at different points, viz. A and B in Fig. 2.18(b). This displacement of the respective contact points, together with the lesser convexity of the tooth profiles in case of positively corrected gears, results in creating the backlash between the profiles of the mating teeth.

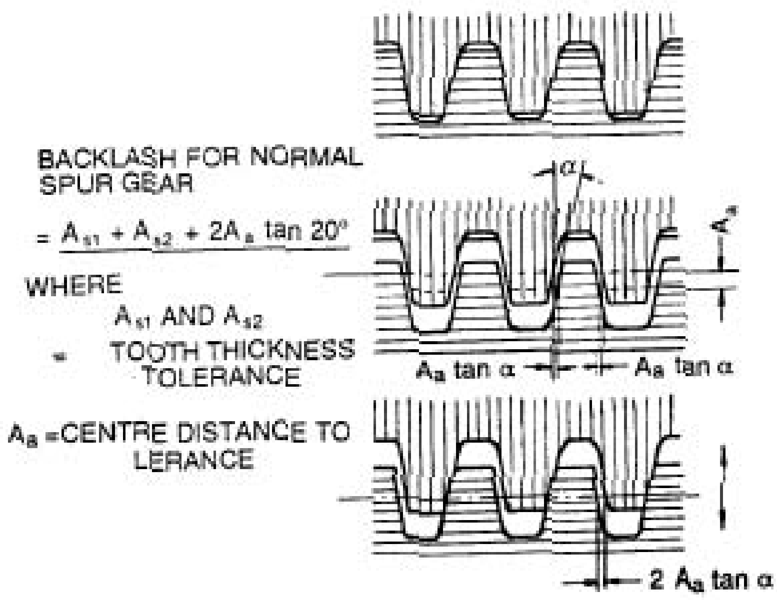
In Fig. 2.30 the conditions which prevail when two standard gears meshing at the standard centre distance have been shown schematically. As we have seen before, this standard centre distance is given by

$$a_0 = \frac{1}{2} m (z_1 + z_2)$$



(*a*)

EFFECT OF TOOTH THICKNESS TOLERANCE



(*b*)

EFFECT OF CENTRE DISTANCE TOLERANCE

Fig. 2.17 Tolerance versus backlash

The backlash here is zero, that is, not considering the tooth thickness allowances or the tooth distance allowances and the centre distance allowances which cause the creation of backlash automatically, as will be seen later in the section on gear tooth tolerance, etc. Pressure angle a . is the standard pressure angle, sometimes called the "in-built" pressure angle of the cutter, namely 14.5°, 15°, 20°, 25°, 30° as the case may be. In the above condition, the mating gears will operate at this particular standard pressure angle. The pitch circles in this condition meet at the pitch point P and are the standard pitch circles or the cutting pitch circles as the gears were cut at these pitch circles. Note that in this condition, the standard pitch circles and the working or operating circles are identical.

In Fig. 2.18 (a) the two gears have been pulled apart and the centre distance has now been increased. It is clear from the figure that now backlash exists between the mating pair of teeth. Restricting our discussion here only on standard gears and not considering the corrected gears

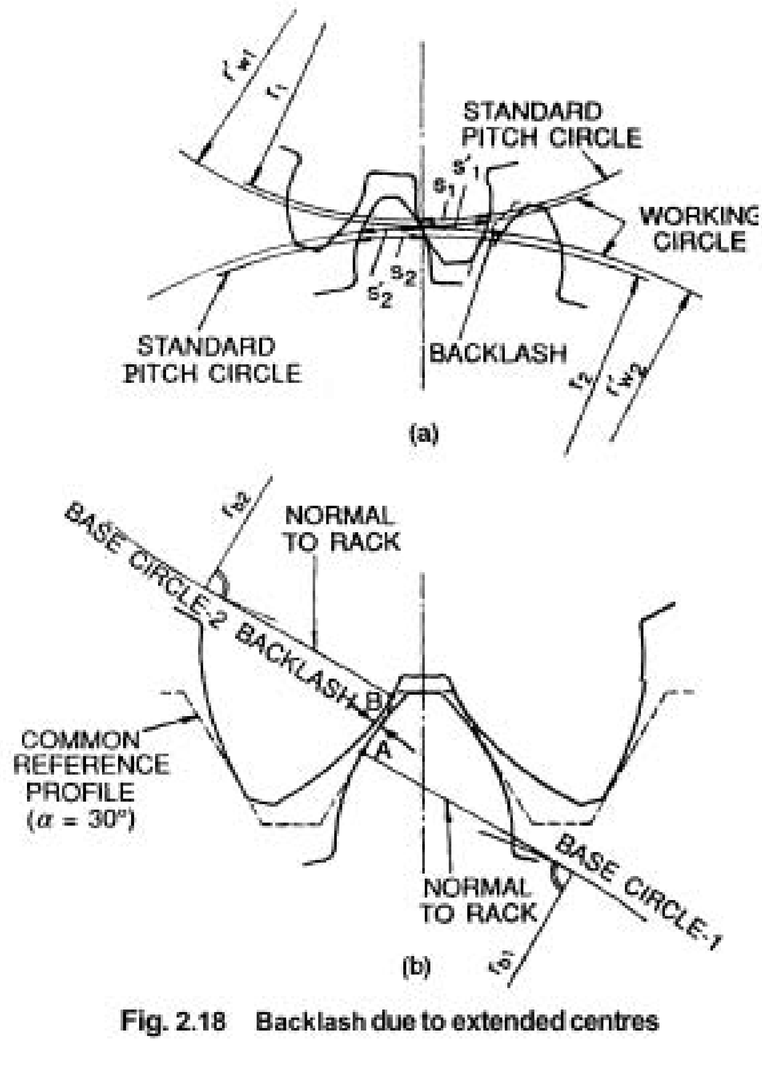


Fig. 2.18 Backlash due to extended centres

(which will be taken up later) where gears are "pushed" to make up for the backlash (as explained in Sec. 2.13), we can arrive at the following relations:

If the gears (now situated at extended centres) are made to mate, the standard pitch circles no longer touch each other. Instead, the tooth profiles touch at a point other than the standard pitch point P. If circles are now passed through the new point of contact, we can call the resulting circles as working circles with radii r'_{w1} and r'_{w2}. (The symbols r_{w1} and r_{w2} are not used here because they are used in connection with corrected gears after "pushing" as detailed in Sec. 2.13.)

The new contact point divides the new centre distance, called a' into segments which are inversely proportional to the angular velocities of the gears. That is

$$\frac{r'_{w1}}{r'_{w2}} = \frac{\omega_2}{\omega_1} = \frac{z_1}{z_2} \tag{2.14}$$

Also

$$a' = r'_{w1} + r'_{w2} \tag{2.15}$$

Joining the two base circles of the gears by a common tangent, we get the new line of action and a new working pressure angle α'_{w}.

Since

$$r_{b1} = r_1 \cos \alpha = r'_{w1} \cos \alpha'_w$$

and

$$r_{b2} = r_2 \cos \alpha = r'_{w2} \cos \alpha'_w$$

$\therefore$ $$(r_1 + r_2) \cos \alpha = (r'_{w1} + r'_{w2}) \cos \alpha'_w, \text{ or } a_0 \cos \alpha = a' \cos \alpha_{w'}$$

or

$$\cos \alpha'_w = \frac{a_0}{a'} \cos \alpha \tag{2.16}$$

From the above equations, the following relations can be established

$$\frac{r_1 \cos \alpha}{r_2 \cos \alpha} = \frac{r'_{w1} \cos \alpha'_w}{r'_{w2} \cos \alpha'_w}$$

or

$$\frac{r_1}{r'_{w1}} = \frac{r_2}{r'_{w2}},$$

Again,

$$\frac{a_0}{a'} = \frac{\cos \alpha'_w}{\cos \alpha} = \frac{r_1}{r'_{w1}} = \frac{r_2}{r'_{w2}}$$

$\therefore$ $$\frac{r_1}{r'_{w1}} = \frac{r_2}{r'_{w2}} = \frac{a_0}{a'} \tag{2.17}$$

Due to faulty mounting or otherwise, sometimes the standard gears are operated at such extended centre distance, in which cases the determination of backlash may become necessary. An expression for such backlash can be arrived at as shown under:

The sum of tooth thicknesses measured on the working circle + Backlash
= Circular pitch measured on the working circle

That is, refemng to Fig.2.18 (a),

$$s_1' + s_2' + \text{Backlash} = p' = \frac{2\pi r'_{w1}}{z_1} = \frac{2\pi r'_{w2}}{z_2}$$

Recalling Eq. 2.8, we have

$$s_1' = 2r'_{w1}\left[\frac{s_1}{2r_1} + \text{inv } \alpha - \text{inv } \alpha'_w\right] = \frac{r'_{w1}}{r_1}s_1 - 2r'_{w1}(\text{inv } \alpha'_w - \text{inv } \alpha)$$

$$s_2' = 2r'_{w2}\left[\frac{s_2}{2r_1} + \text{inv } \alpha - \text{inv } \alpha'_w\right] = \frac{r'_{w2}}{r_2}s_2 - 2r'_{w2}(\text{inv } \alpha'_w - \text{inv } \alpha)$$

From Eq. 2.17, we have

$$\frac{a_0}{a'} = \frac{r_1}{r'_{w1}} = \frac{2\pi r_1}{z_1} \times \frac{z_1}{2\pi r'_{w1}} = \frac{p}{p'}$$

$$\therefore \quad p' = \frac{a'}{a_0} \times p = \frac{a'}{a_0}\pi m$$

wherep andp' are the circular pitches on the pitch circle and the working circle respectively.

$$\therefore \quad \text{Backlash} = \frac{2\pi r'_{w1}}{z_1} - s_1' - s_2' = p' - s_1' - s_2'$$

$$= \frac{a'}{a_0}\pi m - \frac{r'_{w1}}{r_1} \times s_1 + 2r'_{w1}(\text{inv}\,\alpha'_w - \text{inv}\,\alpha)$$

$$- \frac{r'_{w2}}{r_2} \times s_2 + 2r'_{w2}(\text{inv}\,\alpha'_w - \text{inv}\,\alpha)$$

$$= \frac{a'}{a_0}\pi m - \frac{a'}{a_0}(s_1 + s_2) + 2a'(\text{inv}\,\alpha'_w - \text{inv}\,\alpha)$$

$$= \frac{a'}{a_0}[\pi m - (s_1 + s_2) + 2a_0(\text{inv}\,\alpha'_w - \text{inv}\,\alpha)] \qquad (2.18)$$

For standard gears,

$$s_1 = s_2 = s = \frac{\pi m}{2}$$

$$\therefore \quad \text{Backlash} = 2a'(\text{inv}\,\alpha'_w - \text{inv}\,\alpha) \qquad (2.19)$$

It has been mentioned before that two kinds of backlash measurements are done—the normal backlash j_n and the torsional backlash j_t. The magnitude of both the kinds of backlashes will depend upon the tolerances on tooth thickness and centre distance. These aspects have been dealt with in detail in Secs *2.27* and *2.28*. When these grades and tolerances are fixed from various design considerations, the magnitudes of the backlashes become automatically fixed. General guidelinesfor determination of the proper amount ofbacklash will be given later in this section. Expressions ofbacklashes for spur gears are given in Eqs 2.20 to 2.23. These are to be read along with Secs *2.27* and *2.28*. Subscripts U and L stand for the upper and the lower values of the tolerance which is represented by the general symbol A, 1 and *2* for pinion and gear, S for tooth thickness and a for centre distance. Thus A_{SL2} denotes the lower value (L) of tooth thickness (S) tolerance (A) of gear (2).

$$j_n\,(\min) = -(A_{SU1} + A_{SU2})\cos a + 2A_{sL}\sin\alpha_w \qquad (2.20)$$

$$j_n\,(\max) = -(A_{SL1} + A_{SL2})\cos a + 2A_{sU}\sin\alpha_w \qquad (2.21)$$

$$j_t\,(\min) \approx -(A_{SU1} + A_{SU2}) + 2A_{sL}\tan\alpha_w \qquad (2.22)$$

$$j_t\,(\max) = -(A_{SL1} + A_{SL2}) + 2A_{sU}\tan\alpha_w \qquad (2.23)$$

It is important to note here that all the tolerances are to be entered in the above formulae with the proper algebraic sign they carry (+ or –). The centre distance tolerance A_a, for example, carry ± sign (see Appendix L).

For general engineering applications, the values of backlash as per IS: **4460** given in Table **2.2** serve the purpose adequately. Otherwise the amount of backlash is determined from the tolerances on tooth thickness and centre distance, as indicated earlier.

For specifying the proper amount of backlash, the undermentioned salient points are to be kept in mind.

1. The maximum permissible run-out is one of the most important criteria for the determination of the magnitude of backlash, followed by allowable errors in profile, pitch, tooth thickness and helix angle.
2. Gears which rotate in slow speed generally require the least backlash.
3. Maintenance of the proper lubricant film is another important consideration. Backlash should be so chosen that there is adequate clearance for an oil film. To avoid generation of heat and excessive loading on teeth, oil should not be allowed to be trapped at the root of the teeth.

Table 2.2 Backlash for gears for normal applications

(All dimensions in mm)

Pitch line velocity				
Up to 8m/sec			Above 8m/sec	
Module	Backlash Min	Backlash Max	Module	Backlash
20	0.75	1.25	8	0.40
16	0.50	0.85	7	0.38
12	0.35	0.60	6	0.36
10	0.30	0.51	5	0.28
8	0.22	0.40	4	0.23
6	0.20	0.33	3.5	0.22
5	0.15	0.25	3	0.21
4	0.13	0.20	2.75	0.20
3	0.10	0.15	2.5	0.19
2.5	0.08	0.13	2	0.18
2	0.08	0.13	—	—
1.5 and finer	0.00	0.10	—	—

4. When tolerances on tooth thickness are not determined as per the specified tables, it is customary to distribute the backlash allowances evenly amongst the two members of the gear pair. However, when the number of teeth of the pinion is very low, the thinning of teeth should be made on the gear only. This way the pinion teeth are not weakened.

5. If necessary, the design should be aimed at controllable backlash. That is, provision should be made so that relative adjustment can be made between the positions of the mating components after assembly. Such backlash adjustments are required in case of bevel gearing and screw compressor drives.

6. In case of helical and spiral bevel gears, the greater the angle, the higher should be the transvers backlash for a particular normal backlash.

7. For gears with pressure angles more than 20°, backlash required is more on the pitch circles to have a given value of backlash in the direction perpendicular to the mating tooth profiles.

8. Determination of backlash is a direct function of the heat-treatment of the gear teeth which follows.

9. Backlash is influenced by machining errors, mounting misalignments, errors in bearings and allied factors.

10. Gears used in timing and indexing devices, and in certain precision instruments require minimum backlash. The ideal "zero backlash", however, is extremely difficult to achieve. It involves employing special machines and costly techniques.

It has been mentioned before that the amount of actual backlash obtained after mounting will be governed by the magnitude of the tooth thickness allowances of the two gears and by the value of the centre distance allowance.

Equations **2.20** to **2.23** are the expressions for the minimum and the maximum values of the two kinds of backlashes discussed in this section. However, this does not mean that during running, the gear-set can have a backlash which may very between the minimum and the maximum limits. These two values simply show the two extreme permissible limits of the backlash for the quality and the zone which have been chosen for the case in question.

The variation in backlash which may be permitted during the actual running have a definite value, and this value must lie within the range stipulated by the two (min and max) limits. The following example will amply illustrate the point.

EXAMPLE 2.1 : Given $-z_1 = 24, z_2 = 60, m = 2$, quality and zone = $9d$, $\tan \alpha_w = 0.36$.
To find the relevant values of the torsional backlash.

Solution: From Appendices J and L (zone J) we get the following values in micrometres (keeping in mind that for minus numerals, those having lesser absolute values are considered to be higher because they are nearer to zero, e.g. $-90 > -135$).

$$A_{SU1} = -90 \qquad A_{SU2} = -112 \qquad A_{aU} = +56$$

$$A_{SL1} = -135 \qquad A_{SL2} = -168 \qquad A_{aL} = -56$$

From Eqs **2.22** and **2.23** we have

$$j_t\,(\min) = -[(-90) + (-112)] + 2(-56) \times 0.36 = 162$$

$$j_t\,(\max) = -[(-135) + (-168)] + 2(+56) \times 0.36 = 343$$

Therefore, the permissible limits of the torsional backlash are **162 (min)** and **343** (max). The maximum value of the variation which may be allowed in this running gear-set is found as follows

Range of deviation of pinion = 135 − 90 = 45
Range of deviation of gear = 168 − 112 = 56
Total deviation = 45 + 56 = 101

Hence, the permissible variation in backlash in the running gear-set is 101 micrometers. This means that at the limiting ranges, the torsional backlash may vary

between 162 and 263 (=162 + 101), and
between 242 (=343 − 101) and 343

Any intermediate values, of course, are allowed within the limiting ranges, provided the variation does not exceed 101 micrometres.

2.9 Interference and Undercutting of Gear Tooth

In Sec. **1.5** it has been mentioned that the involute curves begin at the base circle and extend outwards to form the gear tooth profiles. Obviously, there is no involute inside the base circle. From Fig. 1.13, we know that the line of action of the two inter-meshing gears is tangent to the two base circles. The two points of tangency represent the two extreme limiting points of the length of action. These two points, T_1 and T_2 in Fig. 1.13 are called the "interference points".

It has been emphasised in Sec. 1.5 that to have and maintain conjugate action, the mating teeth profiles of the gear pair must consist of involute curves (when involute curves are used as teeth profiles). Any meshing outside of the involute portion will result in non-conjugate action. That portion of the tooth profile which lies between the base circle and the root circle comprises non-involute curve.

Referring to Fig. 2.19 T_1 and T_2 are the points of tangency. It may so happen that the mating teeth are of such proportion that the beginning or the end of contact or both occur outside of the

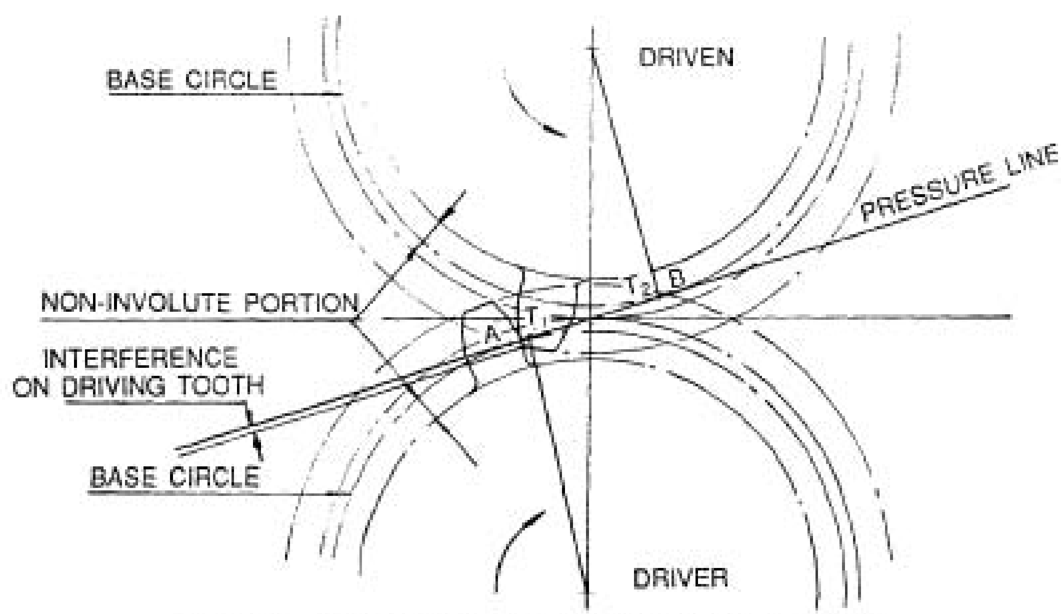


Fig. 2.19 Interference on flank of driving tooth ($\alpha = 14\frac{1}{2}°$)

interference points on the path of contact. Then the involute portion of one member will mate with the non-involute portion of the other member. In the case in question, the initial contact takes place at point **A**. At this point the flank of the tooth of the driver is forced into contact with the tip of the tooth of the driven gear. It can be seen from the figure that the tip of the driven tooth comes in contact below the base circle of the driver. Consequently, no conjugate action takes place. This phenomenon has been termed as "interference" in gear technology.

In the example, contact also occurs on the non-involute portion (at B) when the teeth leave the mutual meshing. In short, if the contact takes place outside of the path T_1T_2 during the course of action, the teeth will interfere with each other as they come in and out of mesh.

Interference in gear toothing is undesirable for several reasons. Due to interference, the tip of one tooth of the gear of the pair will tend to dig into portions of the flank of the tooth of the other member of the pair. Moreover, removal of portions of the involute profile adjacent to the base circle may result in serious reduction in the length of action. All these factors weaken the teeth and are detrimental to proper tooth action. Interference can, of course, be eliminated by using more teeth on the gears, but such solution is seldom resorted to because this calls for larger gears with their ensuing problems, such as, increased pitch-line velocity, noise, reduced power transmission, etc.

If gears are manufactured by one of the generation processes, the interference is automatically eliminated as the cutting tool also removes the interfering portion of the flank.

From Sec. **8.3** on gear cutting processes, it will be seen that in a generation process, the cutting action is such as if the two components—the gear being produced and the cutter-are in mesh like two gears rolling on their pitch cylinders. While generating gear teeth, if there is interference of the cutter, then a recess is cut at the root of the tooth. The profile thus generated deviates from the theoretical tooth profile. This happens when the cutter extends beyond the base circle of pinions having small number of teeth. This removal of material at the root of the gear tooth is called "undercutting".

The tooth is already at the weakest region in the vicinity of the root. By undercutting, the tooth becomes further weakened. Hence, although interference can be avoided by the generation process because the corresponding recess is made at the tooth root by the cutter with the consequent absence of fouling, this is not considered an acceptable solution because of its tooth weakening effect. The problem of interference is simply substituted by another problem caused by undercutting.

There are several practical ways of tackling the problem of interference and undercutting. One of them is to use a larger pressure angle. Since

$$d_b = d \cos \alpha$$

larger pressure angle results in a smaller base circle, the pitch circle diameter remaining the same. This in turn allows more of the tooth profile to be made of involute curve. Thus small pinions of 25° pressure angle system are preferred by some designers inspite of the fact that frictional forces and bearing loads are more.

Another practical way of eliminating interference is to limit the addendum of the driven gear so that it passes through the interference point as shown in Figs 2.20 and 2.21 so that the whole depth of the tooth consists of the normal depth minus the shaded portion, making it a stub-tooth.

Referring to Fig. 2.21 where a rack and a pinion have been shown in mesh, the point of tangency or the interference point is at **T**. This point fixes the maximum addendum for the rack having the pressure angle shown. If the rack has the addendum as shown in the figure, the contact begins at **A**. Undercutting takes place as shown by the dotted line. If now the shaded

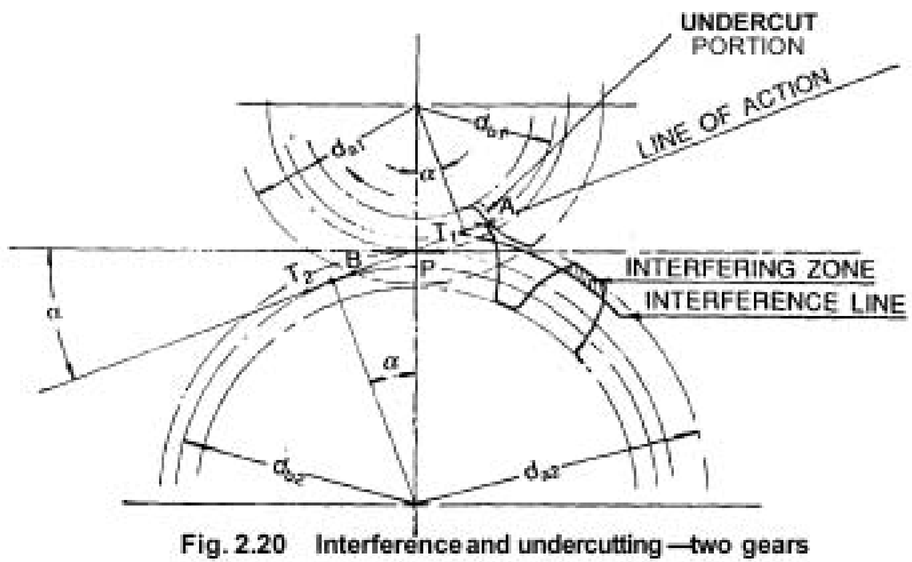


Fig. 2.20 Interference and undercutting—two gears

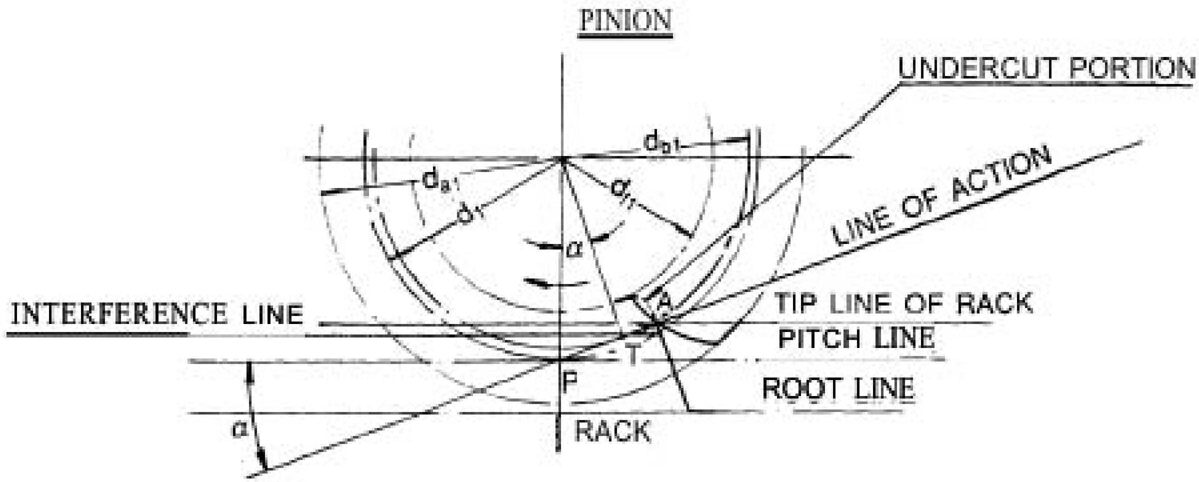


Fig. 2.21 Interference and undercutting—rack and pinion

portion is removed, point A merges with point T. In other words, the point of initial contact is now T and interference is obviously eliminated. The same conditions for cases where two gears mesh are shown in Fig. 2.20.

One important conclusion which can be drawn is: Instead of a rack, if a gear with the same modified addendum as the rack now meshes with pinion, the point of initial contact will lie on the line of action at a point somewhere between the pitch point P and the interference point T. Thus, there is no possiblity of interference.

We can, therefore, conclude that if the number of teeth of pinion is such that it meshes with a rack without interference, it will also mesh with any other gear having the same or a greater number of teeth without any chance of interference taking place.

Determination of Undercutting Radius

If a gear is undercut for one reason or another, it may become sometimes necessary to know the magnitude of the undercutting radius.

In Sec. 2.10, an equation for the minimum number of teeth to avoid undercutting is given as

$$z_{min} = \frac{2}{\sin^2 \alpha}$$

The above equation is valid for standard gear tooth with the addendum of the rack being equal to the module m. The general expression is

$$h_{ca} = \sin^2 \alpha \, m \, \frac{z}{2}$$

The undercut-free minimum number of teeth is given by

$$z_{min} = \frac{2 h_{ca}}{m \sin^2 \alpha}$$

where h_{ca} = The addendum of the rack-cutter without tip fillet-rounding.

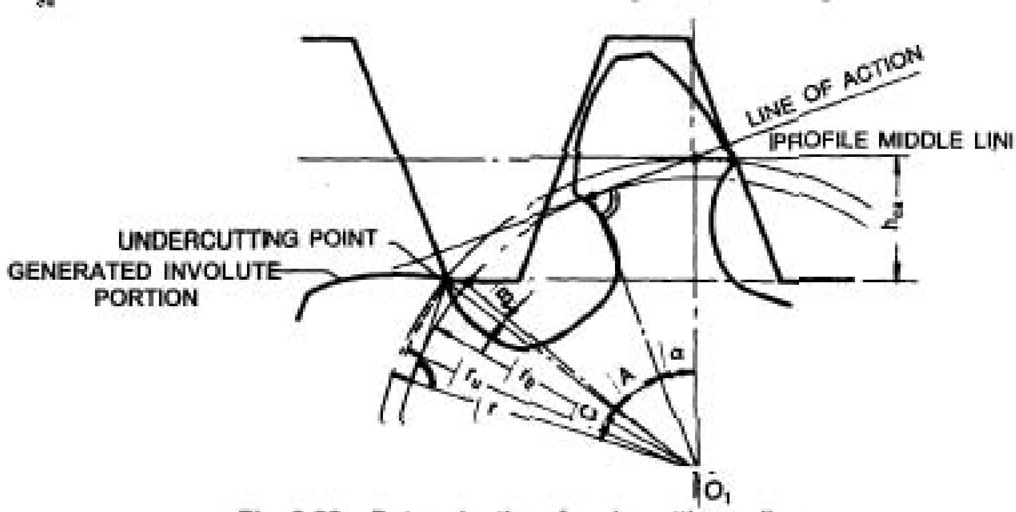


Fig. 2.22 Determination of undercutting radius

The undercutting radius can be determined with the help of geometrical and trigonometrical relations. These relations are given in subsequent paragraphs with reference to Fig. 2.22.

Let r_u = Undercutting radius

r = Pitch circle radius

r_b = Base circle radius

α = Standard pressure angle

A, B, and C are angles (in radians) the values of which can be determined from the following relations.

$$\cos C = \frac{r_b}{r_u}$$

$$B = \text{inv } C = \tan C - C$$

Angle A is determined from the following equation

$$\frac{A}{\sin (A + B)} = \frac{r_u}{r_b}$$

By transposing and inserting the relevant values, we get the following relations

$$h_{ac} = r - r_u \cos (\alpha + A + B)$$

$$r_u = r_b \frac{A}{\sin (A + B)} = r_b \frac{\dfrac{1}{\cos \alpha} - \dfrac{h_{ac}}{r_b}}{\cos (\alpha + A + B)}$$

2.10 Minimum Number of Teeth to Avoid Interference

In Sec. 2.9 the phenomenon of interference and undercutting and conditions thereof have been discussed. It was observed that a pinion which meshed with a rack without interference would also mesh with a gear of the same size as the pinion or with a larger one without interference. Such type of gear action will ensue if the tooth proportions are same in both the cases.

For gears with standard tooth proportions, the minimum number of teeth which a pinion can have to mate with a rack without interference can be calculated. Referring to Fig. 2.23(b), this limiting case can be solved by passing the addendum line of rack through the interference point T of the pinion. The following relations can be arrived at

$$\sin \alpha = \frac{PT}{r} = \frac{m}{PT}$$

whence

$$\sin^2 \alpha = \frac{PT}{r} \times \frac{m}{PT} = \frac{m}{r}$$

Now

$$r = \frac{d}{2} = \frac{1}{2} mz$$

$\therefore$

$$\sin^2 \alpha = \frac{m}{r} = m \frac{2}{mz} = \frac{2}{z}$$

or

$$z = \frac{2}{\sin^2 \alpha} = z_L \text{ or } z_{min} = \text{the limiting or minimum number of teeth} \quad \textbf{(2.24)}$$

For **20°** full-depth tooth system, $z_{min} \approx 17$. For other systems, the values are : **32** for **14½°** full-depth, **14** for **20°** stub and 12 for 25° full-depth system.

Since these values were determined for a pinion meshing with a rack, they can also be taken as the minimum values for a pinion meshing with any gear without having the danger of interference. The generation process by hobbing is analogous to the tooth action between a rack

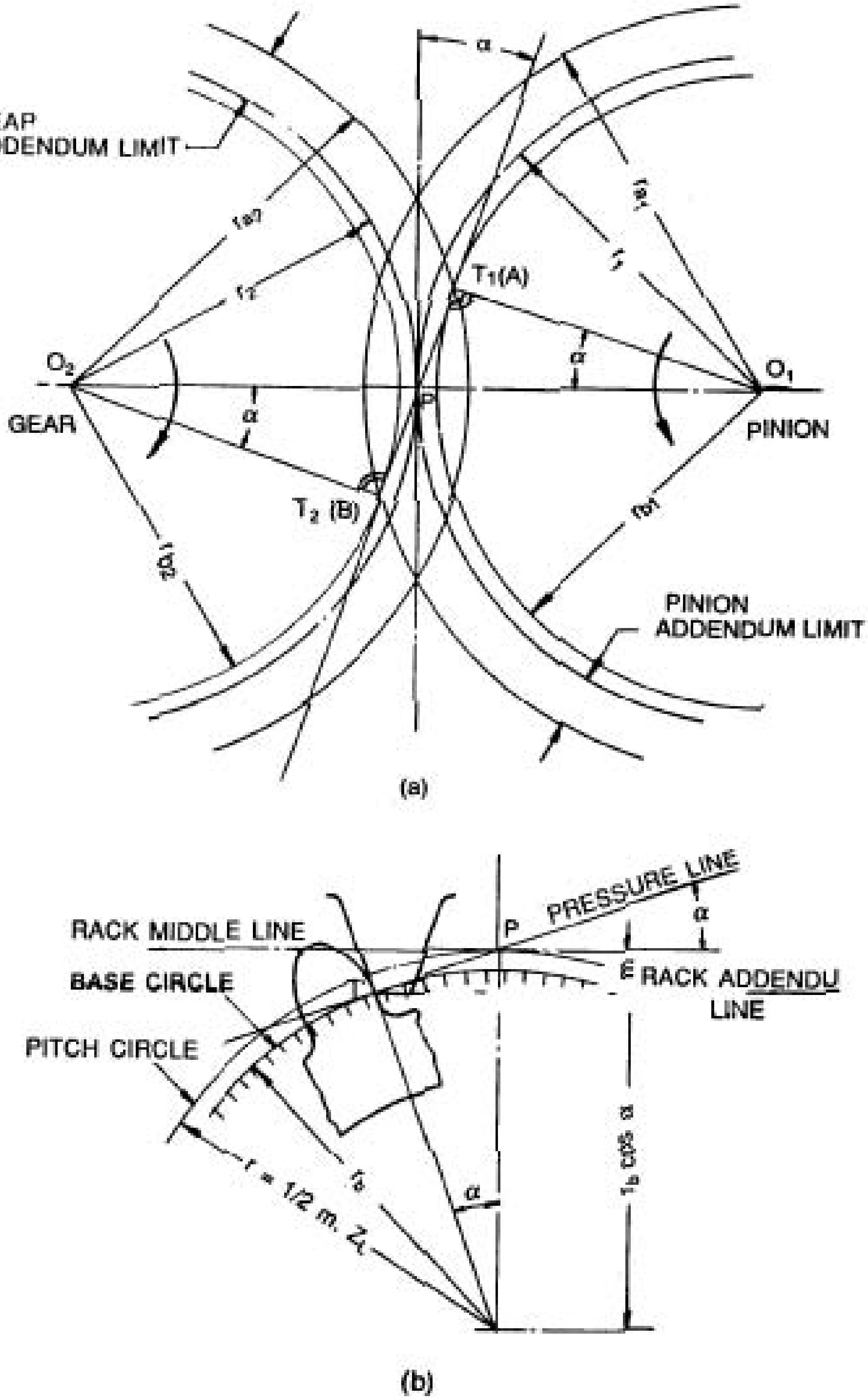


Fig. 2.23 Limiting conditions for undercut-free toothing

and a pinion in mesh. Hence, the minimum values of the number of teeth of pinions are also the minimum which can be generated by a hob without undercutting.

There are other generation processes where a hob is not used. For example, Fellows shaping method uses a pinion-type cutter. The above minimum number of teeth of pinion for such cases will not work. A generalised expression, therefore, is derived for a pinion and a gear in mesh for determining the limiting number of teeth of pinion.

Referring to Fig. 2.23 (a), the condition where interference is just avoided has been represented. T_1 and T_2 are the points of tangency where the line of action, passing through the point of contact of the two pitch circles or the pitch point P, is tangent to both the base circles. To avoid interference, it is obvious that the limiting conditions will be such that the two tip circles must also pass through T_1 and T_2. Therefore, the initial and the final points of contact, A and B, merge with T_1 and T_2 respectively. Considering 20° full-depth teeth with addendum = m, we have from the figure

$$r_{a2} = O_2T_1 = r_2 + m = \sqrt{(O_2T_2)^2 + (T_1T_2)^2} = \sqrt{(r_2 \cos\alpha)^2 + (T_1P + PT_2)^2}$$

$$= \sqrt{(r_2 \cos\alpha)^2 + (r_1 \sin\alpha + r_2 \sin\alpha)^2} = \sqrt{r_2^2 \cos^2\alpha + (r_1 + r_2)^2 \sin^2\alpha}$$

Squaring and simplifying, we get

$$m^2 + 2r_2 m = r_1^2 \sin^2\alpha + 2r_1 r_2 \sin^2\alpha$$

Putting $r_1 = \frac{mz_1}{2}$ and $r_2 = \frac{mz_2}{2}$, we have

$$m^2 + 2m\frac{mz_2}{2} = \frac{m^2 z_1^2}{4}\sin^2\alpha + 2\frac{mz_1}{2}\cdot\frac{mz_2}{2}\sin^2\alpha$$

Simplifying and transposing, we get

$$z_1^2 + 2z_1z_2 = \frac{4(1 + z_2)}{\sin^2\alpha} \qquad (2.25)$$

Dividing by z_2,

$$\frac{z_1^2}{z_2} + 2z_1 = \frac{4}{z_2 \sin^2\alpha} + \frac{4}{\sin^2\alpha}$$

When z_2 tends to infinity, we have the limiting case;

$$2z_1 = \frac{4}{\sin^2\alpha} \text{ or } z_1 = \frac{2}{\sin^2\alpha}$$

This is the case where a rack (z_2 = infinity) meshes with a pinion of number of teeth = z_1. We have already arrived at the above equation from geometrical stand-point.

The value of z_1 as calculated from the above equation is slightly more than 17, (z_1 = 17.097). Therefore, some people take it as 18.

From Eq. 2.25, it can be seen that as z_2 decreases, z_1 also decreases. If $z_1 = z_2$, then the minimum number of teeth becomes around 13. This is the condition depicted in Fig. 2.23 (a). For gear/pinion ratio of 3 : 1, we get the value of about 15 for z_1 after inserting the value $z_2/z_1 = 3$ in Eq. 2.25 and so on for other ratios. Thus the minimum number of teeth of the pinion to avoid undercutting is a function of the gear tooth ratio.

For gears cut with the shaping-type of generation process, we have seen that the smallest number of teeth for two equal gears for 20° full-depth teeth is 13. Figures for other tooth proportions are: 23 for 14.5° full-depth, 10 for 20° stub and 9 for 25° full-depth systems. It should be noted here that there is a maximum limit in each case for the maximum number of teeth of the second gear, i.e. z_2, which will mesh with the pinion having the minimum number of teeth without interference, when gears are cut by the shaping method. Obviously, the second gear cannot be increased to a rack. Keeping the value of z_1 to the minimum calculated, the largest values of z_2 are as follows: $z_1 = 13$ and $z_2 = 16$ for 20° full-depth, 23 and 26 for 14.5° full-depth, 10 and 11 for 20° stub, and 9 and 13 for 25° full-depth systems.

However, as the tooth profiles are often generated with a rack-type cutter or a hob, it follows that the minimum number of teeth of pinion should be so chosen as to avoid undercutting while meshing with a rack or a gear in service.

Experts are of the opinion that a little undercutting does not adversely affect the smooth running of the gear pair much. Hence, they allow the minimum number of teeth to go down to

$$z_{min} = \frac{5}{6} \times 17 = 14$$

From a practical angle it has been found that the detrimental effect on tooth action is marginal when the minimum number is taken as 14, the resulting undercutting being very slight in magnitude.

2.11 Profile Correction of Gears

In the previous sections we have discussed the problems of interference and undercutting and arrived at the value of the minimum number of teeth of the pinion required to avoid undercutting. We have also seen that interference is a serious defect of the involute system of gearing and should be avoided at all costs. Apart from the fact that interference hampers conjugate action when the involute portion of a tooth mates with the non-involute portion of the other tooth, there is every likelihood that the two meshing gears will not rotate at all. Rather, the gear causing the interference will have a tendency to jam on the flank of the pinion — unless, of course, the pinion tooth-root has already been undercut making room to provide free movement of the gear teeth. Besides, due to interference and in the absence of an undercut, the mating gear will try to scoop out metal from the interfering portion. But since the mating gear is not a cutting tool, in the process the teeth become damaged and it will have an overall detrimental effect on the gearing system.

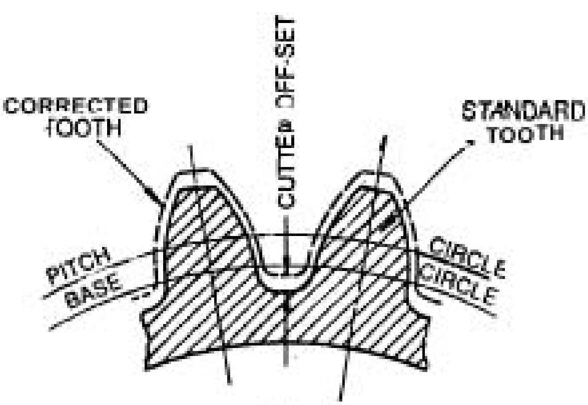
If the situation warrants, a pinion might have to be designed with the number of teeth less than the minimum number stipulated to avoid undercutting. In such cases, the practice which is now universally adopted is what is known as the "profile correction" of gear tooth.

In gear technology profile correction is variously termed as "addendum modification", "profile displacement", "profile shift", etc. by different authors. Also, the correction factor is called as "addendum modification coefficient", etc. In this book, we shall simply use the terms "correction" and "correction factor" respectively.

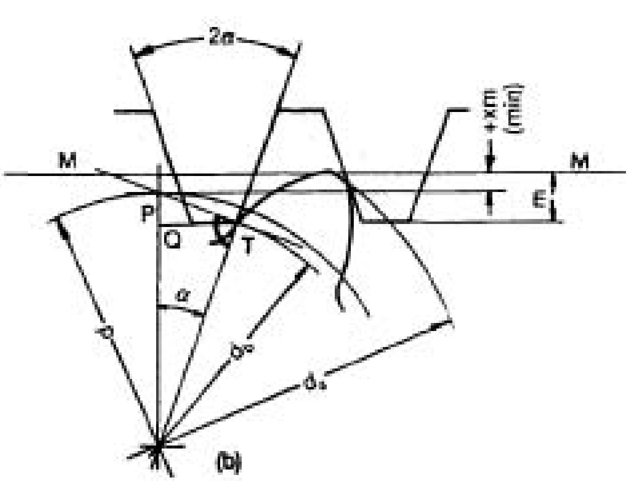
The effects of profile correction are manifold and its various characteristics will be discussed in latter sections. In this section we will restrict our discussion only on finding the appropriate amount of the correction factor just to avoid undercutting.

We have seen before that if the contact between the mating teeth takes place somewhere inside the points of tangency, that is, if the length of action AB is within the line T_1T_2, then no interference will take place. However, if the contact takes place outside T_1T_2, interference will occur. In a generating process, the cutter will take out metal from the interfering zone. Therefore, while generating the pinion, one obvious solution is to make the cutting arrangement in such a way that the cutting tip of the rack-type cutter or hob just touches the point of tangency when the cutting action begins. To effect this, the cutter is withdrawn by a specified distance so that the addendum line of the rack-type cutter or hob just touches the point of tangency when the cutting action begins. To effect this, the cutter is withdrawn by a specified distance so that the addendum line of the rack just passes through the interference point, i.e. the point of tangency. This cutter off-set has been shown in **Fig.2.24(a)**.

As the rack is withdrawn, the outer periphery of the pinion must also be correspondingly increased beforehand. In other words, a larger pinion blank is to be machined, the diameter of which can be calculated by formulae given later. When a standard, uncorrected pinion is cut by the rack, the pitch line (i.e. the reference line of the rack situated at *1m* from the tip line of the rack in standard profile) is tangent to the pitch circle of the pinion at the normal pitch point P. When, however, the rack is withdrawn, this situation alters. This can be seen in Fig. **2.24** (b).



(a)



(b)

Fig. 2.24 Formation of corrected tooth

The rack reference line *MM* is no longer a tangent to the pitch circle. Instead, it is away by an amount equal to xm millimetres.

This amount xm is the profile correction of the gear and the coefficient x is known as the "correction factor". Note that x is dimensionless, but xm is expressed in mm. We shall see later that x can be positive or negative. Positively corrected gear is known as S-plus gear and negatively corrected gear as S-minus gear. To find the correction factor for pinion whose number of teeth lies within the minimum number specified to avoid undercutting, we proceed as per the undermentioned procedure. In Fig. 2.24(b) the rack is withdrawn just enough so that the addendum line of the rack passes through the interference point T of the pinion. This point T is the position from which the involute profile of the pinion tooth starts. Referring to the above figure

$$\sin a = \frac{PQ}{PT} = \frac{m - xm}{PT}$$ for standard, full-depth tooth with addendum = m

Also

$$\sin a = \frac{PQ}{d/2} = \frac{PT}{r} \text{ or } PT = r \sin a$$

Again

$$r = d/2 = mz/2$$

Inserting the values, we get

$$\sin a = \frac{m - xm}{r \sin a} = \frac{2m(1 - x)}{mz \sin a}$$, whence we have $1 - x = \frac{z}{2} \sin^2 a$

From Eq. 2.24, we have

$$z_{min} = \frac{2}{\sin^2 a}. \qquad \therefore 1 - x = \frac{z}{z_{min}} \qquad \text{or } x = 1 - \frac{z}{z_{min}}$$

or

$$x = \frac{z_{min} - z}{z_{min}} \tag{2.26}$$

The above equation gives a theoretical expression of the correction factor which is the minimum value a gear with a number of teeth z must have to avoid undercutting. For a = 20°, we know that the theoretical value of z_{min} is 17. It has also been pointed out that a slight undercutting does not affect the tooth action much and as such the practical limiting number of teeth has been taken as 14. We, therefore, come to the relation which is most commonly used

$$x = \frac{14 - z}{17} \tag{2.27}$$

Putting $z = 14$ in the above equation, x becomes equal to zero—which it should be. Figure 2.25 (a), shows the graphical representation of the above equation. The amount of correction factor necessary corresponding to the number of teeth to avoid undercutting can be directly read from this curve. The figure also shows the negative allowable limit of the correction factor for those gears in which the number of teeth lie above the practical value of z_{min}, and also the boundary line at which the positively corrected tooth becomes pointed or peaked. The limiting number of teeth z_{min} is a function of the pressure angle a and of the helix angle β in case of helical gears. These have been shown graphically in Figs 2.25(b) and (c).

It may be mentioned here that in the opinion of some experts, speed plays a part in determining the magnitude of the allowable undercutting. The recommended values of z_{min} after allowing a marginal, non-detrimental, undercutting vis-a-vis speed are as follows:

Z_{min} = 14 for medium speeds. (This value has been recommended for general case and normal service.)

= 12 for very slow speeds

= 18 for high speeds

2.12 Characteristics of Corrected Gears

The modern trend in gear technology envisages the use of corrected profile gears in most of the power transmitting and other areas. In fact correction factor is now standardised for the purpose of interchangeable gearing (explained in Sec. 8.12 later). Previously, gears were corrected either to avoid undercutting (Sec. 2.11) or to achieve a predetermined centre distance. These reasons are still valid in specific cases wherever required, but irrespective of the above two aspects, pinions as well as gears are now generated with positive correction because of the ensuing beneficial effects which the positively corrected gear profiles offer. Briefly, the reasons of positive correction and the effects thereof are as mentioned hereunder.

1. Avoidance of undercutting: This aspect has been already discussed in Sec. 2.11.

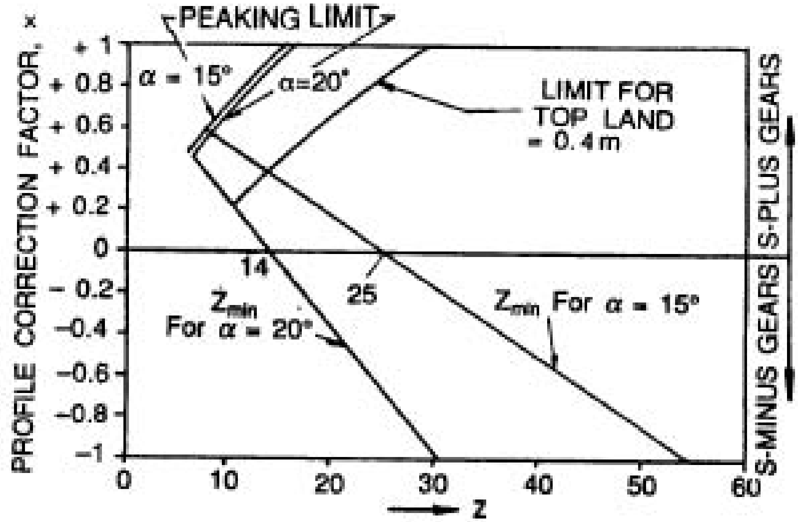
2. Attainment of a pre-determined centre distance: This aspect will be taken up in detail and the relevant formulae will be given in Sec. **2.13.**

3. To increase the strength at the root and flank of the tooth. It will be shown that due to positive correction, the thickness of tooth at the root is increased, resulting in greater load carrying capacity of the teeth. By choosing the proper amount of correction, the designer is in a position to specify gear-sets of higher capacity without entailing the corresponding cost increase for materials of higher strength.

4. Betterment of sliding and contact relations.

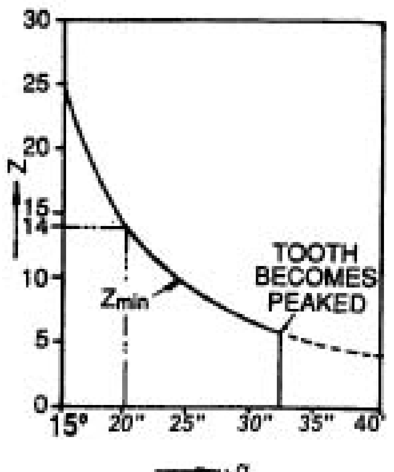
5. To shift the beginning of the effective profile away from the base circle. The highest sliding velocity and the maximum compressive stress occurs at the bottom of the tooth. From discussion on contact stress, it is known that this stress at any point on the tooth profile is a function of the radius vector of the involute curve at that point. From Sec. **1.4** on characteristics of the involute curve, we know that the radius of curvature at the starting point of the curve on the base circle is zero. Moreover, around this region, the curvature rapidly changes and the specific contact pressure is extremely high. Tooth contact at this region is to be avoided as it is detrimental for load transmission and it adversely affects the load carrying capacity. In a positively corrected tooth profile, the start of the active involute where the tooth contact takes place is farther away from the base circle than in the case of an uncorrected profile. This naturally helps to alleviate the above-mentioned difficulties as the tooth contact now takes place on those portions of the involute where the radii of curvature are greater.

It has been shown in Sec. 1.5 how the conjugate action remains unaffected even if the centre distance is changed. This is one of the valuable properties of the gears having involute tooth profile. At extended centres, the gears continue to transmit uniform angular velocity ratio. Due to correction, there is obviously a change in the shape of the gear teeth as shown in **Fig. 2.26.** The active profiles are now formed from the involute curve generated from the same base circle but this time a different portion of the curve, which is farther away from the base circle, is used.



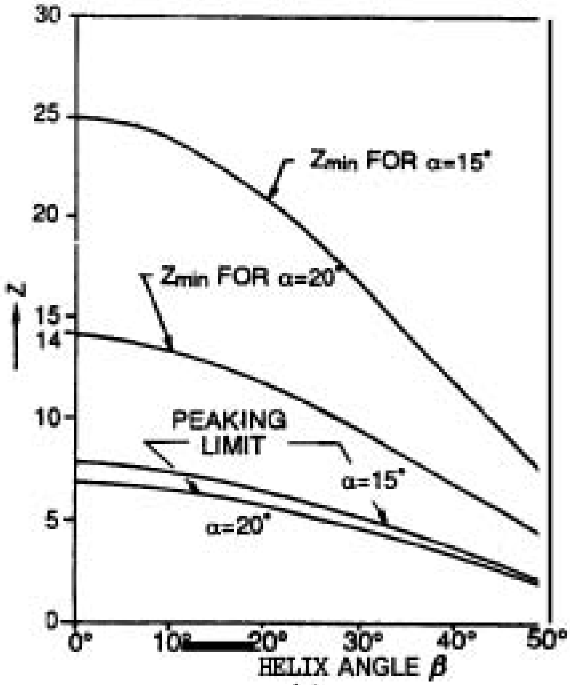
(a)

INFLUENCE OF PROFILE CORRECTION ON LIMITING NO. OF TEETH



(b)

LIMITING NO. OF TEETH FOR SPUR GEARS VS PRESSURE ANGLE



(c)

LIMITING NO. OF TEETH FOR HELICAL GEARS VS HELIX ANGLE

Fig. 2.25 Effect of profile correction factor and limiting number of teeth

Unlike normal gears, the corrected gears since they are custom-built, cannot be used for interchangeable sets unless the correction factor itself is standardised as in the case of the "05" system. Moreover, corrected gears can only be manufactured by the generation processes. They can, of course, be made by formed tools in a milling machine, etc. but this is not economically viable, as the correction varies according to the particular design parameters, and so does the shape of the tool. "his means that a large number of formed tools have to be stocked and each one must be highly accurate. In a corrected gearing system, the contact ratio is somewhat smaller in most cases. These are the disadvantages of the corrected gear tooth system.

In a corrected gear, the following parameters remain unaltered

Base circle diameter $d_b = d \cos \alpha$

Pitch circle diameter $d = m z$

Circular pitch $p = \pi m$

The following parameters change:

Tip circle diameter d_a becomes bigger by an amount of $+2xm$ in case of S-plus gears and smaller by $-2xm$ in case of S-minus gears. (Here, the topping has not been taken into account which will be discussed in Sec. 2.13.) Incidentally, it is customary to express the amount of correction in terms of module, the reason being that since x is dimensionless, it does not convey any physical concept. But xm is in millimetres and as such, it is a tangible amount. It shows the magnitude by which the gear blank is to be made bigger or smaller radially from the centre of the blank, depending on whether x is positive or negative.

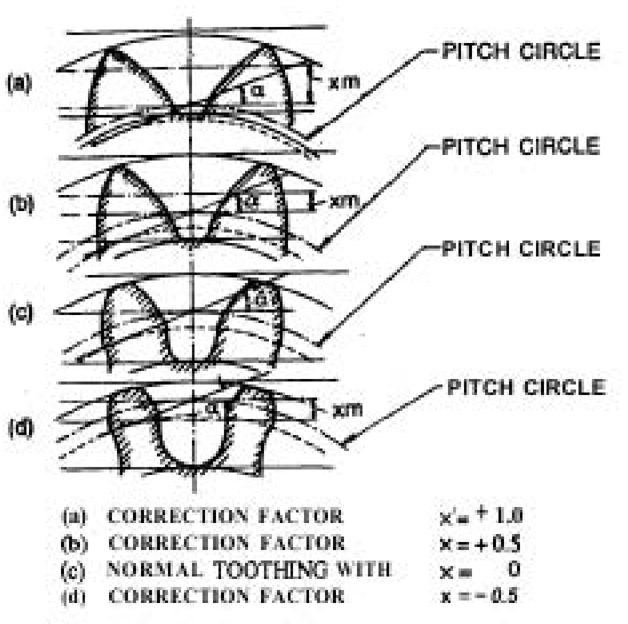


Fig. 2.26 **Effect of profile correction**
Based on Maschinenelemente, Niemann, vol. II, 1965 edition, Fig. No. 37/1, p. 37. Springer Verlag, Heidelberg.

Root circle diameter d_f becomes bigger or smaller by the same amount as in the case of d_a. The whole depth of tooth remains the same unless topping is done.

Tooth thickness s becomes greater and tooth space width e becomes correspondingly lesser in the case of positively corrected gears. The reverse is true in case of negatively corrected gears.

In a positively corrected gear, the addendum is increased by an amount of $x\,m$ (neglecting topping) and the dedendum is correspondingly decreased by the same amount. The reverse is true in case of the negatively corrected gears. Normally, the root-fillet becomes smaller in case of positively corrected gears. This has a detrimental effect on stress concentration problems. Negative correction weakens the teeth and the tooth strength decreases. However, this detrimental effect is largely nullified in case of gears with a greater number of teeth.

The influence of different types and amounts of correction factors on the shape of the tooth profile has been shown in Figs **2.26** and **2.27.** It is clear from these figures that the enlargement of a positively corrected gear is similar in effect to moving the teeth outward radially relative to the pitch circle. Consequently, the tooth thickness at the pitch circle is increased with a corresponding diminution of tooth space as the circular pitch πm remains unchanged. It may be mentioned here that the effect of profile correction on tooth form decreases with increasing number of teeth of gear. With z = infinity, that is, with a rack, the effect is zero. In this connection, see Appendix **A**, Method **4**.

Peaking

One effect of the positive correction is to make the tooth more and more pointed as the correction factor increases. Consequently, the top land becomes correspondingly smaller and ultimately results in a pointed tip. This phenomenon is termed as "peaking". The peaking limit sets a boundary to the amount of positive correction that may be applied. The shape of a peaked tooth has been shown in Fig. 2.26 (a). This figure also shows the relative shapes of S-plus gear (b), normal gear (c) and S-minus gear (d).

We have noticed in Sec. **1.4** that the involute curve becomes increasingly flattened as it moves away from the base circle from which it is generated. It has been stated previously that the profile of a positively corrected tooth (i.e. its face and flank) is composed of that portion of the involute which is farther away from the base circle than in case of an ordinary gear. Moreover, positive correction results in greater tooth thickness at the root. All these aspects combine to make a positively corrected tooth look thicker at the bottom and pointed at the top — more or less like an inverted V.

Though peaking limit determines the maximum value of positive correction, in actual practice the correction is not carried to that limit as in case of a peaked tooth, the top land is zero. Obviously this cannot be allowed. IS: **3756** recommends that the tip thickness should be greater than or equal to 0.4m for hardened gears. In exceptional cases, this may be reduced to 0.25 m. In actual practice, however, the least value of the top land can be brought down to **0.25** m even in gears for normal running without encountering any significant detrimental effects. Nevertheless, it is always advisable to check the value of top land by theoretical calculations where high correction factors are involved.

The relevant formulae have already been given in Sec. **2.3** to determine the tooth thickness at any cylinder (Eqs 2.8 and **2.9**). To calculate the top land thickness s_a of a corrected gear, it is important to remember that in the above equations, the corrected values of the tooth thickness at the pitch circle s (given later in this section), and of the radius at the tip circle r_a are to be

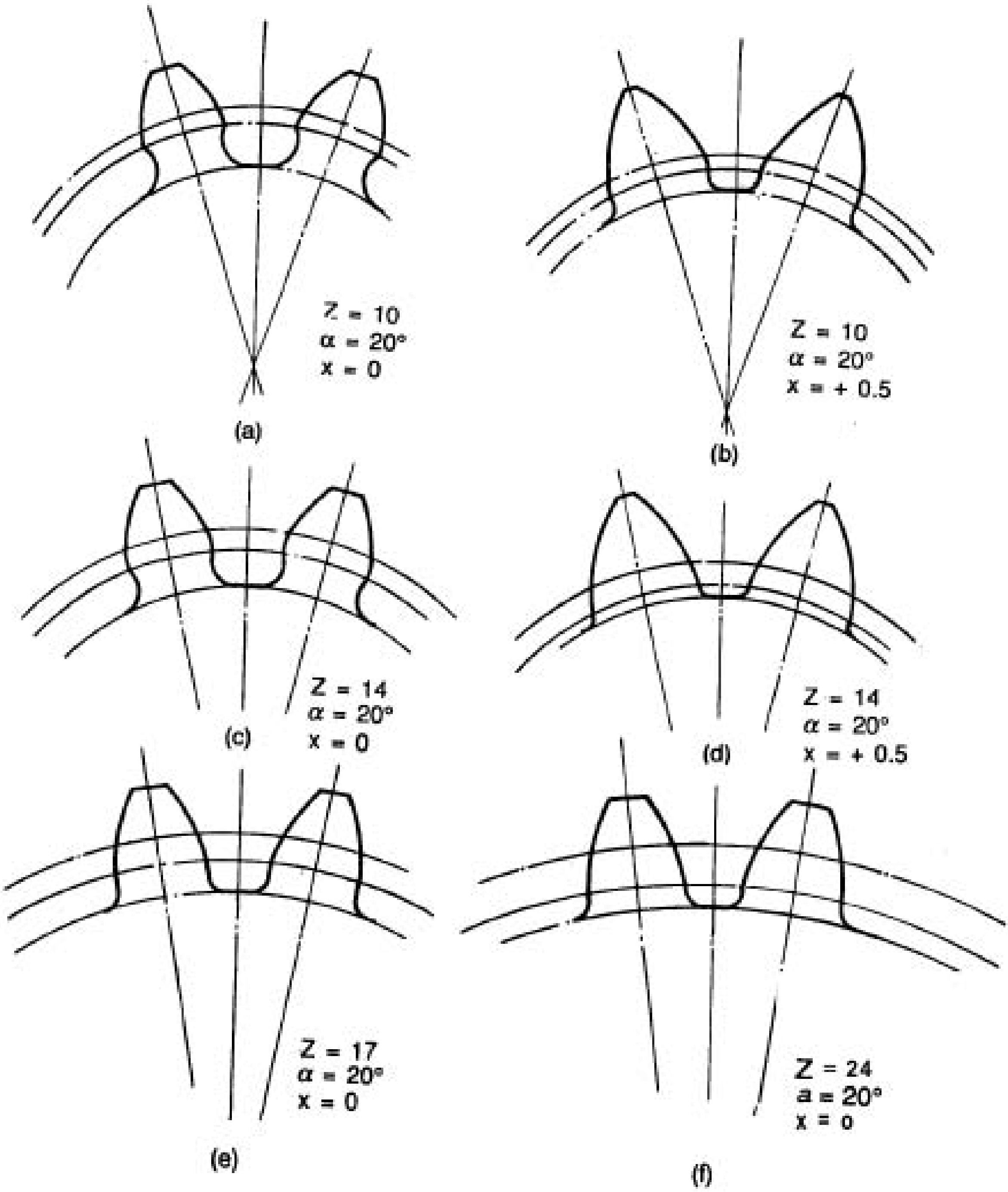


Fig. 2.27 Tooth forms of corrected and uncorrected gears
Based on Grundzuegeder Verzahnung, Thomas, 1957 edition, Fig. No. 11.6, p. 212. Carl Hanser Verlag, Munich

inserted. Also, while finding the value of the pressure angle at the tip circle α_a by means of **Eq. 2.7**, the corrected value of r_a is to be used.

Table 2.3 gives the maximum allowable positive correction factor values in case of a certain selected number of teeth commonly in use. This is based on a top land value of $0.4m$, and the values of correction factors given in the table should not be exceeded if this amount of top land value is to be maintained. Using the relevant equations, values for top lands of gears having other numbers of teeth can be calculated.

Table 2.3 Maximum permissible positive correction factor values vis-a-vis number of teeth

z	14	16	18	20
$+x$	0.43	0.50	0.57	0.62

Determination of Correction Factor for Peaked Tooth

For the purpose of checking, it may sometimes be required to determine the correction factor at which the tooth of a positively corrected gear becomes peaked. Such a case is illustrated in the example below:

Example 2.1 a Given $z = 10, m = 5$, to determine the correction factor at which the teeth of a positively corrected gear become peaked and its outside diameter thereof

Solution

$$s_a = d_a\left[\frac{s}{d} + \text{inv } a - \text{inv } a_a\right] \text{(From Eq. 2.9)}$$

For a corrected gear, as per **Eq. 2.28** (given later), we have

$$s = \frac{\pi m}{2} + 2x\ m\ \tan\alpha = 5\left(\frac{\pi}{2} + 2x\ \tan\ 20°\right) = 5\ (15707963 + 0.7279404\ x)$$

Also, the outside diameter of a positively corrected gear is given by

$$d_a = mz + 2m + 2xm = 5\ (10 + 2 + 2\,x) = 5(12 + 2\,x)$$

For a peaked tooth, $s_a = 0$,

$$\therefore \quad \text{inv}\,\alpha_a = \frac{s}{d} + \text{inv}\,a = \frac{5\ (15707963 + 0.7279404\ x)}{5 \times 10} + \text{inv } 20°$$

$$= 0.1719836 + 0.07279404x = \tan\alpha_a - \alpha_a$$

After inserting the proper symbols and subscripts in **Eq. 2.7**, we get

$$\cos\alpha_a = \frac{d}{d_a}\cos a = \frac{mz}{m\ (12 + 2x)}\cos 20° = \frac{4.6984631}{6 + x}$$

Simplifying, we finally get the following the equations

$$\cos\alpha_a = \frac{4.6984631}{6 + x}$$

and $$\tan \alpha_a - \alpha_a = 0.1719836 + 0.07279404x$$

(Note that in the above expressions, module m plays no part ultimately.)

The above equations cannot be solved by ordinary methods. They can be solved only by using techniques involving higher mathematics, viz., Newton-Raphson method. However, solution can also be obtained by trial and error methods by inserting reasonable arbitrary values of x in the first equation, solving for α_a, inserting this value of α_a in the second equation and then solving for x till the two values of x in the two equations tally. In this case, the value of x is around **0.7**. Consulting Fig. **2.25** (a), if a vertical is drawn corresponding to $z = 10$, it meets the peaking limit curve at a point corresponding to which the value of x is found to be around **0.7.** For this particular gear with peaked teeth, the outside diameter is given by

$$d_a = 5\,(12 + 2x) = 5\,(12 + 1.4) = 67 \text{ mm}.$$

Tooth Thickness of Corrected Gears at the Pitch Circle

The generation of a positively corrected gear has been shown in Fig. **2.28.** The amount of correction is $+xm$ millimetre. The profile reference line of the rack is shifted by an amount of xm from the generating or cutting line which contacts the pitch circle of the gear at point **P**. The generating line of the cutter and the pitch circle of the gear are in rolling contact like two pitch cylinders which are rolling without slipping as per the law of gearing. Had it been a normal gear, they would have rolled at the profile reference line of rack MM, because then $xm = 0$ and the profile reference line would have been the generating line. In the present context it is clear from the figure that the tooth thickness of the gear at the pitch circle (which is equal to $p/2$ or $\pi m/2$ for a standard gear) is now increased by an amount of $2xm \tan \alpha$ due to correction. Here

$$\text{arc } AP = \text{straight length } A'P$$

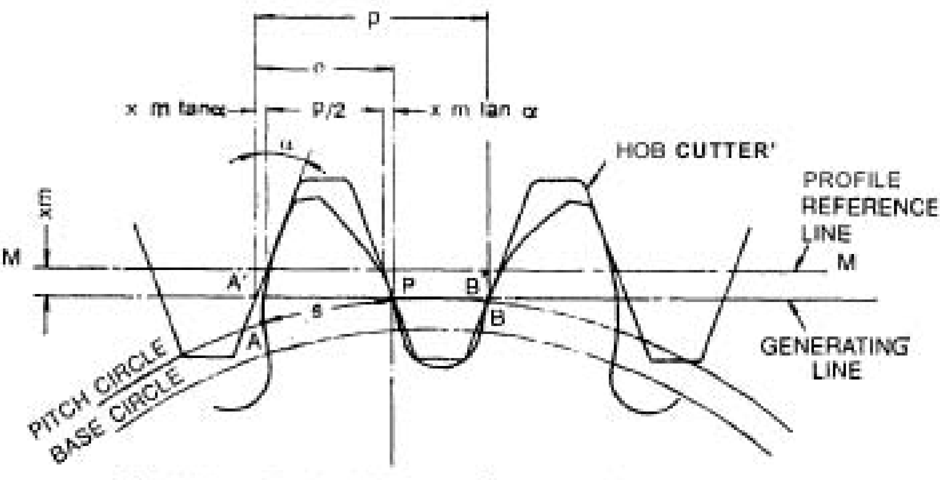


Fig. 2.28 **Tooth thickness of corrected gear**
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 55, p. 59 VEB Fachbuch Verlag, Leipzig

"he expressions for the tooth thickness, therefore, are as follows

For S-plus gear

$$s = \frac{p}{2} + 2xm \tan a = \frac{\pi m}{2} + 2xm \tan \alpha \qquad (2.28)$$

For S-minus gear

$$s = \frac{p}{2} - 2xm \tan\alpha = \frac{\pi m}{2} - 2xm \tan\alpha \tag{2.29}$$

In this connection, see Sec. 8.5 and Appendix A, Method 4.

2.13 Types of Corrected Gearing

Based on the correction aspect, gearing can be classified into two broad categories (1) S_0-gearing, and (2) S-gearing.

S_0-gearing

In S_0-gearing, the two components of the mating pair of gears receive numerically equal correction factors, but these two factors are algebraically of opposite signs. Normally, the pinion is provided with positive correction and the gear with negative correction. In other words

$$x_1 + x_2 = 0 \quad \text{or } x_2 = -x_1 \tag{2.30}$$

That is, the pinion is an S-plus gear and the gear is an S-minus gear. The S_0-gearing is also known as the "long and short addendum" system.

In S_0-gearing, because of the fact that the amount of correction is equal and opposite, their effect is to nullify each other as far as certain dimensions are concerned so that the two pitch circles contact each other at the pitch point *P* and the working pressure angle remains as the standard pressure angle as in the case of uncorrected gear, viz. 20° in usual cases. Also, the centre distance remains unaltered and is equal to the sum of the pitch circle radii. However, unlike uncorrected gearing system, the reference line of the reference profile (the basic rack) does not pass through the normal pitch point P in this case. It is shifted away by an amount equal to the numerical value of xm (mm). In case of the pinion, which normally receives the positive correction, the cutter is moved away from the gear blank centre by an extra amount of xm (mm) while cutting so that an enlarged pinion with its tip diameter increased by an amount of $2xm$ (mm) is produced. In case of the gear, the cutter is moved towards the gear centre by the same amount so that its diameter is smaller by an amount of $2xm$ (mm) than in the case of uncorrected gears. Since topping is not necessary for S_0-gearing, the gear blanks are simply made bigger or smaller, as the case may be, by the amount indicated before feeding them to the gear-cutting machine. (Topping is explained later in this section.)

The S_0-gearing is normally meant where the reduction ratio is large. Thicker pinion teeth are ensured and the gear teeth also do not become significantly weak. However, S_0-gearing is not recommended for small reduction ratios as it tends to weaken the teeth of the gear. The S_0-gearing is also sometimes recommended where for certain specific reasons the normal tooth-thickness of the gear pair or the specific sliding velocities between the meshing teeth flanks are to be changed. Besides, since normally the pinion teeth are weaker than gear teeth when both are made of the same material, they are more vulnerable to breakage and wear. The S_0-system tends to equalise the tooth strength and thereby reduces the susceptibility to such damage.

For this type of gearing, the number of teeth of pinion z_1 is less than the practical limiting number of teeth to avoid undercutting z_{min}. Also, the sum of the teeth of both the gears should be equal to or greater than twice the limiting minimum number of teeth. That is

$$z_1 + z_2 \geq 2z_{min}$$

Table 2.4 gives the expressions for the dimensions of gears in an S_0-gearing.

Table 2.4 Dimensions for S_0-gearing

Description	*Pinion*	*Gear*
Number of teeth	z_1	z_2
Pitch circle diameter	$d_1 = z_1 m$	$d_2 = z_2 m$
Tip circle diameter	$d_{a1} = d_1 + 2m + 2x_1 m$	$d_{a2} = d_2 + 2m - 2x_1 m$
Root circle diameter	$d_{f1} = d_1 - 2(1.25 - x_1)m$	$d_{f2} = d_2 - 2(1.25 + x_1)m$
Tooth thickness on pitch circle	$s_1 = \frac{\pi m}{2} + 2x_1 m \tan \alpha$	$s_2 = \frac{\pi m}{2} - 2x_1 m \tan \alpha$
Centre Distance	$a = a_0 = \frac{d_1 + d_2}{2} = m\frac{z_1 + z_2}{2}$	

Example 2.2: *Given:* transmission ratio $i = 8:1$, $m = 10$, centre distance = 540 mm. To find the dimensions of suitable gears.

Solution: $a = \frac{1}{2}m(z_1 + z_2)$, or $540 = \frac{10}{2}(z_1 + z_2)$, whence $z_1 + z_2 = 108$,

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = 8, \text{ or } z_2 = 8z_1. \text{ Solving, we get } z_1 = 12 \text{ and } z_2 = 96$$

Since z_1 is less than 14, the pinion has to be corrected. Using Eq. 2.27 we get

$$x_1 = \frac{14-12}{17} \; 10.118$$

The centre distance is to remain unaltered. Hence, an S_0-gearing is indicated. $\therefore x_2 = -0.118$. Using Table 2.4 we calculate the required dimensions thus:

$$d_{a1} = mz_1 + 2m + 2x_1 m = (10 \times 12) + (2 \times 10) + (2 \times 0.118 \times 10) = 142.36 \text{ mm}$$

$$d_{a2} = mz_2 + 2m + 2x_2 m = (10 \times 96) + (2 \times 10) - (2 \times 0.118 \times 10) = 977.64 \text{ mm}$$

S-Gearing

In S-gearing the sum of the profile corrections of the two mating gears is not equal to zero. It is either positive or negative. However, the sum is positive in almost all cases in order to take

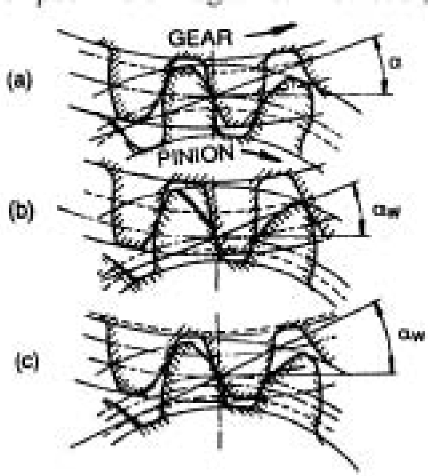


Fig. 2.29 Types of corrected gearing
(a) Normal gearing $x_1 = x_2 = 0$
Pressure angle $\alpha = 20°$
(b) S_0-Gearing $x_1 = -x_2 = 0.5$
Working pressure angle
$\alpha_r = \alpha = 20°$
(c) S-Gearing $x_1 = x_2 = 0.5$
Working pressure angle
$\alpha_r = 25.15°$, $\alpha = 20°$

Based on Maschinenelemente. Niemann, vol. II, 1965 edition, Fig. No. 37/2, p. 37. Springer Verlag, Heidelberg advantage of the beneficial effects of positive correction mentioned before. Usually, the sum is so divided that the pinion gets the bigger share of the positive correction. Sometimes an S-plus pinion may mate with a normal, uncorrected gear. It all depends on how the situation warrants it.

Figure **2.29** illustrates the different types of gearing. In an S-gearing, several parameters change. These changes are explained below along with the procedure for determination of the relevant values.

Determination of the Working Pressure Angle It should be specially noted that in an S-corrected gearing system, the pitch circles of the two meshing gears do not touch each other (Fig. **2.30**). They touch only in case of an uncorrected gearing system or in case of the S_0-corrected gearing system.

In Fig. **2.32** the two corrected gears are in mesh without backlash in an S-gearing system. The circles of the two gears which touch at the pitch point are called working circles. Had it been an uncorrected or an S_0-corrected system the working circles would have been identical to the pitch circles. The pressure line, which is tangent to both the base circles as before and which passes through the pitch point, now makes a new angle α_w instead of the standard pressure angle $\alpha = 20°$. This angle α_w has been termed as the "working pressure angle". We shall now find an expression for the "working pressure angle".

In this connection, it may be mentioned that some authors and gear manufacturers term the working circle and its diameter for a corrected gear as "corrected pitch circle" and "corrected pitch circle diameter". Such nomenclature is a misnomer. The pitch circle and the p.c.d. are theoretical parameters, and irrespective of whether a gear is corrected or not, the p.c.d. remains the same, being given by: $d = m\,z$, and since m and z are not changed in a corrected gear, the pitch circle and the p.c.d. remain unaltered.

Recalling Eq. *2.28* the circular tooth thickness on the pitch circles are given by

$$s_1 = \frac{\pi m}{2} + 2x_1\, m \tan\alpha$$

$$s_2 = \frac{\pi m}{2} + 2x_2\, m \tan\alpha$$

Since the two cylinders are now rolling at the working circles with radii r_{w1} and r_{w2}, we have

$$\frac{\omega_2}{\omega_1} = \frac{z_1}{z_2} = \frac{r_{w1}}{r_{w2}} \tag{2.31}$$

Now, since the two working circles are rolling without slipping, the circular distances covered on the circles by a common point in a particular time are the same (analogous to the circular pitch in a standard gear system). In other words, on these circles, the distance comprising one tooth thickness plus one tooth space of one gear in equal to one tooth space plus one tooth thickness of the other gear—both in circular measure. That is, the two teeth of two gears fit into the corresponding two spaces in the two gears. Using s for thickness and e for gap (as recommended before) we have

Circular pitch on the working circle $= p_\omega = s_1' + e_1' = s_2' + e_2' = \dfrac{2\pi r_{w1}}{z_1} = \dfrac{2\pi r_{w2}}{z_2}$

But

$$s_1' = e_2' \text{ and } s_2' = e_1' \quad \therefore\ s_1' + e_1' = s_1' + s_2' = \frac{2\pi r_{w1}}{z_1} = \frac{2\pi r_{w2}}{z_2}$$

Using Eq. 2.8 for the values of corrected tooth thicknesse, s we have

$$2r_{w1}\left[\frac{s_1}{2r_1} + \text{inv}\,\alpha - \text{inv}\,\alpha_w\right] + 2r_{w2}\left[\frac{s_2}{2r_2} + \text{inv}\,\alpha - \text{inv}\,\alpha_w\right] = \frac{2\pi r_{w1}}{z_1}$$

Dividing by $2r_{w1}$,

$$\left[\frac{s_1}{2r_1} + \text{inv}\,\alpha - \text{inv}\,\alpha_w\right] + \frac{r_{w2}}{r_{w1}}\left[\frac{s_2}{2r_2} + \text{inv}\,\alpha - \text{inv}\,\alpha_w\right] = \frac{\pi}{z_1}$$

Using the relation $2r_1 = m\,z_1$ and $2r_2 = m\,z_2$ and the Eq. *2.31*, after transposing, we have

$$\frac{s_1}{mz_1} + \frac{s_2}{mz_2} \times \frac{z_2}{z_1} = \frac{\pi}{z_1} + (\text{inv}\,\alpha_w - \text{inv}\,\alpha)\left(\frac{z_1 + z_2}{z_1}\right)$$

Multiplying by mz_1

$$s_1 + s_2 = \pi m + m\,(z_1 + z_2)\,(\text{inv}\,\alpha_w - \text{inv}\,\alpha)$$

Using the values of s_1 and s_2 from Eq. *2.28* and equating

$$\frac{\pi m}{2} + 2x_1\,m\,\tan\alpha + \frac{\pi m}{2} + 2x_2 m\,\tan\alpha = \pi m + m\,(z_1 + z_2)\,(\text{inv}\,\alpha_w - \text{inv}\,\alpha)$$

or

$$2\tan\alpha\,(x_1 + x_2)\,m + \pi m = \pi m + m\,(z_1 + z_2)\,(\text{inv}\,\alpha_w - \text{inv}\,\alpha)$$

or

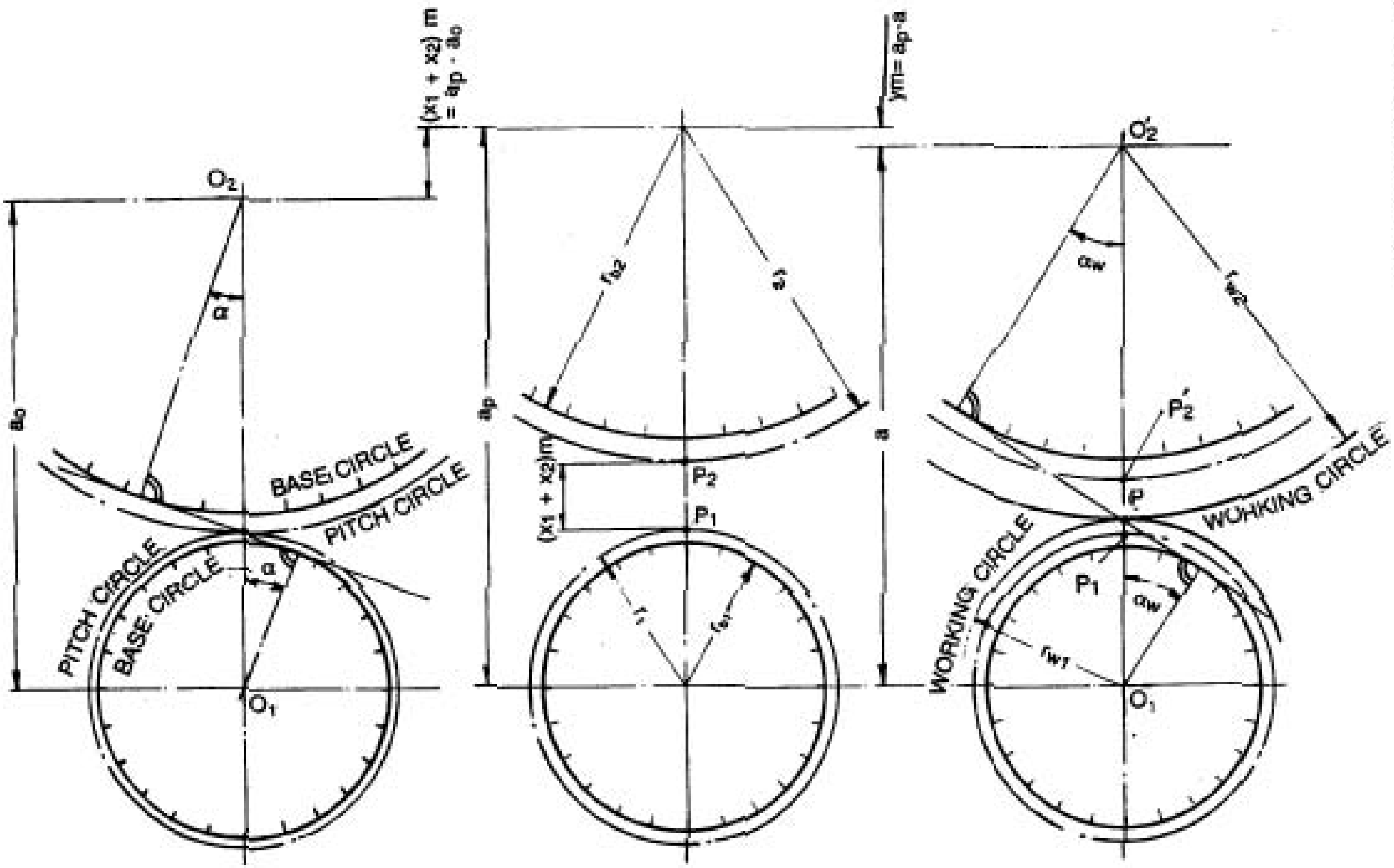
$$x_1 + x_2 = \frac{(z_1 + z_2)\,(\text{inv}\,\alpha_w - \text{inv}\,\alpha)}{2\tan\alpha} \qquad (2.32)$$

and

$$\text{inv}\,\alpha_w = \frac{2\tan\alpha\,(x_1 + x_2)}{z_1 + z_2} + \text{inv}\,\alpha \qquad (2.33)$$

Equations *2.32* and *2.33* are extremely important relations in gear design. Equation *2.32* gives us an expression for the amount of total correction factors the proper distribution of which among the pinion and the gear will be shown in Sec. *2.14*. Equation *2.33* gives us an expression for calculating the value of the working pressure angle α_w. Since the pressure line (or the line of action) is inclined to the horizontal at that angle, it is imperative to know the value of the working pressure angle as it enters into equations involving force analysis, power rating and other important calculations of a gear set.

Actual Centre Distance in an S-corrected Gearing In Fig. *2.30* the centre distance relations in an S-corrected gearing system along with other systems have been shown. Due to the enlargement of the component gears, the centre distance between the two gears is extended. It has been shown in Fig. 2.18 of Sec. *2.8* that because of this extended centre distance, a considerable amount of backlash is created between the mating gears. For proper running of the gear set, this backlash must be eliminated. Incidentally, this backlash is not to be confused with the backlash which is deliberately given and which is a result of the thinning of the tooth profiles caused by imparting tooth-thickness tolerances as well as a result of the centre distance tolerances. This aspect has been discussed in the Sections dealing with backlash and tolerance systems on gears (See Secs 2.8, 2.27 and *2.28*).



(a) UNCORRECTED GEAR

(b) CORRECTED GEAR-BEFORE PUSHING

(c) CORRECTED GEAR- AFTER PUSHING

Fig. 2.30 Centre distance relations in S-corrected gearing
Based on Grundzuege der Verzahnung, Thomas, 1957 edition, Fig. No. 11.8; p. 238, Carl Hanser Verlag, Munich.

To nullify the effect of the backlash created by the extended centre distance and to bring the profiles of the teeth of the mating gears in backlash-free contact, the gears are brought nearer to each other till the tooth surfaces are in contact and the gears are in mesh. This is known as "pushing" since the gears are "pushed" relative to each other. It is obvious that the effect of pushing is to shorten previous centre distance.

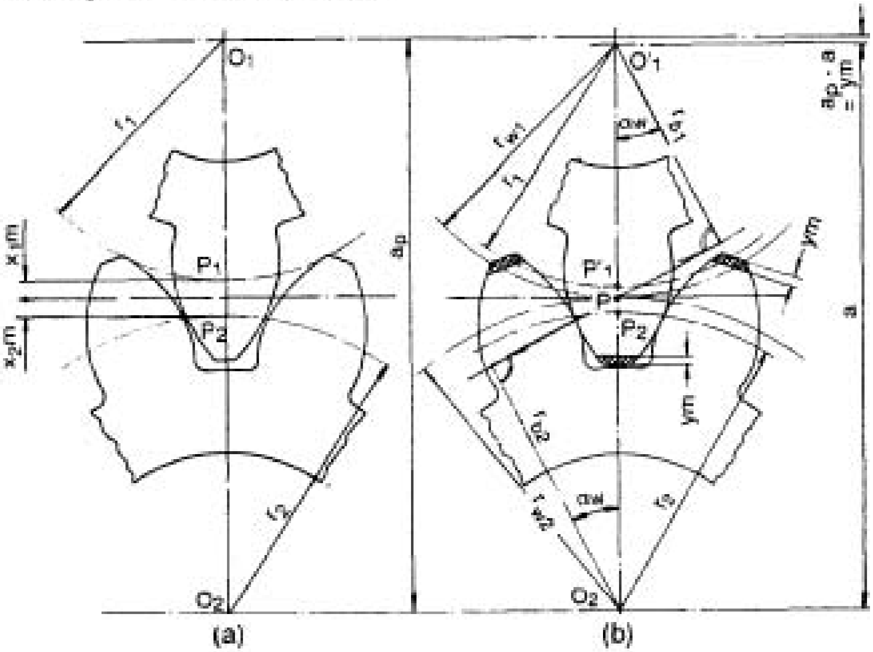


Fig. 2.31 Corrected toothing (S-gearing)
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 61, p. 69, VEB Fachbuchverlag, Leipzig

In Fig. 2.31, the positions of the teeth in an S-gearing have been shown before pushing in (a) and after pushing in (b). Along with these positions, the position of an uncorrected, standard gear has also been shown in Fig. 2.30 for comparison purposes. The following relations can be established from the geometry of the various centre distances from Fig. 2.30.

$$a_p = a_0 + (x_1 m + x_2 m) \qquad (2.34)$$

a_0 = Centre distance in an uncorrected system = $\dfrac{d_1 + d_2}{2} = r_1 + r_2$

Referring to Fig. 2.30 (c), the two cylinders after pushing are now rolling on their respective "working circles" having diameters d_{w1} and d_{w2} or radii r_{w1} and r_{w2} respectively. The common tangent to the base circles through the pitch point P now have a new pressure angle—the "working pressure angle" α_w.

Recalling Eq. 2.7 and suitably transposing, we have

$$2r_{w1} = 2r_1 \frac{\cos \alpha}{\cos \alpha_w} \text{ or } d_{w1} = d_1 \frac{\cos \alpha}{\cos \alpha_w}$$

Similarly

$$d_{w2} = d_2 \frac{\cos \alpha}{\cos \alpha_w}$$

The modified and final centre distance is given by

$$a = r_{w1} + r_{w2} = \frac{d_{w1} + d_{w2}}{2}$$

Now

$$d_{b1} = 2r_{b1} = d_{w1} \cos \alpha_w$$

or

$$d_{w1} = \frac{d_{b1}}{\cos \alpha_w} = \frac{d_1 \cos \alpha}{\cos \alpha_w}$$

Similarly

$$d_{w2} = \frac{d_{b2}}{\cos \alpha_w} = \frac{d_2 \cos \alpha}{\cos \alpha_w}$$

Hence

$$a = \frac{d_{w1} + d_{w2}}{2} = \frac{1}{2} \frac{\cos \alpha}{\cos \alpha_w} (d_1 + d_2)$$

or

$$a = \frac{d_1 + d_2}{2} \frac{\cos \alpha}{\cos \alpha_w} = m \frac{z_1 + z_2}{2} \frac{\cos \alpha}{\cos \alpha_w} = a_o \frac{\cos \alpha}{\cos \alpha_w} \quad (2.35)$$

Equation 2.35 is an extremely important and useful equation from which the final centre distance in an S-gearing can be calculated. The value of the working pressure angle α_w can be calculated from **Eq. *2.33.***

The shortened centre distance a is related to the centre distance before pushing a_p as follows

$$a_p - a = y\, m$$

Note that the above equation is identical to ***Eq. 2.36*** given later in the sub-section on "topping". The coefficient y is called the centre distance modification factor. As in the case of correction, it is customary to express the difference $a_p - a$ in terms of the module m as shown.

Referring to Fig. ***2.30***(c), the common tangent to the base circles is now the new line of action along which the contact takes place and the force is transmitted. It passes through the new pitch point P on the line of centres $O_1O'_2$. Point O'_2 is the new position of the centre of the gear after pushing so that the amount pushed is $a_p - a$. The two working circles with radii r_{w1} and r_{w2} touch each other at P. The working pressure angle is α_w. From geometry, we can establish the following relations

$$\frac{O_1P}{O'_2P} = \frac{r_{w1}}{r_{w2}} = \frac{r_{b1}}{\cos \alpha_w} / \frac{r_{b2}}{\cos \alpha_w} = \frac{r_{b1}}{r_{b2}} = \frac{r_{b1}}{\cos \alpha} \times \frac{\cos \alpha}{r_{b2}} = r_1 \times \frac{1}{r_2} = \frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$$

It, therefore, makes no difference in the angular velocity ratio when the centre distance becomes different from the standard centre distance in case of involute gearing. This is a confirmation of this particular property of the involute curve discussed earlier. This property is of great advantage and significance in gear drive.

Table 2.5 gives standardised values of the centre distance. These values are valid for closed units with module of gears $m \geq 0.5$ mm. The values given in the table are preferred value

Topping (as per **Eq. 2.36** given later in this section) is given by

$$ym = \frac{10}{2}(16 + 24) + 1.163 \times 10 - 210 = 1.63 \text{ mm}$$

Therefore, the required diameters are:

Pinion diameter, $d_{a1} = d_1 + 2m + 2x_1 m - 2ym = (10 \times 16) + (2 \times 10) + (2 \times 0.5652 \times 10) - 2 \times 1.63 = 188.044$ mm

Gear diameter, $d_{a2} = d_2 + 2m + 2x_2 m - 2ym = (10 \times 24) + (2 \times 10) + (2 \times 0.5978 \times 10) - 2 \times 1.63 = 268.696$ mm

Addendum Modification or "topping" It has been explained before that in an extended centre distance system, as in the case of an S-gearing system, the component gears are "pushed" relative to each other to bring the pinion and the gear in a backlash-free contact. The centre distance is shortened due to pushing. Equations for the normal centre distance for an uncorrected gearing as well as for S_0-gearing a_0, that for the S-gearing before pushing a_p, and that after pushing a, have already been given at appropriate places.

Now, one effect of pushing is that the clearance between the tip of one gear and the root of the mating gear becomes less.

Figure **2.31** (a) represents the gear positions before pushing, showing clearly that the backlash exists between the mating teeth. Figure **2.31**(b) depicts the position after pushing has been implemented, thereby eliminating backlash as in the case of Fig. **2.32.** The new centre distance a is less than a_p, as explained earlier. It can be easily seen from Fig. **2.31** that the tips of the teeth have entered the corresponding tooth spaces so much that there is not enough clearance between the tips and the bottom lands. Consequently, normal running of the gear set will be impaired and this may even lead to seizure, unless some remedial measure is taken to avoid such a situation. Moreover, the top of the gear tooth may mate with the non-involute portion at the root region of the tooth of the other gear of the pair, leading to interference and allied problems.

One obvious solution is to "cut-off" the offending tooth tips. This is what is known as addendum modification or topping. It must be emphasised, however, that actually no cutting-off is done after the gear has been made. The blank itself is reduced prior to teeth cutting as per the calculated value determined before. The relevant formulae are given later. Also, in the gear cutting machine, the feed is given accordingly so that a "topped" tooth, (which is obviously of less height than the standard tooth of standard whole depth), emerges. One side effect of topping is that the top land of the tooth is also increased. The topped portion has been shown as cross-hatched in Fig. **2.31** (b).

To achieve the standard clearance c, the tooth to be topped is reduced by an amount given by

$$ym = a_p - a = [a_0 + (x_1 + x_2)\,m] - a \text{ (mm)} \tag{2.36}$$

As in the case of correction, the amount of topping is expressed in terms of module, the unit of measurement being millimetre. The coefficient of topping y is dimensionless, as in the case of correction factor x.

The standard clearance now is given by

$$c = a - \left[\frac{d_{a1} + d_{a2}}{2} + ym - h\right] \text{ (mm)} \tag{2.37}$$

We can now arrive at the value of the outside diameter of the blank after topping numbers, given in the preferred value series. If from operational or design standpoint the calculated values of CD does not tally with the value given in the table, it is not imperative to stick to the standardised value. This can deviate from the standard value and can conform to the calculated value.

Table 2.5 Standard values of centre distance
Based on Zahnraeder, Zirpke, 11th edition, 1980, table no. 2, p. 73.
VEB Fachbuchverlag, Leipzig

Series	*Centre Distance (mm)*																
1			63				100				160				250		
	50		63		80		100		125		160		200		250		315
3	50	56	63	71	80	90	100	112	125	140	160	180	200	224	250	280	315

1		400				630				1000				1600		
2		400		500		630		800		1000		1250		1600		2000
3	355	400	450	500	560	630	710	800	900	1000	1120	1250	1400	1600	1800	2000

If the design conditions permit, the order of preference for the selection of the value of centre distance should be: Series 1 the first choice, followed by Series 2, and lastly Series 3.

Example 2.3 *Given:* $m = 10 \alpha = 20°$, $i = 3:2$, a = 210
m = 10, a = 20' To find the diameters of the appropriate pair of gears. The values of i and a are to be kept exact.

Solution: Normal gearing gives the following results

$$210 = \frac{1}{2} m (z_1 + z_2) = \frac{10}{2}(z_1 + z_2) \text{ or } z_1 + z_2 = 42. \text{ Also, } i = \frac{z_2}{z_1} = \frac{3}{2}$$

Solving the above two equations does not lead to any whole numbers for z_1 and z_2. The nearest value $z_1 + z_2 = 40$ gives $z_1 = 16$ and $z_2 = 24$, satisfying the condition that $i = 24/16 = 3/2$. Since the centre distance is to be kept exact, the solution lies in using an S-gearing.

Using Eq. 2.34, we have

$$a = 210 = 10 \frac{16 + 24}{2} \frac{\cos 20'}{\cos \alpha_w}, \text{ whence } a, = 26' 30'$$

From Eq. 2.33,

$$\text{inv } 26' 30' = \frac{2 \tan 20' (x_1 + x_2)}{16 + 24} + \text{inv } 20'$$

From Appendix H, inv 26° 30 = 0.036069, and inv 20" = 0.014904

Inserting these values and solving, we get $x_1 + x_2 = 1.163$

Using Eq. 2.41 given in the Sec. 2.14, we get

$$x_1 = \frac{1.163.}{1.5 + 1} + 0.5 \frac{1.5 - 1}{1.5 + 1} = 0.5652$$

$$\therefore \quad x_2 = 1.163 - 0.5652 = 0.5978$$

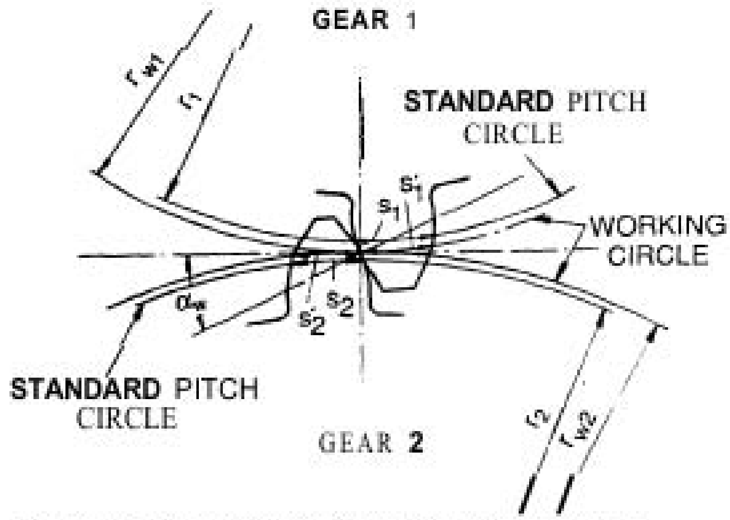


Fig. 2.32 Determination of working pressure angle

$$d_{a1} = [d_1 + 2m + 2x_1 m] - 2ym$$

$$\therefore d_{a1} = [d_1 + 2m + 2x_1 m] - 2\{a_0 + (x_1 + x_2)m - a\}$$

$$= d_1 + 2m + 2x_1 m - 2\frac{d_1 + d_2}{2} - 2x_1 m - 2x_2 m + 2a$$

$$= 2(a + m - x_2 m) - d_2 \tag{2.38}$$

Similarly

$$d_{a2} = 2(a + m - x_1 m) - d_1 \tag{2.39}$$

Sometimes topping may not be considered as necessary because the clearance without topping may be sufficient for certain service conditions.

In that case, the tip diameters are calculated as per the formulae given in case of uncorrected or S_0-gearing.

The important formulae connected with S-gearing have been summarised in Table 2.6.

Table 2.6 **Dimensions for S gearing**

Description	*Pinion*	*Gear*
Number of teeth	z_1	z_2
Pitch circle diameter	$d_1 = z_1 m$	$d_2 = z_2 m$
Tip circle diameter (with topping)	$d_{a1} = 2(a + m - x_2 m) - d_2$	$d_{a2} = 2(a + m - x_1 m) - d_1$
Root circle diameter	$d_{f1} = d_1 - 2(1.25 - x_1)m$	$d_{f2} = d_2 - 2(1.25 - x_2)m$
Tooth thickness on pitch circle	$s_1 = \frac{p}{2} + 2x_1 m \tan\alpha$	$s_2 = \frac{p}{2} + 2x_2 m \tan\alpha$
Topping	$ym = a_0 + (x_1 + x_2)m - a$	

(contd.)

Table 2.6 (Contd.)

Standard centre distance	$a_o = m \frac{z_1 + z_2}{2}$
Actual centre distance (after pushing)	$a = a_o \frac{\cos \alpha}{\cos \alpha_w} = m \frac{z_1 + z_2}{2} \frac{\cos \alpha}{\cos \alpha_w}$
Working pressure angle	$\text{inv}\, \alpha_w = 2 \frac{x_1 + x_2}{z_1 + z_2} \tan \alpha + \text{inv}\, \alpha$
Top clearance	$c = a - \left[\frac{d_{a1} + d_{a2}}{2} + ym - h \right]$
Sum of profile correction factors	$x_1 + x_2 = (z_1 + z_2) \frac{\text{inv}\, \alpha_w - \text{inv}\, \alpha}{2 \tan \alpha}$

2.14 Distribution of Correction Factors

In the previous sections on corrected toothing of gears, the consequences effected by correction on various aspects of toothing have been discussed. It has been emphasised that profile correction of gear teeth is undertaken to avoid undercutting in case of a small number of gear teeth, to suit a pre-determined centre distance when necessary, and above all to increase the load carrying capacity of the gear teeth by providing positive correction whereby the root thickness of the teeth is considerably increased. Better operational properties are also attained by gear correction when compared to those of ordinary, standard gears.

In previous sections, we have also arrived at expressions for corrected toothing systems where the correction appears as total correction on the pinion and the gear. In this section, we shall see how the sum of correction factors can be divided among the two components of the mating pair and the relevant methods thereof. The methods are applicable only in case of spur and helical gears.

If correction is carried out only to avoid undercutting, the formulae given in Sec. 2.11 will suffice. If, however, other factors as stated above are involved which necessitate profile shifting, the total correction should be properly distributed between the pinion and the gear. For convenience some equations relevant to correction are repeated here after necessary transposing

$$x_1 + x_2 = \frac{(z_1 + z_2)(\text{inv}\, \alpha_{wt} - \text{inv}\, \alpha_t)}{2 \tan \alpha}$$

$$\cos \alpha_{wt} = \frac{m_t (z_1 + z_2)}{2a} \cos \alpha_t$$

Here, α_{wt} is the working pressure angle in the transverse plane, α_t is the standard pressure angle in the transverse plane and α is the standard pressure angle in the normal plane—all in case of helical gears. In case of spur gears $\alpha_{wt} = \alpha_w$ and $\alpha_t = \alpha$, $m_t = m_n/\cos \beta$ for helical gears with helix angle $= \beta$; for spur gears $m_t = m_n = m$ as $\beta = 0°$. Having ascertained the total correction factors $x_1 + x_2$ by using the relevant equations, the individual correction factors can be determined by using the following formulae. The underlying idea is that both the pinion and the gear are subjected to equal maximum stresses

$$x_1 = \frac{x_1 + x_2}{i + 1} + \frac{i - 1}{i + 1 + 0.4 z_2} \quad \text{for equal sliding velocity} \tag{2.40}$$

$$x_1 = \frac{x_1 + x_2}{i + 1} + 0.5 \frac{i - 1}{i + 1} \quad \text{for equal root stress} \tag{2.41}$$

$$x_1 \approx \frac{x_1 + x_2}{i + 1} \times \frac{z_1 + 12}{z_1 + 2} + \frac{8}{z_1 + 2} \text{ for equal contact pressure at } i \geq 2 \quad (2.42)$$

Besides the above method, distribution of gear correction factors can also be done as per the steps laid down in IS: 3756 and DIN 3992 which are essentially the same. These standards are applicable for spur and helical gears belonging to the standard basic rack and having number of teeth 10 or more, Figure 2.33 gives distribution of correction factors for reduction gear drives and Fig. 2.34 for step-up gear drives. The undermentioned basic requirements must be satisfied after correction.

(a) A minimum contact ratio of 1:1 should be maintained.

(b) The tip clearance should have a value of at least 0.1 m. Normally, topping is done in case of corrected gears as explained in the section on correction dealing with that aspect so that the tip clearance is around **0.25** *m*.

(c) The top land preferably should not be below **0.4** *m* for hardened gears in normal applications. The value may be brought down to **0.25** *m* in certain cases.

(d) There should be no interference and undercutting.

Figures 2.33 and 2.34 can be used to divide the sum of correction factors equitably as described below. The middle portion of each figure is meant for helical gears and is used for obtaining the equivalent or virtual number of spur gear teeth z_{v1} and z_{v2}. The equivalent number of teeth $z_v = z/\cos^3 \beta$ for helical gears. The equivalent spur gear concept for replacing helical gear for calculation purposes has been explained in the chapter on helical gears. Distribution of correction factors for helical gears is also dealt with in this section along with that for spur gears.

Knowing the sum of number of teeth, i.e. $z_1 + z_2$, the individual correction factors x_1 and x_2 can be ascertained by the procedures explained below and by the examples which follow. The method is applicable for gears with number of teeth up to 150. When the number exceeds 150, a somewhat different method is adopted, but since profile correction in case of gears with a large number of teeth does not have a significant influence on the load carrying capacity of the gear teeth, the method described in this section is adequate for normal applications. The basic idea on which the method is based is to have equalization of load carrying capacity at the roots of the pinion and the gear teeth, balanced conditions, advantageous sliding conditions, etc.

In Figs 2.33 and 2.34 the respective characteristic lines *L*1 to *L*17 and *S*1 to *S*13 represent a compromise between the different requirements and various factors involved. Lines *P*1 to *P*9 in each figure serve the purpose of identification according to tooth properties. These are: (i) zone between *P*3 and P6 for normal applications; (ii) between P6 and *P*9 for higher bending and wear strength; (iii) between *P*1 and P3 for high contact ratio, and (iv) zones marked for special cases—the upper one meant for toothing with greater pressure angle and smaller contact ratio while the lower one designated for toothing with smaller pressure angle and larger contact ratio. The method is well illustrated by the undermentioned examples.

Example 2.4 *Given:* $z_1 = 32, z_2 = 64, m = 3$

To design control gears with high contact ratio for a reduction gear unit. Gears are to be spur gears.

Solution: Line *P*2 is selected from Fig. 2.33 for high contact ratio. Since only spur gears are to be considered the middle portion of the figure is not necessary

$$z_1 + z_2 = 32 + 64 = 96$$

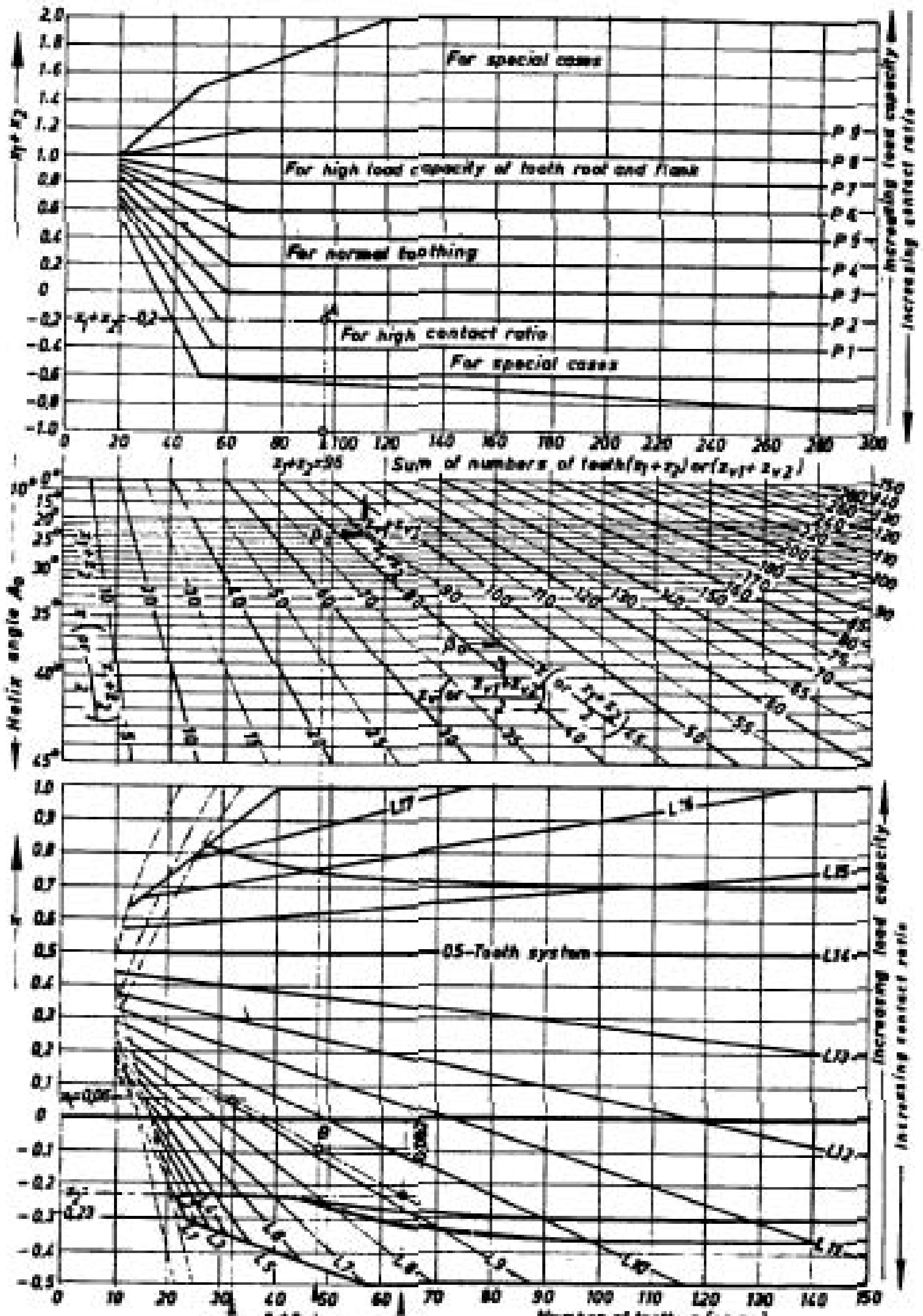


Fig. 2.33 Distribution of correction factors for reduction gear drive

Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 230 & 231 pp 398 & 399 VEB Fachbuchverlag, Leipzig.

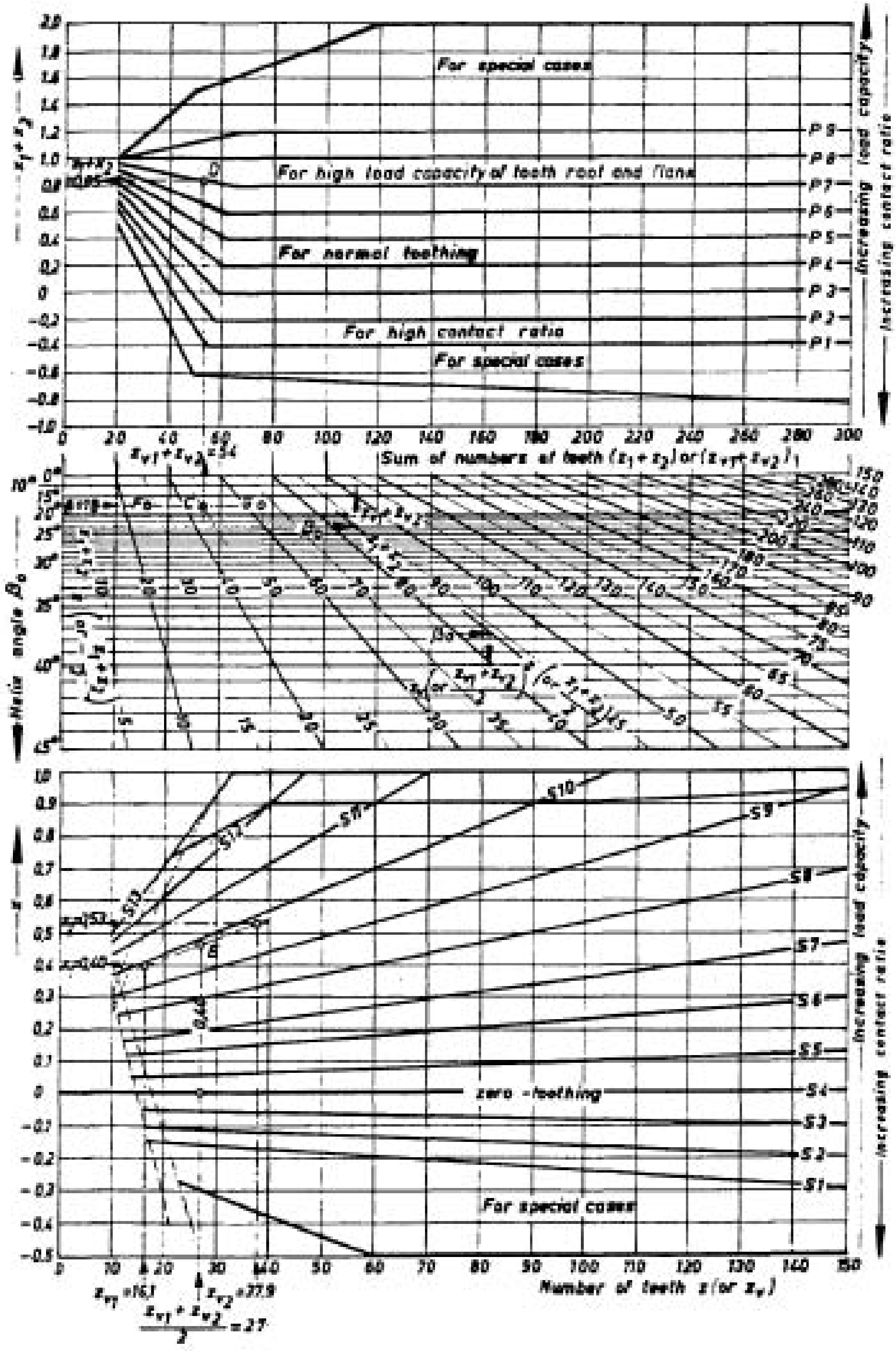


Fig. 2.34 Distribution of correction factors for step-up gear drive

Based on Zahnraeder, Zirpke. 11th edition, 1980, Fig. No. 230 & 232, pp 398 & 400. VEB Fachbuchverlag, Leipzig.

A vertical line is drawn from the horizontal axis of the uppermost diagram at the point denoting 96 till it cuts the horizontal line P2 (shown at point *A* in the figure). This corresponds to $x_1 + x_2 = -0.20$.

Using the two relevant equations given before at the beginning of this section, we get

$$\alpha_{tw} = 19° \ 19' 10''$$

and

$$a = 143.39 \text{ mm}$$

It is desirable to round off the value of the centre distance. Hence

$$a = 143.5 \text{ mm (taken)}$$

Recalculation yields the following values

$$\alpha'_{tw} = 19° 26' 39''$$

and

$$x_1 + x_2 = -0.164$$

A vertical line is then dropped from point A till it meets point B corresponding to the value

$$\frac{x + x_2}{2} = \frac{-0.164}{2} = -0.082$$

One can also raise a vertical from the point $z = (z_1 + z_2)/2 = 48$ to locate point B. This vertical is an extension of the line AB.

A straight line is drawn from point B which matches between the adjacent lines $L9$ and $L10$. Verticals from points: $z_1 = 32$ and $z_2 = 64$ cut this straight line at two points. Horizontals from these two points on the vertical axis give the values

$$x_1 = +0.06 \text{ and } x_2 = -0.23$$

These are approximate values. Hence, keeping $x_1 = +0.06$, the final values are

$$x_1 = +0.06, \text{ and } x_2 = -0.164 - 0.06 = -0.224$$

Example 2.5 *Given:* $z_1 = 14, z_2 = 33$, normal module $m_n = 4.5$, helix angle $\beta = 18°$. To design a set of gears to transmit high load in a step-up gear system.

Solution: This is solved in a similar manner using Fig. 2.34. First, line $P7$ is selected. Line from $\beta = 18°$ meets the line for $z_1 + z_2 = 47$ at C. Vertical from C passes through a value corresponding to the sum of the equivalent number of teeth $z_{v1} + z_{v2} = 54$. Point D on $P7$ is located; $x_1 + x_2 = 0.85$. After finding the working pressure angle and the centre distance as before, the centre distance is rounded off and recalculation is made yielding a value of $x_1 + x_2 = 0.9357$. Point E, below C and D, denotes

$$z_v = \frac{z_{v1} + z_{v2}}{2} = \frac{54}{2} = 27 \text{ and } x = \frac{0.9357}{2} = 0.468$$

Verticals are drawn from $F(\beta = 18°, z_1 = 14)$ and $G(\beta = 18°, z_2 = 33)$ to determine the final values $x_1 = +0.40$ and $x_2 = 0.53$ after drawing line through E in a similar manner as in Example 2.4 using Fig. 2.33.

2.15 Internal Spur Gears

Jn an internal gear, the teeth are cut on the inside of a ring. It is also known as an annular gear since the rim on which the gear is produced is in the shape of an annulus. The internal gears have many applications, the most frequent being the planetary gear systems.

The tooth space on an internal gear more or less corresponds to the tooth of the external gear with which it mates, and the tooth of an internal gear corresponds to the tooth gap of its mating external gear. The tooth and the tooth-space of an internal gear can be proportioned like a standard gear with the addendum and the dedendum in reversed positions, but this is not generally done in order to alleviate the interference effects and also to improve tooth action. Normally, both the internal diameters of the internal gear and the outside diameter of the mating pinion are made slightly larger than the size calculated according to the conventional tooth proportions. Figure. 2.35 shows the internal gear parameters. Usually, the pinion is the driver and the internal gear is the driven one.

The undermentioned characteristics of the internal gears are to be noted vis-a-vis those of the external gears.

1. Since the centre distance is small, a reductian gear unit having internal gear arrangement is more compact for any particular reduction ratio.

2. The tooth forms of internal gears are stronger than those of the corresponding external gears.

3. The action of tooth during operation is much smoother than that of the conventional external gear drives.

4. The sense of direction of tooth travel is the same for the two members comprising an internal set. This results in reduced sliding, low wear and greater efficiency.

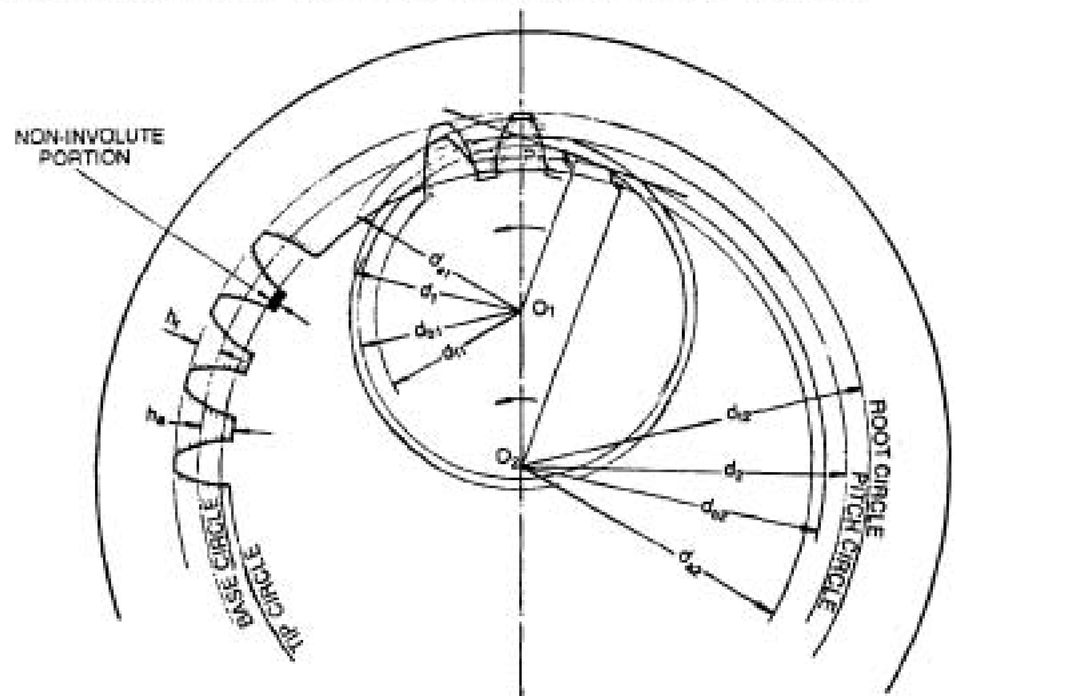


Fig. 2.35 Parameters of internal gear drive

5. The tooth profile of an internal gear more or less wraps around the tooth profile of the external tooth of the pinion. This aspect has several advantages, namely, the contact ratio is greater, load transfer from tooth to tooth is gradual, running is quieter, and most important, since the actual surface in contact is increased because of this wrapping action between the two curved surfaces, a greater load can be taken by the system for the same surface stress and life, or conversely, the surface stress is much less for the same corresponding load. (In this connection see Sec. 2.23 on contact stress.)

6. Due to longer line of action, comparatively more teeth are in contact simultaneously. Consequently, the load-intensity on any one tooth is correspondingly decreased, thus increasing the life of gears.

7. As a consequence of decreased load on one tooth, the impact force at the beginning of tooth engagement is much reduced, resulting in quieter running.

8. Since the shape of an internal gear drive forms a sort of natural guard over the meshing teeth, it is advantageous for some kinds of machines.

In an internal gear drive, two types of interference may take place. First is the usual kind, that is, the mating of involute profile of one member with the non-involute portion of the other member.

The tip circle of the (driven) internal gear does not cut the line of action as the top of the tooth extends inside the base circle where obviously there is no involute profile. Consequently, the part inside the base circle will cause interference which can be avoided by modifying the teeth so that no contact occurs till the "interference point" is reached. The modified internal gear is shown in Fig. 2.36.

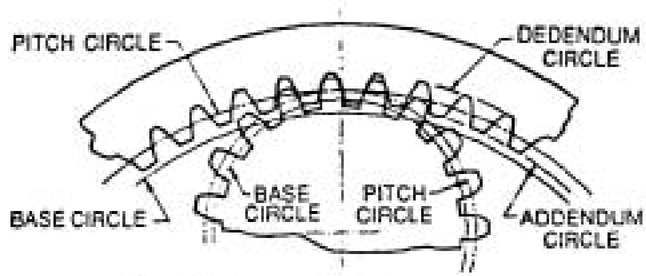


Fig. 2.36 Modified internal gear

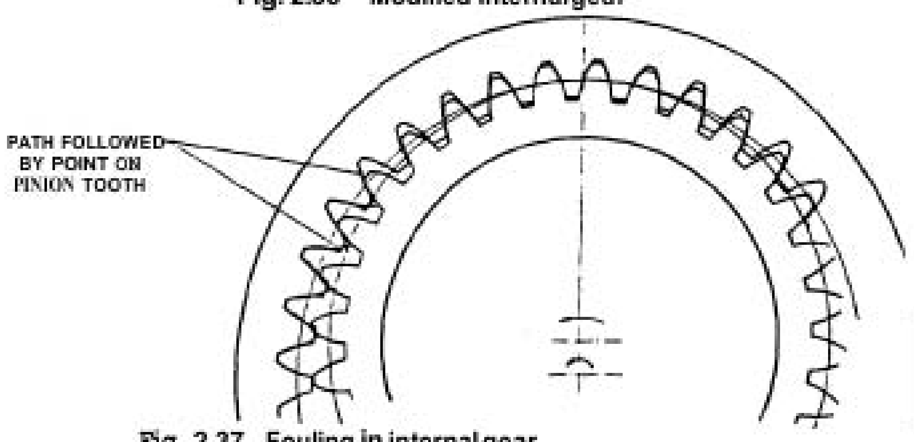


Fig. 2.37 Fouling in internal gear

Based on Grundzuegeder Vezrahnung. Thomas, 1957 edition. Fig. No. 11.7, p. 233. Carl Hanser Verlag. Munich

The second type of interference is called "fouling". This type of interference is peculiar to the internal gear system only. The principal limiting factor underlying the design of an internal gear system is the difference between the numbers of teeth z_2 and z_1 of the mating members. When this difference is too small and the detrimental effect is not taken care of by suitable corrective measures, fouling or tip interference may occur. That is, fouling may take place between the tips of the pinion and the internal gear in the region where their respective tip circles cross as shown in Fig. 2.37.

The tooth profile of an internal gear is concave instead of convex as in the case of external gears. Because of this shape, the fouling type of interference occurs between the inactive profiles as the teeth of the mating members go in and out of mesh. Fouling occurs when the pinion is large compared to the internal gear.

Fouling can be avoided if a minimum difference in tooth numbers between the gear and the pinion is maintained. This is 12 teeth for 14.5° full-depth system, 10 for 20" full-depth system and 8 for **20"** stub tooth system. **As** a general rule, the pinion diameter should not be larger than about two-thirds of the pitch diameter of the internal gear for 20' full-depth teeth. The following general rule applicable for gears at standard centre distance is also used by some designers.

$$(z_2 - z_1) = 10 + \frac{z_1}{5}$$

In each case, however, modifications in the teeth profiles may be necessary. When such modifications are undesirable for any reason, then the difference in tooth numbers has to be greater. Another method of avoiding fouling is to generate the internal gear with a Fellows cutter having two teeth less than the number of teeth of the gear. This automatically relieves the tips **of** the internal gear teeth to prevent fouling.

For internal gear drives, the standard 20" involute full-depth form is mostly used. In principle design technique of an internal gear is the same as used for external gears. The basic rack forms are the same in both the cases. Measures, however, are to be taken while ascertaining the dimensional parameters of an internal gear drive system in order to ensure effective tooth action by avoiding various types of interferences.

The processes available for manufacturing internal gear teeth are limited; e.g., casting, shaping or milling with formed tooth or cutters. An internal gear cannot be hobbed. However, for accurate production of internal gear teeth, only the generation method using a pinion-type or Fellows cutter is employed. But this process too has its limitations as regards the cutter size. Internal gears of small size can be produced by broaching.

To avoid interference and to ensure free and smooth movement of incoming and outgoing teeth of the meshing pair, two methods are generally employed. The first method involves the enlargement of internal (addendum circle or tip circle) diameter of the internal gear as well as of the tip circle diameter of the mating external pinion. For this, the following formula may be **used.**

Internal diameter of the internal gear $d_{a2} = mz_2 - 1.2\,m$

Tip diameter of the external pinion $d_{a1} = mz_1 + 2.5\,m$

Note that in both the cases, the diameters are somewhat larger than the normal values given by

$$d_{a1} = d_1 + 2m = mz_1 + 2m$$

and

$$d_{a2} = d_2 - 2m = mz_2 - 2m$$

The second method is to reduce the addendum of the internal gear teeth by addendum modification or topping. It has been stated before that in 20° full-depth toothing system the fouling type of interference can be avoided if the minimum difference in numbers of teeth of the gear and the pinion is maintained as $z_2 - z_1 = 10$. However, it is a good design practice to allow some topping so that the designer is sure of doing away with all types of interference. The guidelines given in Table 2.7 can be used for cases where $z_2 - z_1 = 10$. If $z_2 - z_1$ is less than 10, the addendum of the internal gear teeth are to be made still smaller. In that case, the relevant values are to be determined by making drawings of actual teeth.

Table 2.7 Addendum values of internal gears where $z_2 - z_1 = 10$

$z_2 =$	20-22	23-26	27-31	32-39	40-51	52-74	75-130	over 130
$h_a =$	0.60 m	0.65 m	0.70 m	0.75 m	0.80 m	0.85 m	0.90 m	0.95 m

In case $z_2 - z_1$ is greater than 10, no interference is normally encountered and hence no topping is necessary.

The relationship between the different gear parameters for an internal drive are summarised in Tables 2.8, 2.9 and 2.10. These are standard formulae and do not take into account the special considerations, e.g. enlargement of gears, special amount of topping, which are connected with an internal drive, and which have already been discussed earlier in this section. Only the standard type of topping to provide the standard top clearance in case of corrected gears has been considered and included. The reader should compare the parameters of internal gears with those of external drives.

Table 2.8 Dimensions of internal spur gear drive for uncorrected gears

Description	*Pinion*	*Internal gear*
Number of teeth	z_1	z_2
Pitch circle diameter	$d_1 = z_1 m$	$d_2 = z_2 m$
Tip circle diameter	$d_{a1} = d_1 + 2m$	$d_{a2} = d_2 - 2m$
Root circle diameter	$d_{f1} = d_1 - 2 \times 1.25m$	$d_{f2} = d_2 + 2 \times 1.25\ m$
Centre distance	$a = \dfrac{d_2 - d_1}{2} = m\,\dfrac{z_2 - z_1}{2}$	
Tooth thickness on the pitch circle	$s = \dfrac{p}{2} = \dfrac{\pi m}{2}$	

In an S_0-corrected internal drive, both the pinion and the internal gear receive the same amount of positive correction. That is $x_2 = x_1$. This is unlike an external S_0-corrected drive where $x_2 = -x_1$.

Table 2.9 Dimensions of internal spur gear drive for S_0-corrected gears

Pitch circle diameter	$d_1 = z_1 m$	$d_2 = z_2 m$
Tip circle diameter	$d_{a1} = d_1 + 2m + 2x_1 m$	$d_{a2} = d_2 - 2m + 2x_1 m$
Root circle diameter	$d_{f1} = d_1 - 2(1.25 - x_1)m$	$d_{f2} = d_2 + 2(1.25 + x_1)m$
Centre distance	$a = \frac{d_2 - d_1}{2} = m\frac{z_2 - z_1}{2}$	
Tooth thickness on the pitch circle	$s_1 = \frac{p}{2} + 2x_1 m \tan\alpha$	$s_2 = \frac{p}{2} - 2x_1 m \tan\alpha$

Table 2.10 Dimensions of internal spur gear drive for S_{plus}-corrected gears

Pitch circle diameter	$d_1 = z_1 m$	$d_2 = z_2 m$
Tip circle diameter	$d_{a1} = d_1 - 2(a - m - x_2 m)$	$d_{a2} = d_1 + 2(a - m - x_1 m)$
Root circle diameter	$d_{f1} = d_1 - 2(1.25 - x_1)m$	$d_{f2} = d_2 + 2(1.25 + x_2)m$
Centre distance	$a = m\frac{z_2 - z_1}{2}\frac{\cos a}{\cos\alpha_w}$	
Working pressure angle	$\text{inv}\,\alpha_w = 2\frac{x_2 - x_1}{z_2 - z_1}\tan a + \text{inv}\,a$	
Tooth thickness on the pitch circle	$s_1 = \frac{p}{2} + 2x_1 m \tan a$	$s_2 = \frac{p}{2} - 2x_2 m \tan a$
Top clearance	$c = \frac{d_{f2} - d_{a1}}{2} - a = \frac{d_{a2} - d_{f1}}{2} - a$	

2.16 Practical Design Criteria for Gear Dimensions

In solving any gear design problem, the usual practice for the designer is to make a rough, preliminary draft design before proceeding to finalise the design data consisting of the finer aspects of the art of gear design. At the preliminary stage, the design involves such considerations as the types and magnitudes of stresses which the gear is likely to be subjected to, an estimate of the approximate gear size keeping the space and weight restrictions in mind, a rough idea about the power rating to meet the requirements in case it is not initially known and other operational parameters.

Though no dogmatic rules can be given which will enable the designer to make a perfect job, some generalisations can be drawn and guidelines given to determine the practical dimensional and other gear data. In Chap. 1 we have discussed about the gear drive requirements as to the type of gearing, speed, axis orientation and the number of stages. In this section design criteria for gear dimensions and allied aspects will be dealt with.

The geometrical proportions of a gear will depend on several factors. Normally, the following parameters are given from which the designer has to proceed in his task.

1. The output power or the torque required.
2. The transmission ratio.
3. The input speed or the required output speed.
4. The anticipated life of the gear set.
5. The duty or the service conditions.

6. The nature of load which the gear set is expected to encounter during its operational life
7. Other special conditions, such as, space requirements, pre-specified centre distance and price limitations.

We will now take up the individual data of gear parameters for proper design.

Material

Selection of proper material has already been discussed in detail in Sec. 1.8 and at other places in this book. Besides strength considerations, the choice of material mainly depends on the requirements of space and weight and the overall price of the gear drive. Cast iron is good enough for ordinary purposes. Steel and cast steel offer better strength. Pinion of synthetic material is quiet running and vibration damping. Phosphor bronze and similar materials are used to alleviate the loss due to sliding.

Number of Teeth

The number of teeth of the pinion and the gear are to be so chosen that a minimum value of 1.1 for contact ratio is assured. For fast moving set, it should be greater than 1.5. The transmission ratio should not preferably be a whole number to ensure hunting tooth action. Guidelines for the selection of minimum number of teeth of pinion are given in Table 2.11. When the number of teeth is below the minimum specified in Sec. 2.10, then obviously the pinion is to be positively corrected.

Table 2.11 Minimum number of pinion teeth

Type of service	*Minimum number of teeth*
Heavy duty and high speed	16
Medium speed	12
Light duty and low speed	10

For the sum (or difference) of number of teeth of pinion and gear, the following rule holds good

For external gearing $z_1 + z_2 \geq 24$, and for internal gearing $z_2 - z_1 \geq 10$

Transmission Ratio

If the transmission ratio is high, a multi-stage gear set is used to avoid unnecessarily big gears. In general, the transmission ratio per stage is given by

$i \leq 7$ for general purpose drive
≈ 10 for maximum value in special cases
≈ 4 for maximum value for change-gear sets

If the given transmission ratio cannot be strictly adhered to for some technical ground, the following deviations are allowed:

For all types of drives excluding worm-drive when $i \leq 250$, the allowable deviation $= \pm 3\%$

For all types of drives including worm-drive when $i > 250$, the allowable deviation $= \pm 5\%$

Pitch Circle Diameter

The final value of pitch circle diameter will depend on many and different factors. Only broad outlines can be given for initial calculations. These are

$d_1 \geq 1.5 \times$ shaft diameter in case of a pinion shaft, i.e. when the pinion and the shaft are made from one stock

$\geq 2 \times$ shaft diameter in case of key-fitted or shrink-fitted pinion

Neglecting the effect of bending and considering torsion only, we can arrive at a rough, initial value of d_1, using the well-known formula from mechanics: torque $T = \frac{\pi}{16} d^3 \tau_p$, and inserting a conservative value of the allowable torsional stress $\tau_p \approx 12 \text{N/mm}^2$

$$\text{Shaft dia} = d(\text{mm}) = 160 \sqrt[3]{\frac{P_1\ (\text{kW})}{n_1 (\text{rpm})}} \tag{2.43}$$

Width of Tooth

In selecting the width, the type and the quality of bearing are the deciding factors, among other considerations. The following guiding values can be given:

Straddle mounted, i.e. bearing on both sides

$$b/d_1 \leq 1.2 \tag{2.44}$$

Overhung, i.e. bearing on one side only

$$b/d_1 \leq 0.75 \tag{2.45}$$

Putting $b_{max} = \lambda m$, Table 2.12 shows the relation between the tooth width and the module.

Table 2.12 Factor λ for different service conditions

Tooth surface condition	*Type of bearing*	λ
Cast clean and smooth	Bearing fitted on steel constructions	10
Machined smooth or ground	Bearing fitted on steel constructions, e.g. beams etc.	15
	Bearing with overhung pinion	15
	Bearing fitted in gear box casings and similar cases	25
	Anti-friction bearings or journal bearings fitted on rigid base and using shafts of sufficient stiffness	30

Normally, the width of pinion is made 3 to 4 mm greater than that of the gear to ensure complete engagement during service, so that the effects of straying of gears on the shafts and misalignment are averted.

Module

Selection of proper module will, of course, depend on the strength considerations. Small modules permit noiseless running. The limiting factors comprise strength of tooth root, quality of bearing and manufacturing constraints on the lower side, and the PCD and number of teeth on the higher side.

The relevant relations are:

$$m_{min} = \frac{b}{\lambda} \tag{2.46}$$

$$m_{min} = \frac{\text{PCD of pinion}}{\text{Minimum number of pinion teeth}} \tag{2.47}$$

2.17 Types of Gear-body Construction

Depending upon the size, material, type of application, cost and other deciding factors, such as nticipated stress conditions and operational parameters, the gears may have different constructional designs and shapes. Gear blanks may be machined from a solid raw-stock or may be manufactured by casting, forging and fabricating by welded construction. Often the designer has to reckon with such considerations as machining facility and availability of heat-treatment measures.

Small pinions are often made integral with the shaft. In such a design, which is normally referred to as a pinion shaft, the key is dispensed with and the provision of an axial-locating device is also eliminated.

Gears are also made by drop-forging and die-casting. Steel gears with diameters up to 500 mm are usually made full without recess. Large gears are generally of cast construction. Very large and wide gears are usually of two-walled variety and are either of cast or welded construction. For saving costly materials, composite designs of gears are sometimes resorted to. In such designs, the gear rim of quality steel is press-fitted or shrink-fitted on to the gear-hub which is made of comparatively inferior material. Grub screws are sometimes fitted between the rim and the hub for extra securing. To avoid fatigue failure, gear teeth are often chamferred sideways or are rounded off laterally.

To effect reduction in weight, the gear crown or the rim may be joined to the central hub through arms or spokes. The following dimensional guidelines are given for cast gear bodies.

Rim thickness (excluding tooth height)	= 1.6 to 2 m
Hub thickness	= (0.4x shaft diameter) + 10 mm, for cast iron
	= (0.3x shaft diameter) + 10 mm, for steel casting
Hub length	≥ 1.5x shaft diameter
Number of arms	= 1/7 to 1/8 of $\sqrt{\text{pitch circle diameter in mm}}$ = 4 to 8 (generally)
Arm thickness	= 1.6m
Arm width	= 8 m to 11 m near the hub, tapering to 6.5 m to 9 m at the rim

The designed gear blank must be rigid and the hub must be of enough thickness for the maintenance of the proper fit, for the provision of the keyway of standard dimensions and for proper torque transmission. If two keys or splines are used, the hub thickness should be decided upon accordingly.

Gears are sometimes made in two halves. This way it is possible to assemble the two halves without pushing the gear along the shaft from one end till it reaches the desired position on the shaft. Large gears are also made in two halves to alleviate casting difficulties or to facilitate despatch. These split gears have even number of teeth and the parting plane always passes through the tooth-gaps so that the teeth are not weakened. The size and positions of the bolts connecting the two halves are especially calculated. For split gears, the ultimate alignment is of vital importance.

When the pitch line velocity is high, the resulting bending stress induced in the rim may be considerable. For steel gears, this stress may not pose a serious problem, but when the gear material is cast iron, the rim may be vulnerable to bursting. Hence it is prudent to check this stress in case of cast iron gears. This involves complicated calculations, but approximate value can be arrived at by making a few assumptions to simplify the procedure.

We can consider that the portion of rim between two arms is a uniformly loaded beam fixed at the ends by the arms. The length of this portion is the mean length of the arc contained between the two arms. Then, neglecting the effects of curvature of the rim and other factors, the total bending load effective in that sector of the rim is given by

$$F(N) = \frac{m_u L v^2}{r} = \frac{M v^2}{r} \tag{2.48}$$

where

F = Force, N
m_u = Mass of unit length of rim, kg/mm
L = Developed length of the sector, mm
M = Total mass = $m_u L$, kg
v = Pitch line velocity m/sec
r = Mean radius of rim, m

From mechanics, we can write the following formula for uniformly loaded beam fixed at the ends

$$\text{Maximum bending moment,} \quad B_{max} = \frac{FL}{12} \tag{2.49}$$

The stress induced may then be calculated by using the formula

$$B_{max} = \sigma Z$$

where Z is the modulus of section of the rim cross-sectional area. The induced stress σ can then be compared with the allowable stress σ_{bp} for checking.

The loading pattern of the arm is also very complicated and the stress calculations thereof are quite involved. A combination of forces acts on the arms. These are the bending force due to the transmitted torque, the centrifugal force on the rim which in turn produces bending and tension, and a vibrating bending force created by the dynamic load. By neglecting all the factors except bending by the transmitted torque, an approximate formula for the stress can be obtained as explained below.

It is assumed that only one-fourth of all the arms take part in the transmission of load. Taking F_t as the tangential force acting on the tooth surface at the pitch circle and Z as the section modulus of the cross-section of the arm, we have the bending moment

$$B = F_t L_a \tag{2.50}$$

where

L_a = Effective length of the arm, that is, the length from hub to rim. Equating for stress, we have

$$B = \frac{\text{Number of arms}}{4} \times Z \times \sigma_b, \tag{2.51}$$

where σ_b is the bending stress produced, the allowable values for which are given by

$$\sigma_{bp} \approx 30 \text{ N/mm}^2 \text{ for cast iron}$$
$$\approx 60 \text{ N/mm}^2 \text{ for steel casting}$$

Large gears are often made of welded construction. Welded design is resorted to for the undermentioned reasons.

1. It is cheaper than the cast design.
2. For cast items, patterns are to be made. If the number required is not large, then the pattern making for a few pieces is not economically viable. Hence, welded design is more economical.
3. The total time required is less.
4. Welded construction is lighter in weight.
5. For the same load carrying capacity, material required is less.

Examples of different designs of large gears have been illustrated in Figs 2.38 to 2.40. In each of these figures, the left design is of cast type and the right design is of welded or fabricated type.

2.18 Force Analysis for Spur Gears

One of the fundamental parameters to be considered, analysed and checked for designing a gear system is the load-transmitting capability of gear teeth. For this the circumferential force effective on the tooth at the pitch circle of the gear when in mesh, must be known.

Depending on the given data, this force F_t, known as the tangential force or transmitted load, can be derived from the following standard equations

$$F_t = \frac{2000T}{d} = \frac{191 \times 10^5 \times P}{d \times n} = \frac{1000P}{v} \tag{2.52}$$

where

F_t = Tangential force in newtons

T = Transmitted torque in newton metres

d = Pitch circle diameter in millimetre

P = Power in kilowatts

n = Speed in rpm

v = Circumferential velocity in metres per second

Irrespective of the value of the contact ratio, for calculation the total gear forces are taken to be effective on a single pair of teeth in mesh. Referring to Fig. 2.41, normal force F_N acts along the pressure line. The normal force due to the pinion produces an equal and opposite reaction at the gear tooth as shown in the figure. Since the pinion is mounted on its shaft, force R acts at the centre of the shaft which is equal in magnitude but opposite in direction to F_N. The same thing happens in case of the gear.

Normal force F_N is resolved into two components—F_t in the tangential direction and F_r in the radial direction. So far as the transmission of power is concerned, component F_r plays no part. The driving component is F_t. The tangential component of shaft reaction R and F_t constitute a couple which produces the torque on the pinion which in turn drives the gear set. The magnitudes of the components of the normal force F_N are given by

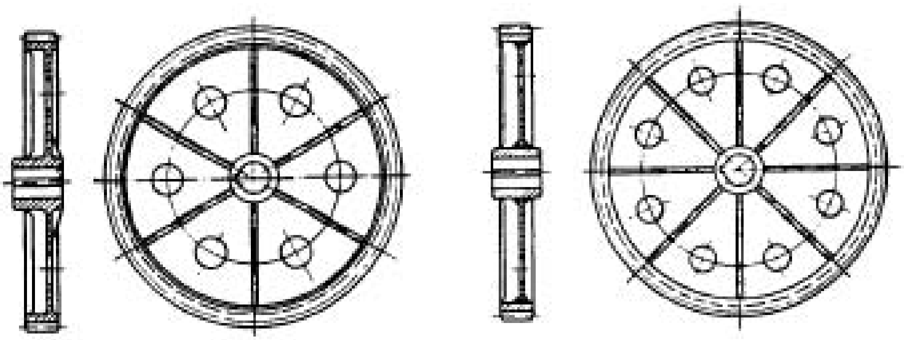


Fig. 2.38 Types of gear body construction
Basedon Maschinenelemente.Niemann,vol,II, 1985 edition, Fig 72, p. 72 Springer Verlag, Heidelberg

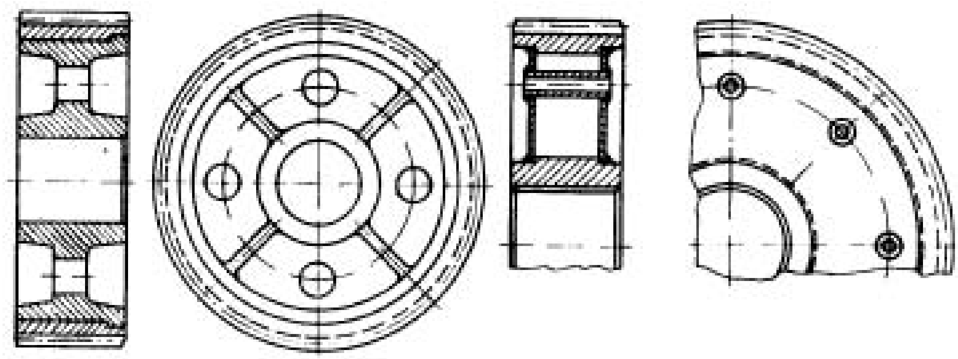


Fig. 2.39 Types of gear body construction

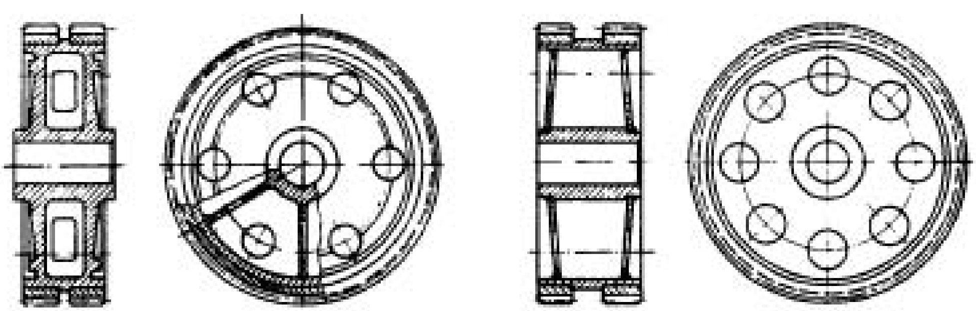


Fig. 2.40 Types of gear body construction

$$F_t = F_N \cos \alpha \quad (2.53)$$

$$F_r = F_N \sin \alpha = F_t \tan \alpha \quad (2.54)$$

where α is the pressure angle. In case of correctedgears with non-standard centre distances, the working pressure angle α_w should be inserted in the above equations instead of α.

If the metric technical system of units is used, the following relations are of relevance.

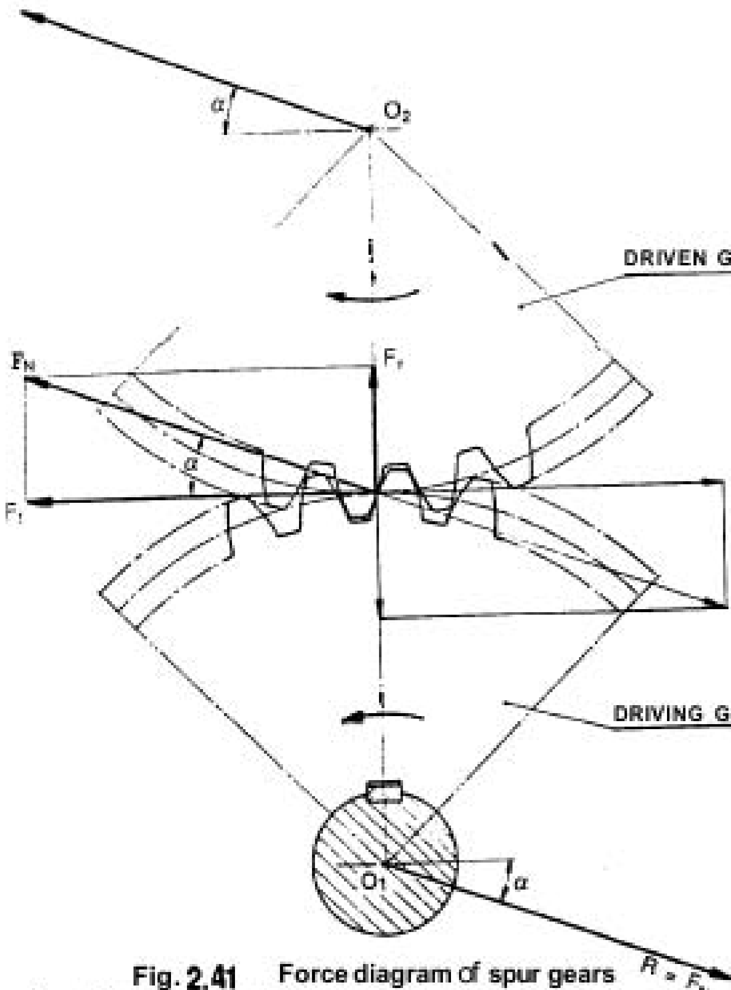


Fig. 2.41 Force diagram of spur gears

F_N = Force imposed on tooth of driven gear
F_t = Tangential driving force (Transmitted load)
F_r = Radial component (Separating force)
R = Reaction of shaft to force exerted by the driving gear

$$F_t\ (\text{kgf}) = 102\frac{P(\text{kW})}{v\ (\text{m / sec})} = 195\times 10^3\frac{P(\text{kW})}{d\ (\text{cm})\ \times\ n\ (\text{rpm})} = 75\frac{P(\text{metric HP})}{v\ (\text{m / sec})}$$

$$= 143\times 10^3\frac{P(\text{metric HP})}{\text{d (cm)}\ \text{x}\ n\ (\text{rpm})} \qquad (2.25)$$

Circumferential velocity

$$v(\text{m / s}) = \frac{d(\text{cm})\times n\ (\text{rpm})}{1910} \qquad (2.56)$$

The above force analysis is, of course, a simplified one. In actual practice, dynamic forces (as discussed later in Sec. 2.22), deflection of shaft due to the action of the radial force F_r, power losses due to bearing and shaft-seal friction, lubrication effects, etc., are also to be taken into consideration for precise calculations. Distribution of gear forces on shafts for different orientation of the driving and driven gears has been illustrated in Fig. 2.42.

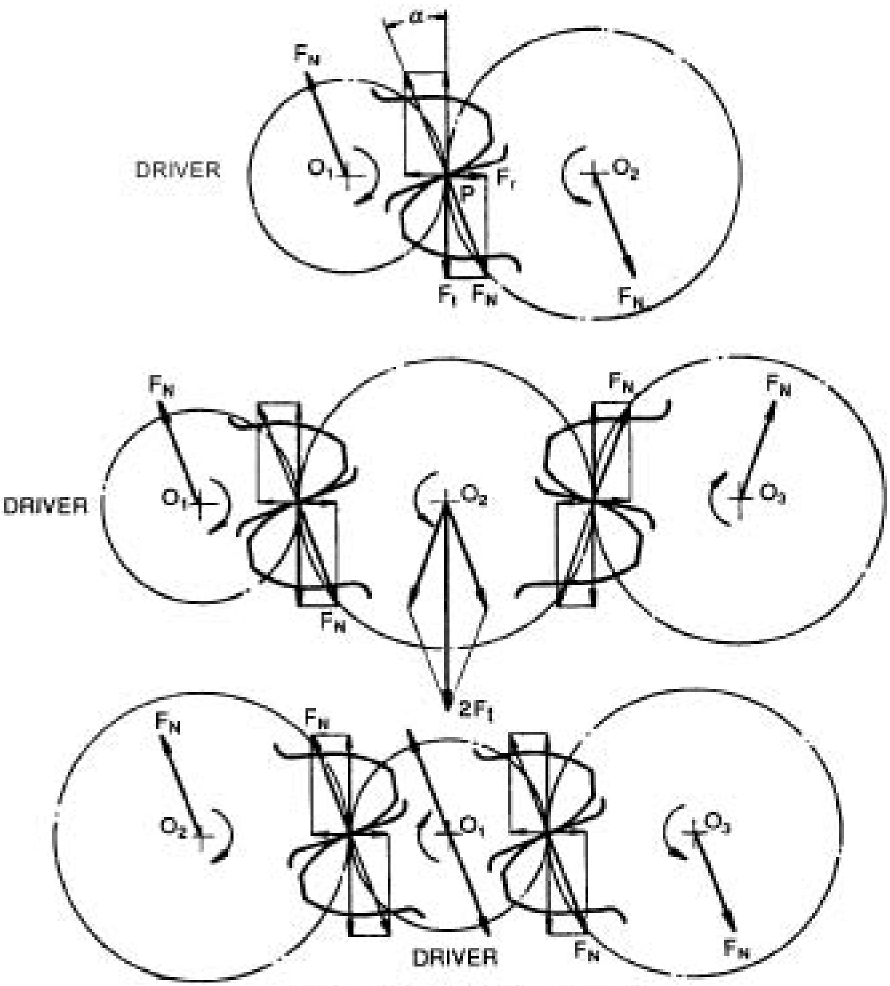


Fig. 2.42 Distribution of **forces** fon **shafts**

Basedon Die Tragfaehig keit der Zahnraeder, Thomas and Charchut, 7th edition, 1971, Fig. No. 7,8 & 9, p. 34 & 35. Carl Hanser Verlag, Munich

2.19 Spur Gear Bearing Loads

Gears impose loads on bearings on which the shafts carrying the gears are mounted. Depending on the arrangement of the gear drive, the magnitude of the gear-tooth drivingforce and the direction, the bearing loads are distributed on the individual bearings accordingly. In the case of helical gears, apart from the resultant radial forces acting on the bearings, thrust forces axially along the shafts are also created, as will be discussed in Chap. 3 on helical gears. For selection of the type of bearing, that is, journal or anti-friction bearing, ball, roller or thrust bearings, etc., and determination of its size will depend, among other factors, on the load it has to carry. In case of anti-friction bearings, the catalogues and manuals of the standard bearing manufacturers give details about the load carrying capacities of these types of bearings and their selection procedures, so that the selection of the most proper and suitable type and size of bearing poses no problem once the bearing loads are known. In case of journal bearings, however, a proper bearing has to be designed after considering bearing loads, material of bearing, allowable stress, type of lubrication, etc. In any case, a complete load analysis is required before the right type and size of bearing is selected or designed, as the case may be. For anti-friction bearings, load ratings and life required by design calculations should correspond with those given in the manufacturer's catalogues. For reduction gear units in general, a service life of 20,000 hours is the usual criterion.

It has been shown in Sec. 2.18 that force F_N which is normal to the tooth curve and which acts along the line of action can be resolved into two components, F_t which is tangential to the pitch circle of the gear and which is actually the driving force, and F_r which is the radial component directed towards the axis of the gear and as such, serves no useful purpose.

In the following diagrams, bearings loads in several cases of gear-drive orientations are shown with their respective values in terms of known forces. In all cases subscript B stands for the ultimate load on the bearing, letter L with appropriate subscripts, such as I, II, etc., stands for the respective distances, and subscripts N, t and r have the usual meanings as given before. In each case the resultant load on a bearing is the vector sum of all the individual force components acting on that bearing.

Straddle Mounted

The straddle mounted type of arrangement is shown in Fig. **2.43** (a). The resultant force acting on bearing BZ and BII are given by

$$F_{BI} = \text{The resultant force on bearing } BI = \sqrt{(F_{BIt})^2 + (F_{BIr})^2} \quad \textbf{(2.57)}$$

$$F_{BII} = \text{The resultant force on bearing BIZ} = \sqrt{(F_{BIIt})^2 + (F_{BIIr})^2} \quad (2.58)$$

F_{BIt} and F_{BIr} are the tangential and the radial components of the main gear force F_N acting on bearing Z. Their counterparts on bearing ZZ are F_{BIIt} and F_{BIIr}.

The magnitudes of the two resultant bearing forces will depend upon the position of the gear as it is mounted on the shaft, and are given by

$$F_{BI} = \frac{F_N L_{II}}{L} \quad \textbf{(2.59)}$$

$$F_{BII} = \frac{F_N L_I}{L} \quad \textbf{(2.60)}$$

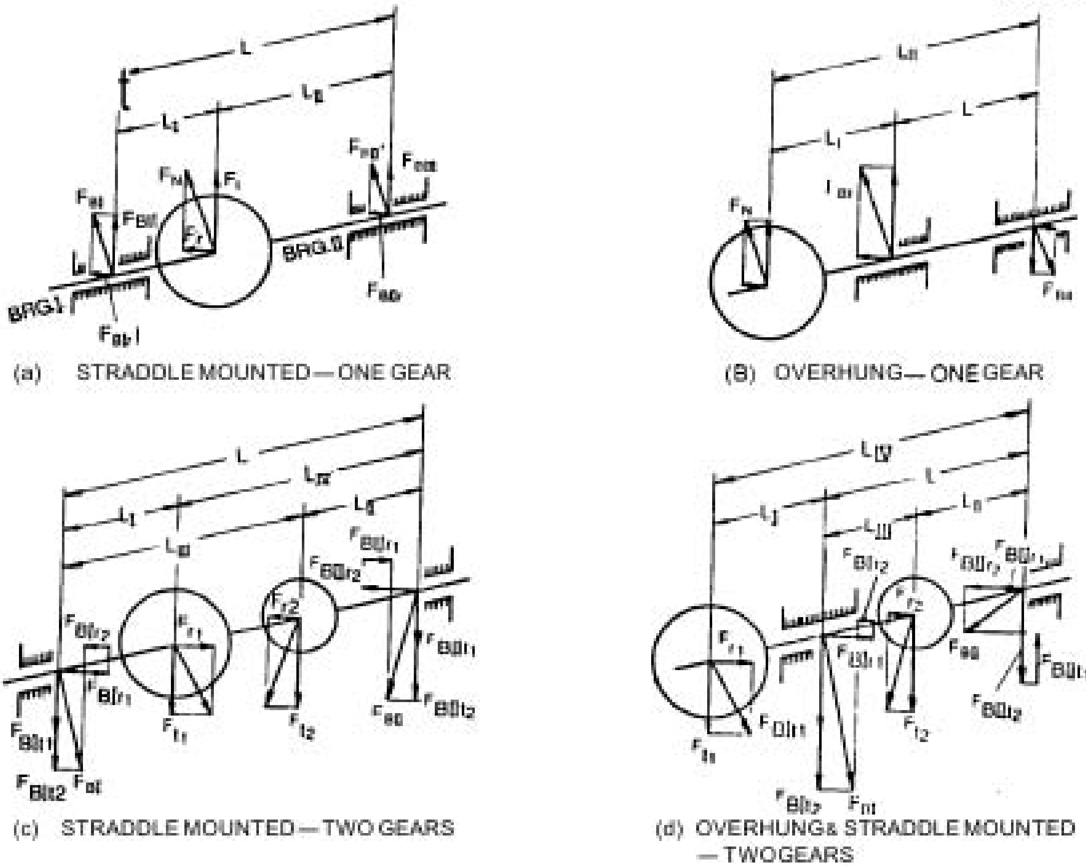


Fig. 2.43 Distribution of gear forces on bearings
Based on Die Tragfaehigkeit der Zahnraeder. Thomas and Charchut. 7th edition. 1971, Fig. No. 11, 12, 13. & 14, p. 35 & 36. Carl Hanser Verlag, Munich

Overhung

Figure 2.43(b) shows the overhung type of arrangement. In this case Eqs 2.57 to 2.60 are also applicable. It is recommended that the following relations in distances are adhered to

$$L\ (\text{min}) = 2.5 \times \text{Pitch diameter of} \geq 2.5 \times L_{I} \tag{2.61}$$

Two Gears Mounted on the Same Shaft

When two gears are mounted on the same shaft, the orientation of the different forces shown in Figs 2.43(c) and 2.43(d).

For Fig. 2.43(c)

$$F_{Blt} = F_{Blt_1} + F_{Blt_2} = \frac{F_{t_1} L_{IV} + F_{t_2} L_{II}}{L} \tag{2.62}$$

$$F_{Blr} = F_{Blr_1} - F_{Blr_2} = \frac{F_{r_1} L_{IV} + F_{r_2} L_{II}}{L} \tag{2.63}$$

$$F_{BII} = F_{BIIt_2} + F_{BIIt_1} = \frac{F_{t_2}\, L_{III} + F_{t_1}\, L_I}{L} \tag{2.64}$$

$$F_{BIIr} = F_{BIIr1} - F_{BIIr_2} = \frac{F_{r_1}\, L_I - F_{r_2}\, L_{III}}{L} \tag{2.65}$$

For Fig. 2.43 (d)

In this case the equations for bearing I (Eqs 2.62 and 2.63) are applicable. Equations for bearing II are different and are given by

$$F_{BIIt} = F_{BIIt_2} - F_{BIIt_1} = \frac{F_{t_2}\, L_{III} - F_{t_1}\, L_I}{L} \tag{2.66}$$

$$F_{BIIr} = F_{BIIr_1} + F_{BIIr_2} = \frac{F_{r_1}\, L_I + F_{r_2}\, L_{IV}}{L} \tag{2.67}$$

After ascertaining the tangential and the radial components, the resultant forces acting on the bearings can be found by using the general equations 2.57 and 2.58, which are the vector sums of the components forces.

Example 2.6 *Given* : Straddle mounted corrected pinion of a S-gearing system with the following data

P, = 150 kW, n_1 = 1450 rpm, $z_1 = 23$, m = 5, working pressure angle $\alpha_w = 24°\,41'$,

distance between bearings = 200 mm, pinion is situated at 70 mm from one end.

To find the bearing loads and the maximum bending moment.

Solution: From Sec. 2.13, we have the relation

Working circle diameter

$$d_w = d_1 \frac{\cos a}{\cos a_t} = m\, z_1 \frac{\cos a}{\cos a_t} = 5\times 23 \times \frac{\cos 20°}{\cos 24°\ 41'} = 118.93 \text{ mm}$$

Torque

$$T_1 = 9550 \times \frac{P_1}{n_1} = 9550 \times \frac{150}{1450} - 988 \text{ N m}$$

Circumferential force on the pinion

$$F_t = \frac{2 \times T_1}{d_{w1}} = \frac{2 \times 988}{0.11893} = 16615 \text{ N}$$

Normal force on the tooth profile

$$F_N = \frac{F_t}{\cos a_t} = 18286 \text{ N}$$

Referring to Fig. 2.43 (a)and using Eqs 2.59 and 2.60, we get

$$F_{BI} = \frac{F_N L_{II}}{L} = 18286 \times 130/200 = 11886 \text{ N}$$

$$F_{BII} = \frac{F_N L_I}{L} = 18286 \times 70/200 = 6400 \text{ N}$$

Maximum bending moment

$$B_{max} = F_{BI} \times L_I = 11886 \times 70 = 832 \text{ N m}$$

Journal bearings: For the purpose of designing journal bearings, the allowable bearing loads are to be known according to the material used. Besides, the following data are of relevance in this context:

$$L = 0.8 \text{ to } 1.2\,d$$

where L is the length of the bearing and d is the diameter of the journal portion of the shaft.

$$\text{Bearing pressure } p_p = \frac{\text{Bearing load}}{\text{Projected area}} = \frac{F_B}{dL} \quad (2.68)$$

The allowable bearing pressure values for common bearing materials for different velocities are given in table 2.13.

Table 2.13 Allowable bearing pressure on journal bearings

Bearing material	3 (max)	40	30	20	10	5	2
	Circumferential velocity of journal v in m/s						
	Permissible bearing pressure p_p in N/cm²						
Cast iron	50 to 100						
Bronze or white metal		50	75	100	150	200	300

2.20 Journal and Journal Bearing

A journal is defined as that portion of the shaft which is encased inside a bearing. To determine the size of the journal, the diameter is first calculated on the basis of the bearing force in question and then it is checked against the bending stress produced. The force to be carried by the bearing is given by

$$F_B(\text{N}) = dL\,p \quad (2.69)$$

where d is the diameter of bearing in mm, L is the length of the bearing in mm, and p is the specific bearing pressure produced in N/mm². The expression dL represents the projected area of the loaded journal.

For safe working, p must be equal to or less than the allowable bearing pressure p_p. The value of p_p depends on the service conditions, circumferential velocity of the shaft, duration of operation, material and type of lubrication and cooling. Estimation of the dynamic load carrying capacity is rather involved. For simplified calculations, the values of allowable bearing pressure for common bearing materials at different velocities given in Table 2.13 can be used.

Journals may be mounted on anti-friction bearings or on radial bearings known as sleeve bearings or journal bearings. For the proper selection of anti-friction bearings, the reader may refer to catalogues of standard bearing manufacturing companies. In this section only journal bearings will be discussed.

Journal bearings are simple in construction, easy to instal, cheaper in cost, require less space and normally noise-free and impact damping. However, the starting friction is very high and the system requires considerably more lubricant. Modern design practice calls for shorter journal bearings having length to diameter ratio($L:d$) between 0.4–0.8 and 1.0–1.2. Though ideally L should be equal to or less than d, this condition may not be possible to adhere to because of the specific bearing pressure p produced in case of different bearing materials. Shaft being made of steel, the following relations can be used for different materials.

L = 1-1.8 d for bronze or gun metal
= 1.5-2 d for white metal
= 2-3 d for cast iron

Example 2.7 *Given:* Bearing load = 20000 N, speed = 1500rpm. To determine the relevant parameters for the bearing.

Solution: The following tentative selections are made with the help of Table *2.13*.

Material: shaft—Fe **490**, bearing—white metal.

$$L:d = 2 \text{ or } L = 2\,d,\ p_p = 100\ \text{N/cm}^2, F_B = dLp_p \text{ or } 20000 = d \times 2d \times 100, \therefore d = 10\ \text{cm}$$

$$v\,(\text{m/s}) = \pi dn/60 \times 100 = 3.14 \times 10 \times 1500/60 \times 100 \approx 8.$$

This velocity is well within the value specified in the table. The bending moment within the journal portion is given by

$$B = F_B L/2 = 20000 \times 2012 = 200000\ \text{N/cm} = Z\sigma_b = 0.1 \times d^3 \times \sigma_b,$$

or

$$200000 = 0.1 \times 10^3 \times \sigma_b,\ \text{whence}\ \sigma_b = 2000\ \text{N/cm}^2 = 20\ \text{N/mm}^2$$

This stress value is also within the normal permissible value for steel selected. Hence, the dimensions and the materials selected are acceptable.

2.21 Shaft Design

In order to design the shaft on which the gear is mounted, many diverse factors have to be taken into account. Stress concentration is a major factor for which the reader may refer to standard books on mechanics where this aspect is dealt with in detail. In Appendices **M** and N, the standardised values of shaft diameters and the fillet radii in case of stepped shafts have been given. These data should be made use of while fixing the shaft dimensions.

By and large the main determining factors for the calculation of the diameter of the shaft are the maximum bending moment and the torque to which it is subjected. The determination of bending moment has been dealt with in Sec. 2.19 for spur gears and in Sec.3.13 for helical gears. Expression for torque has been given in Sec. 2.25 and at other places.

The next step is to calculate the equivalent bending moment for the combined effect of bending and torsion. This is given by

$$B_e = \sqrt{B^2 + 3\,(\alpha T)^2} \leq \sigma_{bp}\ Z \tag{2.70}$$

where B and T are the maximum values of bending moment and torque respectively. Equation **2.70** is based on the maximum distortion-energy theory, which is also known as the shear-energy theory or the von Mises-Hencky theory, and this theory is normally applicable for materials used for shafts.

Section modulus $Z = 0.1d^3$ for a solid shaft of diameter d and α is a stress relations factor given by

$$\alpha = \frac{\sigma_{bp}}{1.73\,\tau_p} \tag{2.71}$$

where σ_{bp} = Permissible bending stress, and τ_p = Permissible torsional shear stress. The following allowable values can be taken for shaft materials

σ_{bp} = 40 to 60 N/mm² for Fe **490**
= **60** to **100** N/mm² for Fe **620** and alloy steels

As a rough guideline, σ_{bp} can be taken as 20% of the endurance limit of the shaft material in reversed bending.

τ_p = **40** to **80** N/mm² for steel having ultimate tensile strength less than 500 N/mm².
= 60 to 100 N/mm² for steel of UTS greater than **500** N/mm².

The corresponding equivalent stress is given by

$$\sigma_e = \sqrt{\sigma^2 + 3(\alpha\,\tau)^2} \tag{2.72}$$

where σ and τ are the bending and the torsional stresses produced respectively. The diameter of the shaft is given by

$$d \geq \sqrt[3]{\frac{B_e}{0.1 \times \sigma_{bp}}} \tag{2.73}$$

Besides strength considerations, shafts are sometimes checked for stiffness, deflection and critical speed. For long shafts, a certain angle of twist must not be exceeded. For such cases this is the deciding factor for the ultimate selection of the shaft diameter.

The angle of twist is given by

$$\phi \text{ (radian)} = \frac{TL}{I_p G} \tag{2.74}$$

where T = Torque (Ncm), L = Length of shaft (cm), I_p = Polar moment of inertia of the shaft cross-section (cm⁴), and G = modulus of rigidity (N/cm²).

For circular shafts

$I_p = \pi d^4/32$ G for steel is around 8×10^6 N/cm².

Expressing ϕ in degrees, the angle of twist per meter length is given by

$$\phi_m = \frac{18000T}{\pi G I_p}, \text{ whence } d\text{(cm)} = 15.5\sqrt[4]{T/(G\phi_m)}$$

Normally, the permissible value of ϕ_m is 1/4°/m. This gives

$$d(\text{cm}) = 0.4125 \times \sqrt[4]{T} \tag{2.75}$$

Shafts which have to withstand bending forces caused by machine elements mounted on them should be checked against the deflection which results from such loadings. Elements carried by the shaft should be mounted close to the bearings to reduce deflection. The maximum deflection must be kept within the allowable limit which is normally 0.001 to 0.003 times the span, i.e. the distance between the bearings.

In case of high speed shafts (with n greater than 1500 rpm) of certain types of machines, the critical speed should be checked. Imbalance in the system causes vibration. The vibration amplitude may reach such values as to cause ultimate failure of the shaft. Resonance occurs when at a critical shaft velocity, the frequency of vibration of external forces and of the shaft system coincide. The speed at which resonance sets in is called the "critical speed". To avoid the disturbance caused by this phenomenon, the operating speed must be widely away from the critical speed which should lie at least 10% above or sometimes widely below the operating speed.

The reason for resonance vibrations lies mainly in the centrifugal force which is caused by improper balancing and deflection of the shaft. Arriving mathematically, the critical speed is given by

$$n_c(\text{rpm}) = 300\sqrt{1/f} \tag{2.76}$$

where f = the maximum static deflection (in cm) of the total rotating and vibrating masses.

An indication of the impending failure of the shaft is its excessive vibration. However, the shaft does not fail all of a sudden. Proper balancing, therefore, is of utmost importance for high speed shafts together with the machine elements mounted on them.

2.22 Dynamic Loads on Gear Teeth

The forces acting on gear teeth are normally determined from parameters like power and speed. These forces which are known as transmitted loads, however, give only the theoretical values. In actual practice, these forces may be exceeded considerably because of effects due to moving and swinging masses, machining errors of teeth caused by cutting inaccuracies, deformation and deflection of teeth under load causing periods of acceleration and deceleration of short durations during the course of action, inertia forces required to change the velocity of the inertial-masses of the meshing gears, shaft misalignment and impact loads. These additional forces which are superimposed on a steady (transmitted) load are generally termed as "dynamic loads".

Although considerable research has been carried out by experts in the field viz. Erle Buckingham, G. Niemann and others, the dynamic load remains as one of the least understood phenomena of gear teeth in action. The vibration characteristics induced by the dynamic loads are so complex that no completely satisfactory methods of computation of such forces have evolved, and calculations based on considerations of rigid bodies lead to inaccurate results.

Considering the practical aspect, therefore, the gear transmission phenomenon is far from what one concludes from the conventional formulae. What actually happens when gear teeth come in contact depends on diverse factors. Briefly, these may be described as follows: Gear teeth in mesh have a number of imperfections, namely, the tooth profiles are never perfect involutes, the tooth spacings are never uniform; kinematic perfection is never attained because teeth deflect under load; the shaft and mountings also deflect under load; the tooth elements are not perfectly parallel to the axis in case of spur gears; and uniform distribution of load never takes place across the face and flank of teeth. The result of all these defects and errors is the dynamic load which is in addition to the load derived from the usual formulae.

Dynamic load gives rise to shock and impact. Research has shown that the worst effect occurs when the load is transferred from one mating pair of teeth to the next pair. They bounce apart and mesh again with an impact. The dynamic load is momentarily at the maximum peak at the instant of impact. Though perfect determination of the amount of the dynamic loads is not possible, approximate values can be derived, the magnitude depending upon the moment of inertia of the rotating masses, the composite error in spacing and profiles of teeth, speed and material of gears.

Pattern of the Dynamic Load

Rigorous analysis of dynamic load is extremely difficult and uneconomical for common design purposes. Often a more or less satisfactory method of proper selection of gear class is based on the desired noise level. A gear set will never run without producing some noise, but since the magnitude of the noise generated is directly related to the accuracy with which the gears have been cut, noise can be reasonably taken as a criterion or indicating factor of the accuracy of the gear set. Obviously noise can be reduced by exercising greater accuracy.

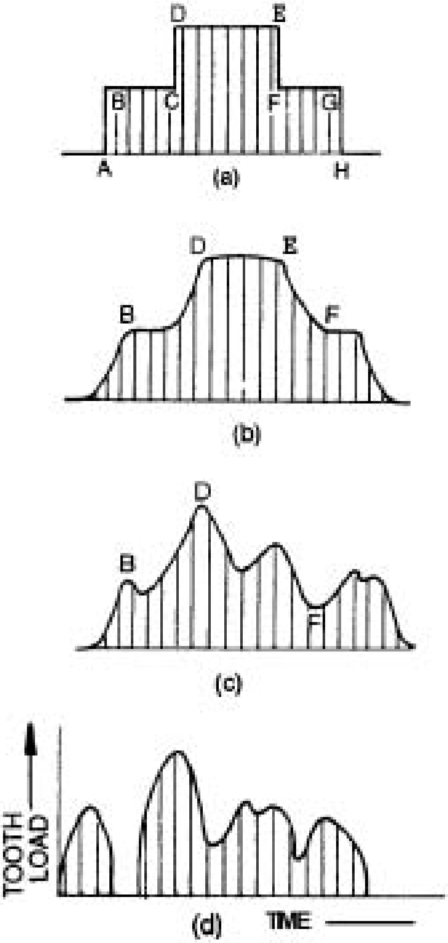


Fig. 2.44 Tooth load versus time characteristics

Figure 2.44 shows the load *vs.* time relation of a pair of meshing gears. During the course of teeth engagement, there will be a period when the applied load is carried by two pairs of teeth, at other period the entire load is on one pair only. This aspect has already been discussed in Secs 2.5 and 2.7. Figure 2.44(a) shows an idealised representation of the load–time characteristics in case of a spur gear pair running at low speed. Recalling Secs 2.5 and 2.7, a pair of tooth comes in contact and instantly this pair takes the load represented by the line *AB*. Since at that instant, one previous pair is still in mesh, the new pair shares about half of the total load while the older pair takes the remainder of it. This continues till point *C* is reached whereupon the load on the new pair is suddenly increased by *CD*. At this stage the older pair has gone out of engagement and the new pair takes the full load. This is represented by the line *DE*. At *E* there is a sudden reduction in load-carrying because another new pair has come into mesh in the meantime. The rest of the load sequence is a reversal of the initial one. A more realistic picture of load — time curve is shown in Fig. 2.44(b) where the elasticity of the teeth and their convex profiles modify the vertical lines and round off the corners. At high speed the curve takes the form as shown in Fig. 2.44(c). Here the most important feature is point *D* where the load overshoots and the maximum force on the tooth becomes greater than the force which corresponds to the nominal torque.

The foregoing case deals with teeth which are properly cut as regards tooth profile and spacing. This, however, is not the case in practice as pointed out before, and consequently the angular velocity ratio never remains constant throughout the period of engagement. All types of errors make a considerable difference in the load–time characteristics. Oscillograph records show large variations from the idealised tooth-force curves. The load–time curve may take a shape as depicted in Fig. 2.44(d). It may be seen from this figure that for a short period the teeth remain out of engagement due to acceleration or deceleration of the rotating masses and then they again come in contact with considerable impact, shooting to a maximum momentary force. It is this excess of maximum transient load over the nominal load which quantifies the dynamic load. "his has also been termed as "dynamic increment" by some authors.

Since no clear-cut or wholly satisfactory method has been evolved so far, the undermentioned procedures can be adopted for deriving approximate results on the dynamic load aspects:

Peak Loads

In case of big gear boxes with considerable self weights of moving parts, failure may occur during starting or sudden braking of the system giving rise to instantaneous peak loads. In case of direct coupling of electric motors, the starting torque can amount to a lot more than the nominal torque. In hoisting mechanisms, a sudden actuation of the main coupling may result in a torque reaching around 2 to 4 times the nominal torque of the motor. It is, therefore, imperative that in such cases a thorough calculation should be made, taking into account such factors as the starting, braking as well as any special and critical service conditions which the gear boxes may be subjected to.

In cranes, hoisting devices and similar applications, the acceleration torque is calculated and added to the nominal torque for the determination of maximum bending and contact stresses.

The acceleration torque is given by

$$T_a = I_m \alpha = I, \frac{\omega}{t} \tag{2.77}$$

The expression for T_a can be expanded as follows

$$T_a(\mathrm{Nm}) = \frac{2\pi n\ (\mathrm{min}^{-1})}{4g(\mathrm{ms}^{-2})} \times \frac{W\ (\mathrm{N})\ D^2\ (\mathrm{m}^2)}{t\ (\mathrm{s})} = \frac{2\times 3.14\times n\ (\mathrm{s}^{-1})}{60\times 4\times 9.81\ (\mathrm{ms}^{-2})} \times \frac{W(\mathrm{N})\,D^2(\mathrm{m}^2)}{t\ (\mathrm{s})}$$

More conveniently

$$T_a\ (\mathrm{Nm}) = \frac{1}{375} \times \frac{WD^2 n}{t} \tag{2.78}$$

Here,

T_a = Acceleration torque in N m

WD^2 = An expression which has been termed as the "fly-wheel moment or "moment of gyration" of moving masses in Nm^2. Some authors and manufacturers of motors and couplings use the German expression GD^2

W (or G) = Weight of the rotating masses in N

D = Diameter of gyration of rotating masses in m = 2 x k (radius of gyration)

We have the following relation

$$I_m = \frac{WD^2}{4g} \tag{2.79}$$

where

I_m = Mass moment of inertia in newton metre seconds squared

g = Acceleration due to gravity in meters per second squared

ω = Angular velocity in radians per second

α = Angular acceleration in radians per second squared

t = Starting or braking time in seconds

n = Speed in rpm

Values of I_m and D for common geometrical solids can be found in standard books of mechanics. The WD^2 values of motors and couplings of standard make are normally given in the catalogues of the manufacturers of those items.

For proper selection of the motor torque in actual working conditions, it is necessary to add the acceleration torque to the rated torque of the motor. To achieve this, the equivalent fly-wheel moment of the whole rotating system is to be calculated with respect to the motor shaft. This is done by using the following equation

$$WD^2_{eq} = (WD^2)_I \times \left(\frac{n_1}{n_1}\right)^2 + (WD^2)_{II} \times \left(\frac{n_2}{n_1}\right)^2 + (WD^2)_{III} \times \left(\frac{n_3}{n_1}\right)^2 + \ldots \tag{2.80}$$

where $(WD^2)_I$ is the fly-wheel moment of the motor or of the motor plus the coupling, and $(WD^2)_{II}$, $(WD^2)_{III}$, etc., and n_2, n_3, etc., are the WD^2 values and the speeds of the other rotating masses of the system respectively, e.g. pinion shafts, gears, other shafts, other couplings, rotating parts of the driven machine and any other rotating inertial mass that has to be accelerated to bring the system to the required speed.

When elaborate calculation is not possible or necessary, the design torque can be taken to be around 10 to 25% higher than the rated torque of the motor. The value of the torque found from the rated power and speed by using the conventional formulae plus the acceleration torque determines one of the design criteria by which the proper motor is selected. The reader is advised to refer to Sec. 2.26 in this connection.

Different Aspects of Dynamic Load

The dynamic load, which has been defined as the maximum instantaneous load acting on the meshing teeth of gears during action, may be created due to outside factors like service conditions, number of starts, nature of duty, type of drive, and also due to internal factors, such as different types of tooth errors which lead to speed fluctuations which in turn give rise to additional forces due to momentary acceleration or deceleration and deformation of teeth under load. Besides, as each gear is a rotating mass and, therefore, possesses inertia, it follows that the mass requires the application of force to change its velocity. Both external and internal factors influence the magnitude of dynamic loads.

External factors : These factors can be taken care of by using the values given in Table 2.16. The values of the service factor ϕ have been arrived at by experience, taking into consideration the type of driving machine, starting and running conditions, nature of duty and other operational parameters. The maximum tooth load will occur when

$$T_{max} = T_{rated} \times \phi \tag{2.81}$$

where T_{max} and T_{rated} are the maximum and the rated torque parameters respectively, and ϕ is the service factor.

Internal factors : These factors are so complex that to arrive at a reasonably accurate value of the additional load due to the dynamic forces acting on the tooth profiles, a large number of complicated calculations are to be made. This is justified when calculations for master gears are involved. Moreover, in case of comparatively smaller speed ranges, the effects of tooth errors, and other detrimental factors are less pronounced than those in case of high speed gears.

Non-uniform and jerky operation of a gear is the effect of tooth errors, such as profile error, or pitch error, which are caused by faulty manufacture. Teeth also deform under load as stated earlier. The gear ratio i may be considered to be of constant magnitude during one complete cycle, but in actual practice instantaneous values of this ratio remain changing all the time resulting in non-uniform rotation of the driven gear. This gives rise to angular accelerations which in turn produce a hammering effect during engagement causing additional dynamic load. Noise and vibration are created, and smooth and useful transmission of energy is impaired. Obviously, a larger dynamic load will be created if the machining accuracy is low, velocity is high and the rotating masses are comparatively greater.

In majority of the cases, the approximate method developed by Prof. G. Niemann is sufficient to calculate the dynamic forces.

Based on experimental research work, the following relation has been derived

$$Ft_{dyn} = K_a \, K_v \, b \, v \tag{2.82}$$

where,

F_{tdyn} = Additional tangential force on the tooth caused by dynamic loads, N

K_a = Factor taking into account the amount of load in relation to the effective tooth errors, newtons per centimetre

K_v = Factor taking care of the influence of the circumferential velocity, seconds per metre

b = Face width of tooth, centimetres

v = Circumferential velocity, metres per second

K_a is determined from the relation:

$$K_a = \frac{F_t}{b} \times \phi + 26f \tag{2.83}$$

where

F_t = Tangential force on the tooth, N

f = The numerical value (expressed in micrometers) of the largest existing tooth error e.g. f''_i, f_f, f_p etc. as the case may be.

After ascertaining K_a, the value of K_v is read off from Table 2.14

Table 2.14 Factor K_v in relation to factor K_a and velocity v

K_a in N/cm	v in m/s										
	1	2	4	6	8	10	12	14	16	18	20
	$K_v \times 10^{-2}$ in s/m										
500	5.7	5.6	5.6	4.6	3.4	2.9	2.6	2.1	1.9	1.9	1.6
1000	5.0	4.9	4.8	4.1	3.4	3.0	2.7	2.4	2.1	1.9	1.8
2000	3.8	3.7	3.2	3.0	2.7	2.5	2.4	2.3	2.2	2.1	2.0
3000	3.3	3.0	2.5	2.4	2.3	2.3	2.3	2.2	2.2	2.1	2.1
4000	3.1	2.9	2.5	2.3	2.3	2.3	2.3	2.2	2.2	2.2	2.2
5000	3.0	2.9	2.5	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3
10000	3.0	2.9	2.4	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3
12000	2.9	2.8	2.4	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3

Based on Die Tragfaehigkeit der Zehnraeder. Thomas and Charchut, 7th Edition, 1971, table no 6, p. 30 Carl Hanser Verlag, Munich.

To calculate the gear parameters when the dynamic load is considered, the total tangential force and the total torque are to be determined. These are given by

$$F_{t\,tot}\,(\text{N}) = F_t \times \phi + F_{t\,dyn} \tag{2.84}$$

$$T_{tot}\,(\text{Nm}) = T_{rated} \times \phi + \frac{d}{2} \times F_{t\,dyn} = F_t \times \frac{d}{2} \times \phi + F_{t\,dyn} \times \frac{d}{2} \tag{2.85}$$

$$= (F_t \times \phi + F_{t\,dyn}) \times \frac{d}{2} = F_{t\,tot} \times \frac{d}{2}$$

where $F_{t\,tot}$ and T_{tot} are the total tangential force and the total torque respectively, and d is the pitch circle diameter in cm.

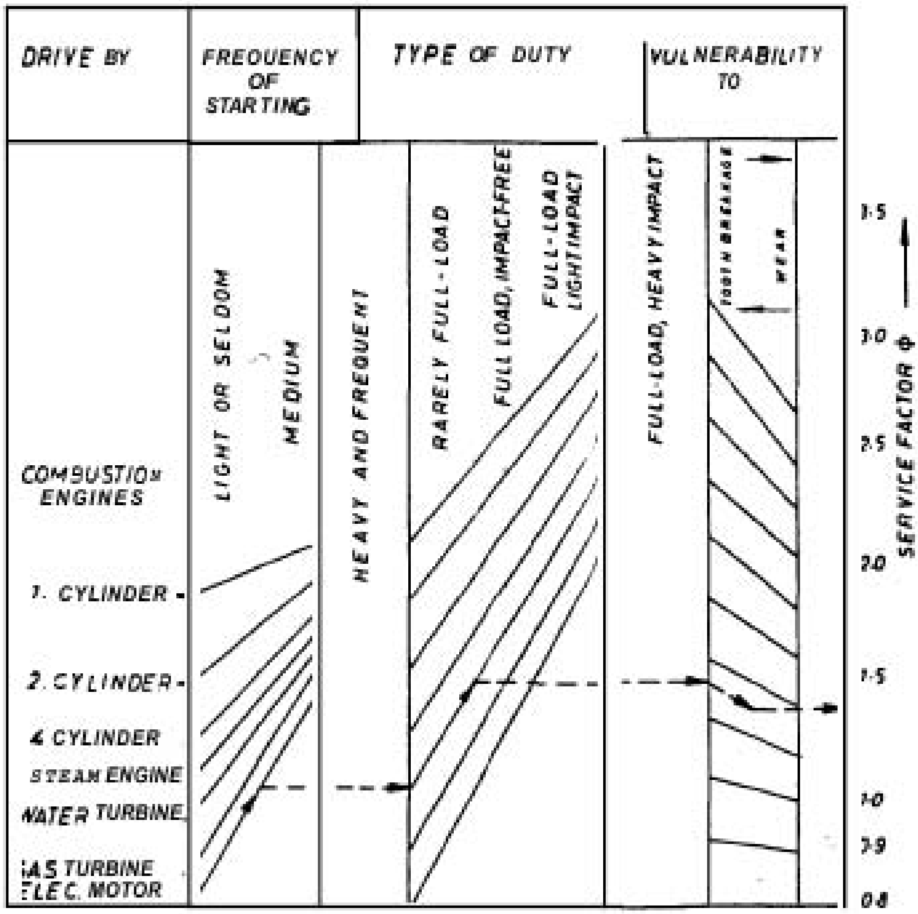
In Table 2.15 the values of allowable circumferential velocity in relation to manufacturing processes and quality or grade has been shown. These are guiding values only and are given to facilitate gear calculations for general engineering applications. The value of the maximum tooth error f is to be calculated on the basis of Table 2.23 given in Sec. 2.27.

Table 2.15 Allowable circumferential velocity of gears in relation to the manufacturing processes and quality

v m/sec	*Manufacturing process*	*Quality*
0–0.8	Cast, coarse cut, gas cut, flame cut	10–12
0.8–4	Smooth finished	8–10
4–12	Ground	6–8
12-60	Scraped, shaved, fine ground	4–6

The effect of the additional load due to dynamic causes can be neglected in case of small circumferential velocity υ less than 1m/sec. Normally, a linear relationship exists between the magnitude of the dynamic load caused by the inner factors and the velocity up to around υ = 50 m/s. Beyond this value the increase in dynamic load is only marginal. The dynamic load in case of a helical gear can be taken as about 75% that of the spur gears for calculation. Table 2.16 gives the service factors for different service conditions.

Table 2.16 Service factor for different service conditions



Based on Die Tragfaehigkeit der Zahnraeder, Thomas and Charchut, 7th Edition, 1971, table no. 5, p. 29. Carl Hanser Verlag, Munich.

Example 2.8 A mixing drum is operated by an electric motor through a one-stage gear drive. The pinion is keyed to the shaft and is mounted at the end of the shaft of diameter 45 mm. The following data are given: rated power = 10kW, speed of pinion = 500 rpm, reduction ratio = around 4, drive is medium, impact-free full load and normal vulnerability to tooth failure. The system is subjected to dynamic loading. Find the total power required for the system disregarding the acceleration torque required for starting or braking. Take pinion material as 60 C 4.

Solution: For initial calculation of a normal gear drive, the following relation holds good $d_1 \approx 2 \times$ Shaft diameter $= 2 \times 45 = 90$ mm, $v = d_1\,(\text{cm}) \times n_1\,(\text{rpm})/1910 = (9 \times 500)/1910 = 2.36$ m/s.

From the given service conditions, the service factor $\phi = 1.4$ is selected from Table 2.16.

$$F_t(\text{N}) = 2 \times T_1\,(\text{N cm})/d_1\,(\text{cm}) = 2 \times T_1/9 = T_1/4.5$$

Also $\quad T_1 = P_1\,(\text{kW}) \times 955000/n_1\,(\text{rpm}) = 10 \times 955000/500 = 19100$ N cm

$$\therefore \quad F_t = \frac{19100}{4.5} = 4244.$$

Referring to the simplified calculation of strength given in Sec. 2.25, a rough estimate is now made about the value of the module as follows:

To avoid undercutting, z_1 is first assumed to be 15. Next, using Eq. 2.111, we get,

$$p\,(\text{cm}) = 3\sqrt{\frac{2\pi\, 955000\, P_1}{z_1 \Lambda c n_1}}$$

$$c = c_o K_1 K_2 K_3, c_o = 3000/(2.36 + 10) = 243$$

The following values of the coefficients are selected:

$$K_1 = 3, K_2 = 0.8, K_3 = 1$$

$$\therefore \quad c = 243 \times 3 \times 0.8 \times 1 = 583$$

$$\Lambda = 3 \text{ (selected for machined teeth)}$$

$$P_1 = 10\text{kW}, n_1 = 500 \text{ rpm and } d_1 = 90 \text{ mm } \Lambda = 9 \text{ cm} = 0.09 \text{ m}$$

Inserting the relevant values, we get

$$p = 3\sqrt{\frac{2 \times 3.14 \times 955000 \times 10}{15 \times 3 \times 583 \times 500}} = 1.66 \text{ cm} = 16.6 \text{ mm}$$

$$p = \pi m = 16.6 \text{ mm}$$

$$\therefore \quad m = 16.6/3.14 = 5.28 \text{ mm}$$

The nearest standard module **5.5** is taken from Appendix D.

Taking $z_1 = 17$ for greater safety against undercutting and recalculating, we get

$$d_1 = m \times z_1 = 5.5 \times 17 = 93.5 \text{ mm}$$

Taking quality = 8 which is commensurate with the velocity range as per Table 2.15, and consulting the various tables given for different kinds of error in Sec. 2.27, we find for m = **5.5**, $d_1 = 93.5$ and quality = 8, the error $f_i'' = 36\ \mu$m.

Now $\quad \Lambda = b/\pi m$, or $b \approx 10 \times 5.5 = 55$ mm $= 5.5$ cm

New value of tagential force is given by

$$F_t = \frac{2 \times 19100}{9.35} = 4086 \text{ N}$$

From Eq. 2.83, we have,

$$K_o = \frac{F_t}{b} \times \phi + 26f = \frac{4086}{(5.5} \times 1.4\Big) + 26 \times 36 = 1976 \text{ N / cm}$$

$$\upsilon = d_1 n_1 / 1910 = 9.35 \times 500/1910 = 2.44 \text{ m/s}$$

The new value of υ makes marginal difference in the value of c_o. Hence, $m = 5.5$ is kept.

$K_\upsilon = 3.6 \times 10^{-2}$ s/m, found by interpolation from Table 2.14.

$$F_{t\,dyn} = K_o K_\upsilon b\upsilon = 1976 \times 0.036 \times 5.5 \times 2.44 = 955 \text{ N}$$

The final values are

$$F_{t\,tot} = (F_t \times \phi + F_{t\,dyn}) \times \eta$$

$$= (4086 \times 1.4 + 955) \times 0.98$$

$$= 6542 \text{ N (assuming 98\% efficiency)}$$

$$T_{tot} = F_{t\,tot} \times \frac{d_1}{2} = 6542 \times 9.3512 = 30584 \text{ N cm} = 305.84 \text{ Nm}$$

Therefore

$$\text{Total power required} = \frac{\text{Torque (Nm)} \times \text{Speed (rpm)}}{9550}$$

$$= 305.84 \times 500/550$$

$$= 16 \text{ kW}$$

2.23 Contact Stress and Surface Durability

When gears mesh, the region of contact is theoretically a line. The curvatures of the individual mating surfaces at the points of contact will vary according to the given dimensions of the tooth profile of the mating gears as well as to the instantaneous positions of the point of contact on the line of action as the gear tooth surfaces roll and slide during the course of action. The nature of contact is, therefore, analogous to that of two contacting cylinders of constantly changing radii of curvature. In the case of external gears, both the surfaces in contact are convex while for internal gears they are convex and concave.

Though theoretically it is a line contact, the line actually develops into a band of certain width along the length of the teeth due to mutual compressive pressure. As the tooth surfaces move relative to each other with a combination of rolling and sliding, this band is also continuously on the move. The stress pattern developed within this band is quite complicated. According to Dudley, this is what happens when the gear tooth surfaces are in action — a point of maximum compressive stress is created in the centre of the hand of contact. Just below this point, a maximum subsurface shear stress develops. Due to the mutual sliding motion coupled with friction, additional stresses come into action. In the direction of sliding, each tooth surface develops a subsurface compression towards the leading end of its moving contact band while a region of tensile stress follows the trailing end.

Since the tooth surfaces undergo fluctuating, repeated and cyclic stresses of all kinds during the course of action, fatigue failure of the surface ensues. Though various kinds of gear tooth failures are attributable to fatigue, pitting is quite common (see Sec. 1.8). The continuous stress reversal plus its ever varying magnitude leads to fatigue which, augmented by heavy load, results in cracks, plastic flow and ultimately rupture of the metal, Lubricants under pressure enter these cracks, and in turn, help loosen bits of metal on the gear tooth surface, thereby producing pits.

It has been observed that pitting mostly occurs in the vicinity of the pitch line. This may be due to the fact that the direction of sliding velocity changes at the pitch line, making the effect of the compressive stress more damaging around this region. Also, this region sustains the maximum dynamic load. Since the pinion is usually the driver which makes more revolutions relatively, it is more vulnerable to pitting than the mating gear. Besides, the nature of sliding motion also promotes pitting of the driving member of the gear pair.

The phenomenon of pitting, which is essentially a type of gear tooth surface failure due to fatigue, is also generally known as wear. There is a considerable difference of opinion as to the exact definition of wear. Some gear experts assign the term "wear" exclusively to fatigue failure by pitting caused by contact stress, and use the term "abrasive wear" in case of the general thinning of tooth caused by rubbing or engendered by fine particles carried in the lubricant or embedded on the tooth surfaces. Others prefer to use the word wear in a general sense to encompass all kinds of tooth damage and surface failure which include pitting, scoring, tooth breakage due to fatigue and all kinds of abrasion. Still others restrict the term to denote removal of tooth material, layer after layer, by abrasion and other means, and do not include pitting as a wear phenomenon. In the absence of a precise definition, we will use the nomenclature "wear" in this book to indicate failure by pitting only, since this is the most common type of surface fatigue failure. Other kinds of failures will be defined, described and discussed in Sec. 8.7.

The German physicist, Heinrich Hertz, developed expressions for the stresses which are created when two curved surfaces are in contact. These surface stresses are universally known as contact stresses or Hertz stresses or Hertzian stresses. The symbol to denote the contact stress or pressure is p, while σ is used for bending stress. Both the symbols p and σ carry the appropriate subscripts to conform to the expressions or equations which are relevant to the context. For example p_{sf} stands for the surface fatigue strength and its values for common gear materials are given in Appendix E, p_c denotes contact stress in general, p_{cp} stands for permissible contact stress and p_P is the contact stress developed at the pitch point P of two mating gear tooth profiles.

Contact stress is generally the deciding factor for the determination of the requisite dimensions of gears whose tooth surfaces are not hardened. Research on gear action has confirmed the fact that beside contact pressure, sliding velocity and viscosity of lubricant as well as other factors like frictional forces also influence the formation of pits on the tooth surface. However, sufficient information and experimental data are lacking to take all these design parameters into account while making the calculations. Hence, the calculations are mainly based on Hertzian equations.

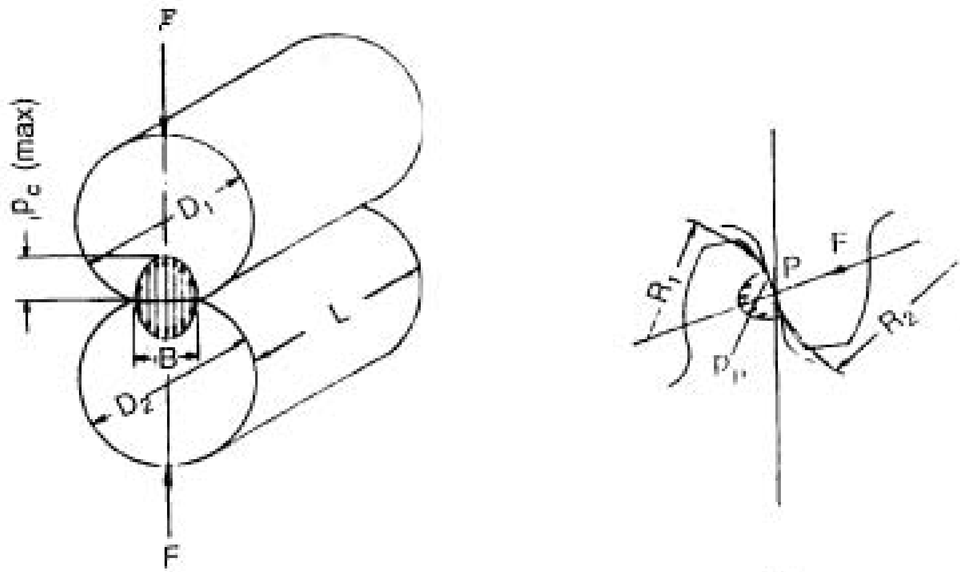
In machine design, problems frequently occurs when two members with curved surfaces are deformed when pressed against one another giving rise to an area of contact under compressive stresses. Of particular interest to the gear designer is the case where the curved surfaces are of cylindrical shape because they closely resemble gear tooth surfaces.

In Fig. 2.45 (a) two cylinders are shown in contact under compression. In Fig. 2.45 (b) two gear teeth are shown in mating condition at the pitch point. Referring to Fig. 2.45 (a), the area of contact under load is a narrow rectangle of widh B and length L. The stress distribution pattern is elliptical across the width. The maximum value is given by

$$p_{c\,(\max)} = \frac{4\,F}{\pi\,B\,L} \tag{2.86}$$

where

$$B = \sqrt{\frac{8\,F}{\pi\,L} \times \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{[(1/D_1) + (1/D_2)]}} \tag{2.87}$$



(a) (b)

Fig. 2.45 Contact **stress**

Here, F is the applied force, ν_1 and ν_2 are the Poisson's ratios of the two materials of cylinders with diameters D_1 and D_2, and E_1 and E_2 are the respective modulii of elasticity.

Combining Eqs 2.86 and 2.87, and assigning a value of 0.3 to Poisson's ratio, we have the following simplified version after replacing the diameters by the respective radii

$$p_{c\,(\max)} = \sqrt{0.35\,\frac{F\,(1/R_1 + 1/R_2)}{L\,(1/E_1 + 1/E_2)}} \tag{2.88}$$

The Hertz equations discussed so far can be utilised to calculate the contact stresses which prevail in case of tooth surfaces of two mating spur gears. Though an approximation, the contact aspects of such gears can be taken to be equivalent to those of cylinders having the same radii of curvature at the contact point as the load transmitting gears have. This assumption yields reasonably accurate results in the region of the pitch line. However, it is to be kept in mind that the radius of curvature changes continuously in case of an involute curve, and it changes sharply in the vicinity of the base circle. In fact near the base circle this radius approaches zero and the surface stress approaches infinity. Hertz equations do not produce accurate stress values in this region, and contact in this region is to be avoided to avoid the deterimental effects of high surface stresses. Contact stress equations are valid only on the assumption that the elastic limit is not exceeded and that the contact band is subjected to only compressive stress.

For calculation, the surface at the pitch point is generally taken as the criterion as regards surface loading of gear teeth. To find this stress, the following relations are inserted to make use of **Eq. 2.88**

$$F = \frac{F_t}{\cos \alpha},\ L = b,\ R_1 = d_1 \sin \alpha / 2 = r_1 \sin \alpha,\ R_2 = d_2 \sin \alpha / 2 = r_2 \sin \alpha$$

where F_t is the tangential force or transmitted load, b is the tooth width, R_1 and R_2 are the radii of curvature at the pitch point, and d_1 and d_2 are the pitch circle diameters of the gears

Putting

$$\frac{1}{E} = \frac{1}{2}\left(\frac{1}{E_1} + \frac{1}{E_2}\right),\ \text{or } E = \frac{2\ E_1\ E_2}{E_1 + E_2}$$

for pinion and gear materials with different modulii of elasticity, and

$$u = \frac{d_2}{d_1}$$

we arrive at the following relations,

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{\sin \alpha}\left(\frac{1}{d_1} + \frac{1}{d_2}\right) = \frac{2}{\sin \alpha}\left(\frac{1}{d_1} + \frac{1}{u d_1}\right) = \frac{2}{\sin \alpha}\ \frac{1}{d_1}\left(1 + \frac{1}{u}\right) = \frac{2}{d_1 \sin \alpha}\ \frac{u+1}{u}$$

Inserting these values in **Eq. 2.88**, we get the expression for the maximum contact pressure at the pitch point

$$p_p = \sqrt{0.35 \frac{F_t}{\cos \alpha}\ \frac{1}{b}\ \frac{2}{\sin \alpha}\ \frac{1}{d_1}\ \frac{u+1}{u}\ \frac{E}{2}} = \sqrt{0.35 \frac{F_t\ E}{b\ d_1}\ \frac{u+1}{u}\ \frac{1}{\cos \alpha\ \sin \alpha}} \tag{2.89}$$

In case of S-corrected gears with service pressure angle α_w Eq. **2.89** is modified to

$$p_p = \sqrt{0.35 \frac{F_t\ E}{b\ d_1}\ \frac{u+1}{u}\ \frac{1}{\cos^2 \alpha \tan \alpha_w}} \tag{2.90}$$

To simplify calculation, Eq. **2.90** is written in the form

$$p_p = y_m\ y_p \sqrt{\frac{F_t}{b\ d_1}\ \frac{u+1}{u}} \tag{2.91}$$

where y_m is the material coefficient and y_p is the pitch point coefficient, which are given by

$$y_m = \sqrt{0.35\ \frac{2\ E_1\ E_2}{E_1 + E_2}} = \sqrt{0.35 E} \tag{2.92}$$

$$y_p = \sqrt{\frac{1}{\cos^2 \alpha\ \tan \alpha_w}} \tag{2.93}$$

The value of y_p is 1.76 for uncorrected and S_0-corrected gearing. For y_m see table 2.17 for pairing of common gear materials.

Table 2.17 Material coefficient y_m for pairing of common gear materials

Pinion		Gear		
Material	E_1 N/mm²	Material	E_2 N/mm²	y_m $\sqrt{N/mm^2}$
Steel	206,000	Steel	206,000	269
		Cast steel: Grade 30–57 Grade 26–52	 201,000 201,000	 267 267
		Cast iron with spheroidal graphite SG 50017	173,000	257
		Bronze	103,000	219
		Cast iron: FG 260 FG 200	 126,000 118,000	 234 229
Cast steel Grade :30–57	201,000	Grade 26–52	201,000	265
		SG 500/7	173,000	255
		FG 200	118,000	228
SG 50017	173,000	FG 200	118,000	222
FG 200	126,000	FG 200		207
FG 200	118,000		118,000	203

Based on Zahnraeder, Zirpke, 11th edition, 1980, table no. 31, p. 397. VEB Fachbuchverlag, Leipzig.

Recalling Sec. 1.8, the surface durability is the property which determines the ability of the gear surface to resist fatigue type of tooth surface failure caused by contact pressure. For satisfactory service and life, the gears are to be designed in such a way that the surface stresses lie well within the surface endurance limit of the material used. Appendix E gives the values of the surface fatigue strength of common gear materials. In Sec. 1.8 the importance of the compressive strength and hardness of the tooth surface has been emphasised. The power rating of modern industrial gears is primarily based on the surface strength and not on the beam strength. The safe limit of contact stress should, therefore, be carefully chosen, and this limit should never be exceeded to avoid surface failure. Guidelines for selecting the proper design stress are detailed in Sec. 2.25 on strength calculation.

2.24 Stress Concentration

From mechanics, we know that stress concentration plays a major role when a machine member is subjected to fatigue type of loading. As gear teeth have to withstand repeated and fluctuating types of load, their failure is essentially attributable to fatigue phenomenon. Studies through photoelasticity have revealed that stress concentration occurs mainly at the point of loading and at the root fillets. The magnitude of stress concentration at the root fillet depends on the minimum radius of the fillet and also on the general configuration of the part. Generating methods of gear cutting produce fillets which are actually trochoids and not arcs of circles.

The cutter tips produce these fillets and there is no single standard for the curvature of the tips which will give the best result as far as the avoidance of stress concentration is concerned. For precise calculations, stress concentration should also be taken into account. Apart from fillet radius and pressure angle, stress concentration factor is a function of material, root thickness of tooth, load position on the tooth, and other parameters.

Employing photoelastic studies, the following empirical relation for gears with 20° pressure angle has been derived by Dolan and Broghamer*

$$K_t = 0.18 + \left[\left(\frac{t}{r}\right)^{0.15} \times \left(\frac{t}{h}\right)^{0.45}\right] \tag{2.94}$$

where K_t is the theoretical stress concentration factor, r is the minimum fillet radius, h is the height of applied load above the weakest section, and t is the thickness of tooth at the weakest section.

This theoretical value should be modified by the notch sensitivity factor q, the details of which may be found in any standard book on mechanics. The value of q will depend upon the fillet radius, material and other factors. For hardened teeth, the value of q is practically unity, so that the theoretical value holds good. Calling the modified or actual stress concentration factor as K_f, the stress produced at the tooth root should be arrived at after multiplying the stress value (calculated on the basis of the transmitted load or tangential force F_t) by K_f and then proceeding

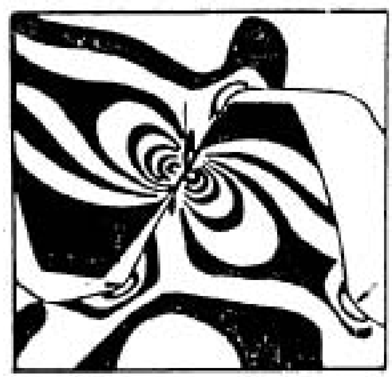


Fig. 2.46 Pattern of stress distribution
(Photoelastic method)
Areas of stress concentration are indicated by arrows

* T.J. Dolan and E.I. Broghamer: *A photoelastic study of the stresses in gear tooth fillets*, Univ. Ill Eng. Expt. Sta. Bull. 1942.335.

accordingly by the usual methods given in the relevant sections dealing with strength calculation. For hardened teeth $K_f \approx K_t$, giving safer stress values. Since the stress concentration enhances the stress, the actual stress may be 1.2 to **2** times the theoretical stress based on conventional equations. Obviously, the actual stress must lie below the allowable stress for the material. Further, since the stress is of dynamic nature, it should be less than the endurance limit to ensure a reasonable margin of safety.

Figure 2.46 shows the pattern of stress distribution on a mating pair of gear teeth using photoelastic method. Under polarised light, the stress pattern becomes visible on transparent plastic models of gear teeth. The resulting alternate layers of bright and dark areas are the isochromatic fringes produced by double refraction. Positions of maximum stress are indicated by a crowding of these layers of bright and dark areas as shown in the figure. By and large the photoelastic method of stress analysis agrees with the actual stress conditions, except in some special cases.

2.25 Strength Calculations and Power Rating

Complete knowledge of the stresses which the gear teeth are subjected to is imperative for the determination of the different parameters of a gearing system. This should be as exact as possible and is a prerequisite for the proper design to avoid damage or failure of the gears within the stipulated life. Hence, the most important stresses which the gears normally encounter should be theoretically checked as regards load-carrying capacity.

It should, however, be kept in mind that the load capacity of a gear pair can be determined reliably only by practical experimentation. The stress conditions are of three dimensional nature and are not readily amenable to theoretical analysis. Running tests on various specimens of different materials and in different simulated service conditions are conducted from which relevant charts, tables and curves are made to facilitate prediction of the behaviour of gears in actual operational life. Factors like tooth shape, root fillet form and stress concentration thereof, surface finish, meshing conditions and other data are analytically assessed. Factors which are generally unpredictable, such as variable loads, peak load, effect of tooth errors, uneven load distribution, and the detrimental effects of improper lubrication are usually taken care of by using semi-empirical relations and similar methods. For high power gear drives, best results are obtained by hardened and precisely ground teeth. These gears have an extremely high load carrying capacity coupled with a high degree of reliability, restriction of size and weight requirements.

From theoretical considerations, appropriate gear dimensions can be reasonably arrived at by using the design data available in books, manufacturer's manuals, journals, and the standard specifications, such as IS, DIN, BS, GOST, and the various American standards. The most important stresses which should be considered for gear designing are:

(*i*) the stress due to the bending of the tooth, and

(*ii*) the stress created by contact pressure, generally known as Hertz stresses.

Besides, gear failure by scoring is also considered.

Basically, failures due to both the above kinds of stresses that is, bending stress and contact stress, may be considered as types of fatigue failures since the stresses are of cyclic nature. Tooth breaks due to the repetition of bending stress of varying magnitude whereas surface failure occurs due to the numerous application of contact stress causing flakes and particles to detach themselves from the tooth surfaces where cracks have formed below the surface (see Sec. 2.23).

In this section, the strength calculation and power rating will be treated in three ways by using:

(a) relations from the first principles of gear drive,
(b) a simplified method for quick and rough calculation, and
(c) methods laid down in IS: 4460.

Relations from the First Principles of Gear Drive

Essentially, a gear tooth in operation can be considered as a cantilever beam under load. For this reason, the ability of the tooth to resist tooth breakage at the root is often referred to as its beam strength. The basic tooth stresses caused by bending were first investigated by Wilfred Lewis and the bending stress equation is still known as the Lewis formula although much modification of the original formula has taken place during the period of nearly 10 years.

Referring to Fig. 2.47, the tooth load F_N is supposed to act at the tip corner as shown. Under this assumption, calculation becomes simplified. Load F_N acts along the line of action at a grip angle of α'_a at the tip corner. When referred to pitch point P, angle a is the pressure angle of the the system. As explaind in earlier sections, force F_N is normal to the tooth profile at pitch point P on the line of action.

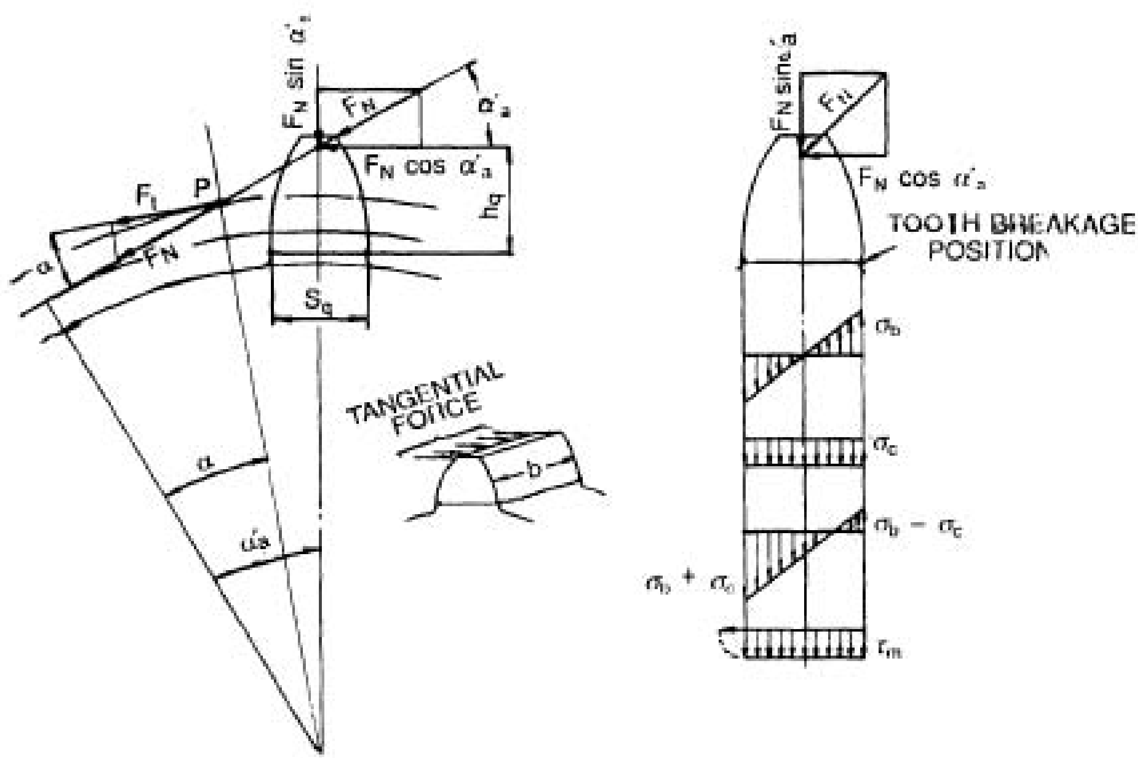


Fig. 2.47 Gear tooth forces

Based on Die Traglaehigkeit der Zahnraeder. Thomas and Charchut, 7th edition, 1971, Fig. No. 25, p. 67. Carl Hanser. Verlag, Munich

If at the intersection of the line of action and the present centre line of the tooth, this force F_N is resolved, then we get

the radial component $= F_N \sin \alpha'_a$

the tangential component $= F_N \cos \alpha'_a$

From Fig. 2.47 we can easily see that the following individual stresses are created.

$$\text{Bending stress } \sigma_b = \frac{\text{Bending moment}}{\text{Section modulus}} = \frac{F_N \cos \alpha'_a h_q}{\frac{1}{6} b S_q^2} = \frac{6F_N \cos \alpha'_a h_q}{b \; S_q^2} \tag{2.95}$$

Compressive stress

$$\sigma_c = \frac{F_N \sin \alpha'_a}{b \; S_q} \tag{2.96}$$

Average shear stress

$$\tau_m = \frac{F_N \cos\alpha'_a}{b \; S_q} \tag{2.97}$$

where S_q is the thickness of the weakest portion at the root of tooth.

Research work in this field has established the fact that for calculation of practical strength the compressive and the shear stresses can be neglected since their effects are marginal. Only the bending stress, therefore, is taken as the determining factor with sufficient accuracy. Now, the tangential force or the transmitted load is given by

$$F_t = F_N \cos a$$

Normally in solving a gear problem, the power and the speed are given. The following formulae are recalled

$$F_t \text{ (N)} = \frac{2T_1 \text{ (Nm)}}{d_1 \text{(mm)}} \times 1000$$

Torque

$$T_1 = 9550 \times \frac{P_1 \text{ (kW)}}{n_1 \text{ (rpm)}} \text{ (Nm)} = 974 \times \frac{P_1 \text{ (kW)}}{n_1 \text{ (rpm)}} \text{ (kgfm)}$$

$$= 716.2 \times \frac{P_1 \text{ (Metric HP)}}{n_1 \text{ (rpm)}} \text{ (in kgfm)}$$

Circumferential velocity

$$v \text{ (m/s)} = \pi d_1 n_1 = \frac{d_1 \text{ (mm)} \times n_1 \text{ (rpm)}}{19100}$$

Also

$$F_t = 1000 \times \frac{P_1 \text{ (kW)}}{v \text{ (m/s)}} \text{ (N)} = 102 \times \frac{P_1 \text{ (kW)}}{v \text{ (m/s)}} \text{ (kgf)} = 75 \times \frac{P_1 \text{ (metric hp)}}{v \text{ (m/s)}} \text{ (kgf)}$$

Substituting $F_t/\cos a$ for F_N in Eq. 2.95, multiplying the denominator and numerator by m, and after rearranging, we have.

$$\sigma_b = \frac{F_t}{b\ m}\frac{6\ m\ h_q\ \cos\alpha'_n}{S_q^2\ \cos\alpha} = \frac{F_t}{bm}q_k \tag{2.98}$$

The expression

$$q_k = \frac{6\ m\ h_q\ \cos\alpha'_n}{S_q^2\ \cos\alpha} \tag{2.99}$$

is used to derive the form factor q_k the values of which have been computed and are presented in Figs 2.48 and 2.49. These figures can be used for spur, helical, uncorrected and corrected gears. The magnitude of the module has no influence on the form factor since h_q and S_q also increase in proportion to m. The effect of topping on the value of q_k is also negligible.

Besides q_k, the equation for the bending stress is to be modified by taking into account another factor called the overlap factor or the contact ratio factor q_e. Equation **2.95** is based on load taken by one pair of mating teeth only. Factor q_e is used to make allowance for the reduced load carried at the tooth tip due to two pairs of teeth making simultaneous contact and thus sharing the load. With sufficient accuracy, q_e can be taken to be equal to the reciprocal of the contact ratio (CR). That is

$$q_e = \frac{1}{CR}$$

The contact ratio can be calculated by using Eqs 2.12 and **2.13** given in Sec. **2.7.** However, as the circumferential force does not distribute itself on the tooth pairs as it ideally should due to inaccuracy in toothing or low load, the value of q_e can be taken as **1**.

We can now finally write the bending stress equation as

$$\sigma_b = \frac{F_t}{b\ m}q_k\ q_e \tag{2.100}$$

Since the number of teeth as well as the materials of the pinion and the gear are usually different, it is advisable to check the bending stress for both the members of the gear set.

Equation 2.100 for bending stress is to be checked against the allowable bending stress of the material. This working stress can be found in different ways and its selection depends on the discretion of the gear designer. Some guidelines are discussed here in this regard.

Permissible bending stress σ_{bp} can be directly taken from Appendix E which shows strength characteristics of common gear materials. It can also be found by using the relation.

$$\sigma_{bp} = \frac{\sigma_e}{\text{Factor of safety}} = \frac{\sigma_e}{2\text{ to }3} \tag{2.101}$$

It has been emphasised before that gear teeth subjected to bending is actually a manifestation of gear fatigue. Moreover, the teeth may be subjected to continuously reversible drives. Keeping all these factors in mind, some designers prefer to select the maximum permissible tooth bending stress according to the number of stress cycles the gear is expected to undergo during its operational life. Such maximum allowable stress values as a function of the number of stress cycles for different materials can be obtained from Fig. **2.50.** If the stress conditions of a drive are known with sufficient accuracy, reliable gear design can be made with a minimum factor of safety.

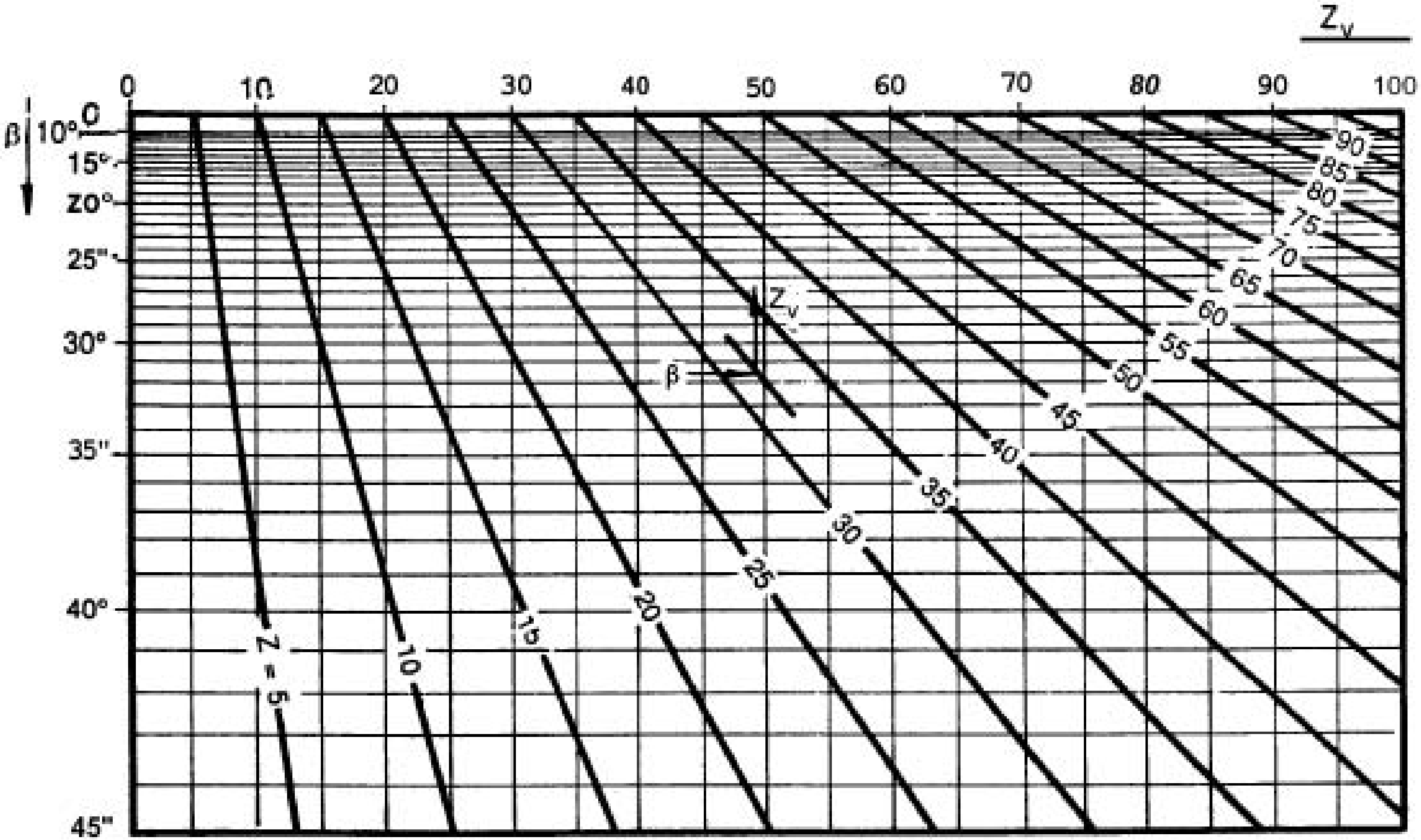


Fig 2.48 Virtual number of teeth Z_v in relation to helix angle β and number of teeth Z

Based on Die Tragfaehigheit der Zahnraeder, Thomas and Charchut, 7th edition, 1971, Fig. No. 28, p. 71 Carl Hanser, Verlag, Munich

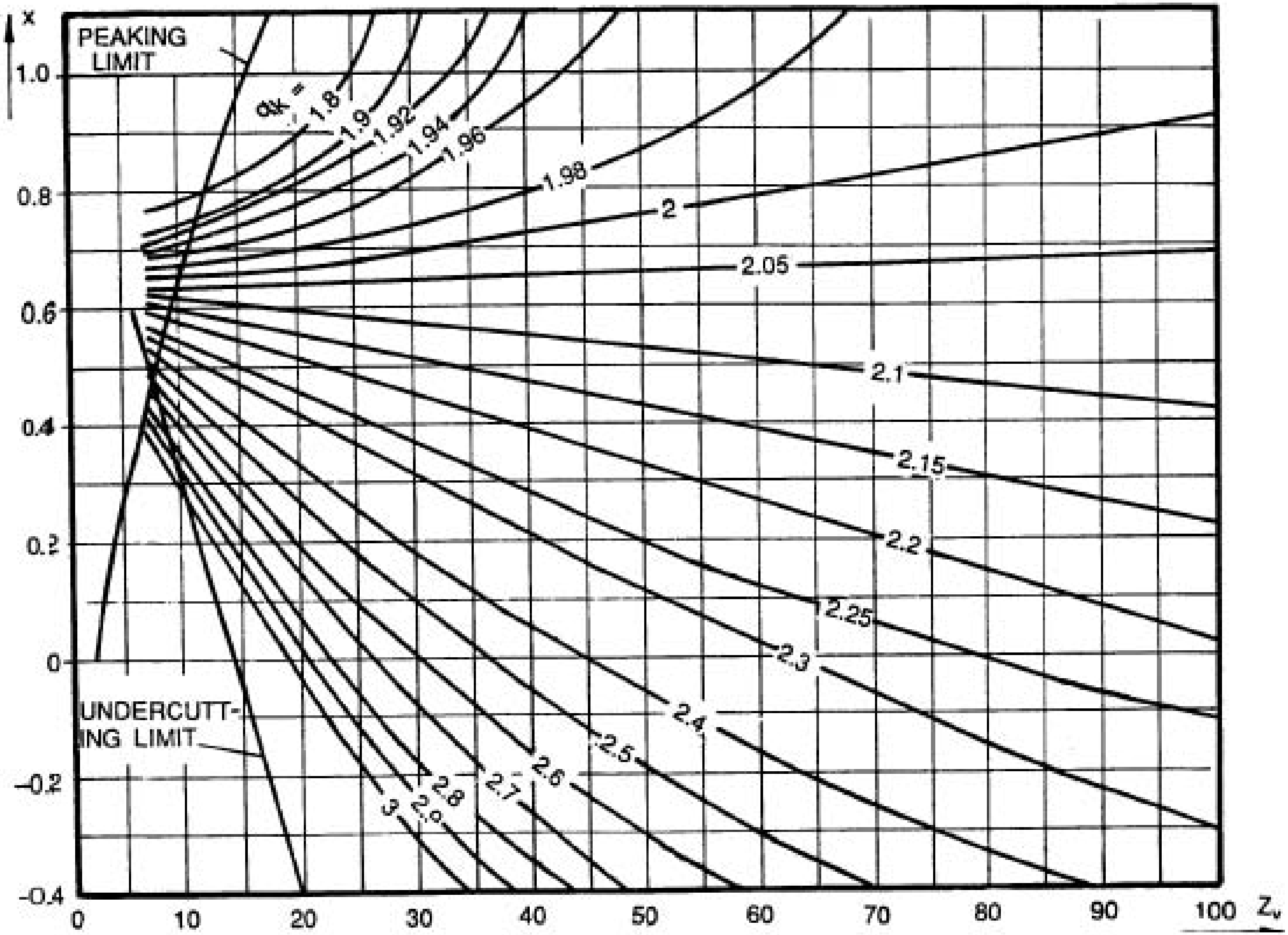


Fig. 2.49 Tooth form factor q_k
Based on Die Tragfaehigkei der Zahnraeder. Thomas and Charchut, 7th edition, 1971, Fig. No. 28, p. 71, Carl Hanser, Verlag, Munich

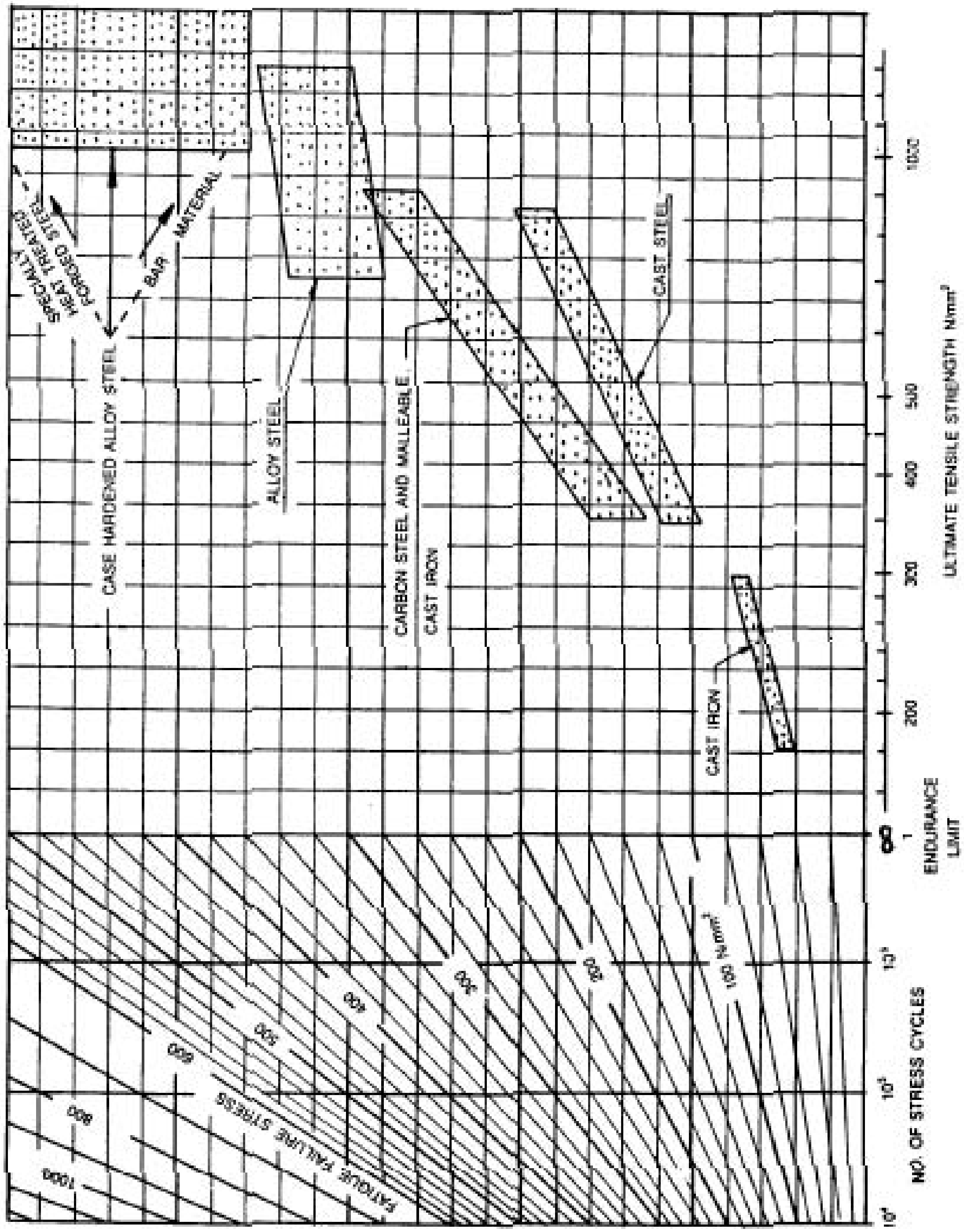


Fig. 2.50 Maximum permissible tooth bending stress
Based on MAAG Handbook of M/s. MAAG Zahnraeder AG, Zurich, page 124

The endurance limits for different materials given in Fig. *2.50* are based on conservative estimates. **For** cases where the stress cycles are large ***or for infinite life,*** the endurance limits in this figure can be taken as the maximum permissible stress for design calculation.

For hardened gears, the bending stress is usually the determining factor. Using several equations given before we can arrive tentatively at the required value of the module.

Thus

$$\sigma_{b1} = \frac{F_t}{b\ m} q_{k1}\, q_e = \frac{2000\ T_1}{d_1^2 \left(b / d_1\right) m} q_{k1} q_e = \frac{2000\ T_1 q_{k1} q_e}{z_1^2\ m^3 \left(b / d_1\right)}$$

Putting σ_{bp} for σ_{b1} and transposing, we get

$$m = \sqrt[3]{\frac{2000\ T_1\ q_{k1}\ q_e}{z_1^2 \left(b / d_1\right) \sigma_{bp}}}$$

A suitable value of the quotient b / d_1 can be obtained from Sec. 2.16. **As** the values of q_{k1} and q_e are also not known initially, they are to be guessed. The average values which can be taken are $q_{k1} = 2.25$ and $q_e = 0.9$. Inserting these values, we can arrive at the following expression for the module

$$m = \sqrt[3]{\frac{4000\ T_1}{z_1^2 \left(b / d_1\right) \sigma_{bp}}} \tag{2.102}$$

As explained earlier, besides the bending stress, contact pressure is also created where the two surfaces of the mating teeth meet. This surface stress, commonly known as Hertz stress, has been discussed in detail in Sec. *2.23*. For unhardened gears, the contact stress is usually the deciding design criterion. Recalling Eq. *2.91* and inserting the permissible surface stress p_{cp} for p_p, we have the following expression after modifying the equation in the same manner as in the case of bending stress

$$p_{cp} = y_m\ y_p \sqrt{\frac{2000\ T_1}{\left(b / d_1\right) d_1^3} \cdot \frac{u + 1}{u}} \tag{2.103}$$

Putting $d_1 = z_1 m$ and transposing **Eq.** *2.103*, we get

$$m = \frac{1}{z_1} \sqrt[3]{\frac{2000\ T_1\ y_m^2\ y_p^2}{\left(b / d_1\right) p_{cp}^2} \cdot \frac{u + 1}{u}} \tag{2.104}$$

Depending on the material pairing, the value of y_m is to be taken from Table 2.17 of Sec. *2.23*. The factor y_p is not generally known beforehand. An average value of $y_p = 1.66$ can be taken for initial calculation.

For calculating permissible surface stress, the following relation can be used for all practical purposes

$$p_{cp} = \frac{p_{sc}}{1.5} \tag{2.105}$$

where p_{sc} is the surface fatigue strength as given in Appendix E.

In case the value of the surface fatigue strength is not readily available, the following simple relationship gives fairly accurate results

For cast iron and heat-treatable steel

$$p_{sc} \approx 0.25 \times HB \tag{2.106}$$

For surface-hardened teeth

$$p_{sc} = 25 \times HRC \tag{2.107}$$

As indicated earlier, gears need not be designed for infinite life. In case of applications involving finite life, the permissible Hertz stress can obviously be taken higher than that meant for infinite life. Experiments have shown that the contact stress at which pitting may be expected to occur for a finite life corresponding to 10^N load cycles is approximately $\sqrt{7/N}$ times the surface fatigue strength, where N is in the range of **3** to **7.** Depending upon the type of loading as to its frequency, the number of load cycles can be expressed as follows

For constant loading

$$L_c = 60 \times n \times L_h \tag{2.108}$$

where L_c = Life in number of cycles
L_h = Life in hours
n = Speed in rpm

For variable loading the, equivalent life in number of cycles is given by

$$L_{c\,eq} = 60\left[L_{h1}n_1\left(\frac{T_1}{T_1}\right)^3 + L_{h2}n_2\left(\frac{T_2}{T_1}\right)^3 + L_{h_3}n_3\left(\frac{T_3}{T_1}\right)^3 + \ldots\right]$$

$$= \frac{60}{T_1^3}\sum T_i^3\ L_{hi}\ n_i \tag{2.109}$$

where, T_1 is the maximum sustained torque acting for L_{h1} hours at n_1 rpm, and $T_2, T_3 \ldots$ are small sustained torques acting for L_{h2}, L_{h3} ... hours at $n_2, n_3 \ldots$ rpm

Example 2.9 *Given:* Input through electric motor, load is of impact nature, gears to be mounted on shafts fitted with anti-friction bearings in a gear box. Nominal power, P_r = **2 kW,** transmission ratio, i = 3, input speed n_1 = **50** rpm, z_1 = **45.** A suitable spur gear pair is to be designed.

Solution: The following data are taken

b/d$_1$ = 0.5, material of pinion **40** Cr **4,** material of gear **45** C 8.

Both members of the gear set are flame-hardened.

$$z_2 = i\,z_1 = 3 \times 45 = 135$$

To have hunting tooth action, we choose z_2 = 137 which is a prime number

$\therefore$ $i = 137/45 = 3.04$. The deviation of i is within $\pm 3\%$ which is the usual allowable limit in this case.

$$n_2 = \frac{n_1}{3.04} = \frac{50}{3.04} = 16.45 \text{ rpm}$$

Nominal torque at the pinion shaft = $9550 \times \frac{P_1}{n_1} = 9550 \times \frac{2}{50} = 382$ Nm

Since the load is of impact nature, an impact factor of 1.5 is taken. Hence, the actual torque is given by

$$T_1 = 382 \times 1.5 = 573 \text{ Nm}$$

From Appendix E, we get

$$\sigma_{bp1} = 200 \text{ N/mm}^2,\ \sigma_{bp2} = 180 \text{ N/mm}^2,\ p_{cp1} = \frac{1620}{1.5} = 1080 \text{ N/mm}^2,$$

$$p_{cp2} = \frac{1640}{1.5} = 1093 \text{ N/mm}^2$$

$$m \approx 3\sqrt{\frac{4000 \times 573}{45^2 \times 0.5 \times 200}} = 2.251 \text{ m} = 2.5 \text{ mm (taken)}$$

$$d_1 = 45 \times 2.5 = 112.5 \text{ mm},\ d_2 = 137 \times 2.5 = 342.5 \text{ mm}$$

$$b = 0.5 \times 112.5 = 56.25 \text{ mm} \approx 56 \text{ mm (taken)}$$

Check : Referring to Sec. 2.16, we have

$$b_{max} = \lambda m = 25 \times 2.5 = 62.5 \text{ mm} > 56 \text{ mm}$$

$$m_{min} = b/\lambda = 56/25 = 2.24 \text{ mm} < 2.5 \text{ mm}$$

The above value are, therefore, within the specified limits.

$$F_t = \frac{2000 \times 573}{112.5} = 10187 \text{ N},\ v = \frac{112.5 \times 50}{19,100} = 0.3 \text{ m/s}$$

In accordance with the velocity, quality 10 is chosen.

Centre distance $a = \frac{d_1 + d_2}{2} = \frac{112.5 + 342.5}{2} = 227.5$ mm

The nearest higher standard centre distance is 250 mm as per Table 2.5 of Sec. 2.13. The reader is advised to recalculate and determine the appropriate correction factors as well as the changed values of d_{a1} and d_{a2} with the help of sections dealing with correction, keeping in mind the limitations imposed by too high a correction factor.

From Fig. 2.49 $q_{k1} = 2.4$ for $z_1 = 45$. Since $z_2 = 135$, we get the value $q_{k2} = 2.15$ by interpolation. Therefore

$$\sigma_{b1} = \frac{F_t}{b\ m} q_{k1}\, q_e = \frac{10,187}{56 \times 2.5} \times 2.4 \times 1 = 175 \text{ (N/mm}^2\text{)} \quad (q_e = 1, \text{ taken})$$

$$\sigma_{b2} = \frac{F_t}{b\ m} q_{k2} q_e = \frac{10,187}{56 \times 2.5} \times 2.15 \times 1 = 156\ (\text{N/mm}^2)$$

y_m = 269 for steel on steel from Table 2.17

y_p = 1.76 for uncorrected gearing.

From Eq. 2.91 (Sec. 2.23)

$$p_p = y_m\ y_p \sqrt{\frac{F_t}{b\ d_t} \times \frac{u+1}{u}} = 269 \times 176 \sqrt{\frac{10,187}{56 \times 112.5} \times \frac{3.04+1}{3.04}} = 694\ (\text{N/mm}^2)$$

Hence, both the bending stresses and the surface stresses are within allowable limits as given earlier.

Simplified Method of Strength Calculation

In case where enoughdata are not available or where approximate values will serve the purpose, the following simplified method of strengthcalculationmay be adopted. Assuming that one tooth only carries the total force at a time, we get the following relation

$$F_t(\text{N}) = bpc \tag{2.110}$$

where

b = Tooth width (cm), p = Circular pitch (cm) = πm, and c = Load factor (N/cm²). Factor c can be expressed as

$$c = c_0 K_1 K_2 K_3$$

Here $c_0 = \sigma_{bp}/Q\pi$ where Q = A factor which depends on the type of toothing, meshing relations and the number of teeth; $Q \approx$ 5.5 to 2.5 with increasing number of teeth. Taking the value of c_0 for cast iron (CI) as the basis for calculation and considering the effects of velocity v and thereby of wear, we get a modified expression of c_0

$$c_0 = \frac{3000}{3\pi} \times \frac{10}{v+10} \approx 300 \times \frac{10}{v+10}\ (\text{Ncm}^2)$$

where σ_{bp} is taken as 3000 N/cm² for CI and the average value of Q is taken as 3. The circumferential velocity v at the pitch line is calculated after assuming a value for d.

Thus

$$v = \frac{\pi\ dn}{60}\ (\text{m/s})$$

Factor K_1 gives the relation betwen the strength of cast iron *vis-a-vis* that of other materials. Thus

K_1 = 1 for CI
= 1.8 to 2.5 for cast steel (average = 2)
= 2.8 to 3.3 for Fe 490, Fe 410, Fe 620 (average = 3)
= 5 to 9 for alloy steels and hardened steels
= 1.7 for phosphor bronze
= 0.8 to 1 for pressed synthetic materials

Factor K_2 takes care of the type and duration of service to which the gear drive is subjected.

K_2 = 0.8 to 1 for normal service conditions
= 0.6 to 0.7 for heavy duty
= 0.5 for heavy duty and shock loads

Factor K_3 is for other influences e.g. tooth form, lubrication, contact ratio.

K_3 = 0.8 for unmachined straight teeth
= 1 for machined straight teeth
= 1.1 to 1.3 for machined helical teeth
= 1.5 for ground helical teeth

$$\text{Let } \Delta = \frac{\text{Tooth width}}{\text{Circular pitch}} = \frac{b}{p} = \frac{b}{\pi m}$$

Its value can be taken as indicated below

Δ = 2 to 3 for cast teeth
= 2 to 5 for unground, hardened teeth
= 3 to 6 for machined teeth
= 6 to 13 for fine-machined teeth flanks, $n_1 \leq 3000$ rpm
= 13 to 27 for teeth with best surface quality, high accuracy of toothing, mounted on rigid bearings, and $n_1 \geq 3000$ rpm

Circumferential force is given by

$$F_t \text{ (N)} = \frac{2T}{d} = \frac{2T}{mz} = \frac{2T}{z}\frac{\pi}{p} = bpc = \Delta\, ppc$$

Here torque T is in N cm, and d and m are both in cm. By transposing, we get

$$p \text{ (cm)} = \sqrt[3]{\frac{2\pi T}{z \Delta c}} \tag{2.111}$$

Putting

$$T = 955{,}000 \times \frac{P \text{ (kW)}}{n \text{ (rpm)}}$$

we can find the value of p from Eq. 2.111 and thereby the required module m.

Calculations in case of helical gears can be made in a similar manner with the value of σ_{bp} taken as 20 to 40% higher than that of spur gears.

Example 2.10 An electric motor of 2.5 kW drives a shaft through a single reduction unit. Motor pinion data $z_1 = 14$, $n_1 = 1000$ rpm, Driven gear $z_2 = 70$.

To find (*i*) speed of driven shaft, (*ii*) module and the width of the gears made of CI with milled teeth, and (*iii*) diameter of the driven shaft, material: 45 C 8.

Solution: (*i*) $n_1 z_1 = n_2 z_2 \quad \therefore n_2 = \frac{n_1 z_1}{z_2} = \frac{1000 \times 14}{70} = 200 \text{ rpm}$

(*ii*) $c = c_0 K_1 K_2 K_3, \; c_0 = 300 \times \frac{10}{v + 10}$

Assumed, d_1 = 100 mm = 0.1 m $\quad \therefore v = \frac{\pi d_1 n_1}{60} = \frac{3.14 \times 0.1 \times 1000}{60} = 5.2 \text{ m/s}$

$c_0 = 300 \times \frac{10}{5.2 + 10} = 200 \text{ N/cm}^2$ $K_1 = 1$ for CI. $K_2 = 1$ for normal service, $K_3 = 1$ for milled teeth $\therefore c = 200 \times 1 \times 1 \times 1 = 200$ N/cm²

$$\lambda = \frac{b}{p} = 3 \text{ (selected)}$$

$$\therefore \quad p = 3\sqrt{\frac{27\pi}{14 \times 3 \times 200} \times \frac{955000 \times 2.5}{1000}} = 1.21 \text{ cm}$$

$$= 12.1\text{mm} = \pi \times 3.85 \quad \therefore m = 3.85 = 4 \text{ mm (taken)}$$

$$d_1 = mz_1 = 4 \times 14 = 56 \text{ mm} \quad d_2 = mz_2 = 4 \times 70 = 280 \text{ mm},$$

$$b = p\,\lambda = \pi 4 \times 3 = 40 \text{ mm}$$

(*iii*) Taking allowable torsional stress to be 2000 N/cm² for the shaft material, we have

Torque

$$T\text{ (N m)} = 9550 \times \frac{2.5\text{ (kW)}}{200\text{ (rpm)}} = 955\,000 \times \frac{2.5}{200}\text{ (N cm)}$$

Also

$$T\text{(Ncm)} = Z\text{ (section modulus)} \times \tau_p\text{ (permissible shear stress)}$$

$$= \frac{\pi}{16} \times d^3 \times 2000$$

whence the diameter of the driven shaft

$$d = 3.1 \text{ cm} = 31 \text{ mm} \approx 35 \text{ mm (taken)}$$

Method for Rating of Machine Cut Gears as per IS: 4460

Method for rating spur and helical power transmitting gears connecting parallel shafts and meant for general engineering applications has been specified in IS: 4460. The method primarily deals with checking of power transmitting capabilities of a gear set when the relevant parameters, such as material, gear dimensions, speed, are known so as to ensure whether the particular drive meets with the requirements. For this purpose, a set of formulae along with the values of the basic stress factors for common gear materials as well as other relevant factors, e.g. speed factors, zone factors, pitch factors, has been specified. The stress factors in tabular form and other factors in graphical form are given at the end of this sub-section (Table 2.18 and Figs 2.51-2.56). In IS: 4460, the conventional metric system has been used. As such, the units are kept unaltered. The reader can refer to Appendix T for conversion into SI units. The symbols for materials are also left as given in IS: 4460. For new symbols see Appendix E.

In this sub-section horse power always means metric horse power only (denoted by the German symbol PS) which is equal to about 0.736 kW, as distinct from the British horse power (HP) which is equal to about 0.746 kW.

The power rating of spur and helic gears is divided into (*i*) horse power for strength, and (*ii*) horse power for wear. These are given by

Horse power for strength

$$\text{(PS)} = \frac{X_b\; YS_b bm^2 nz}{25.4} \times \frac{1774}{10^8} \tag{2.112}$$

Horse power for wear

$$\text{(PS)} = \frac{X_c\; Y_z S_c bmnz}{K} \times \frac{1774}{10^8} \tag{2.113}$$

The Standard also specifies formulae for allowable tangential loads with respect to strength and wear, and these formulae can be used for checking purposes when the tangential load of a gear drive is calculated from the given power, speed and gear dimensions. These are given by.

Allowable tangential load (kgf/mm of tooth width)

$$= X_b Y S_b m \quad \text{for strength} \tag{2.114}$$

$$= \frac{X_c Y_z S_c}{K} \times 25.4 \quad \text{for wear} \tag{2.115}$$

where

X_b = Speed factor for strength

X_c = Speed factor for wear

Y = Strength factor. See Fig. 2.53 for spur gears and helical gears with 30' helix angle. For other helix angles, the strength factors obtained from Fig. 2.53 shall be multiplied by 1.33 $\cos^2\beta$

Y_z = Zone factor. See Fig. 2.54 for spur gears and Fig. 2.55 for helical gears with 30' helix angle. For other helix angles, the zone factor obtained from Fig. 2.55 shall be multiplied by 0.75 x $\sec^2\beta$

K = Pitch factor

S_b = Bending stress factor, this depends on the tensile strength of the material

S_c = Surface stress factor, this depends on the hardness of the material

Other factors, such as b, m, z and β have same meaning as before. The normal rating of a gear is the allowable continuous load for 12 hours running time per day.

The following examples will illustrate the use of the above formulae, tables and graphs.

Example 2.11 A pair of spur gears is required to reduce the speed from 500 to 100 rpm working continuously for 12 hours running time. The pinion is of 0.40% carbon steel, is normalised, and has 20 teeth. The gear is of cast iron, grade 20, IS: 210, and has 100 teeth, m = 8, b = 100 mm, a = 20". The allowable horse power of the pair is to be determined.

Solution: From Eq. 2.112 horse power for strength (PS) = $X_b Y S_b b m^2 n z \times 1774/25.4 \times 10^8$.

For pinion, we get the following values

X_b = 0.3175 from Fig. 2.51
Y = 0.72 from Fig. 2.53
S_b = 14.05 from Table 2.18

Hence PS for pinion = 0.3175 x 0.72 x 14.05 x 100 x 64 x 500 x 20 x 1774/(25.4 x 10^8) = 143
Similarly PS for gear = 0.42 x 0.615 x 4.22 x 100 x 64 x 100x100 x 1774/(25.4 x 10^8) = 48.7.

From Eq. 2.113 horse power for wear (PS) = $X_c Y_z S_c b m n z \times 1774/(K \times 10^8)$. Using the relevant tables and figures, we get the following values after inserting the appropriate data

PS for pinion = 0.305 x 2.2 x 1.125 x 100 x 8 x 500 x 20 x 1774/(2.5 x 10^8) = 42.8
PS for gear = 0.400 x 2.2 x 0.81 x 100 x 8 x 100 x 100 x 1774/(2.5 x 10^8) = 40.5

The allowable horse power rating of any pair of gear is the least of the four values calculated for the pinion and the gear. Hence, in this case the allowable rating is 40.5 PS.

Example 2.12 The long travel mechanism for a steel plant crane is equipped with a single stage helical gear box by the crane manufacturer. The gears are to be checked for strength and wear. The relevant data are as follows.

Crane duty class 3 as per IS: 4137; duration of service 5 hours/day; ambient temperature 55° C; power rating of gear box 29.5 kW at 720 rpm; $z_1 = 20$; $z_2 = 145$; m = 5; $\beta = 10°$; $\alpha = 20°$; tooth width b = 140 mm; material C 40 for pinion and C 30 for gear, both hardened and tempered.

Solution: $i = z_2/z_1 = 145/20 = 7.25 = n_1/n_2 = 720/n_2$ whence $n_2 = 99.31$ rpm

Input torque $T_1 = 974 \times 29.5/720 = 39.9$ kgf m $= 39.9 \times 10^3$ kgf mm

Tangential load on pinion $F_t = 2T/d_1 = 2 \times 39.9 \times 10^3 \left(\dfrac{mz_1}{\cos \beta}\right) = 2 \times 39900 \left(\dfrac{}{}\right)$

$= 786$ kgf

As per IS: 4137 which classifies steel mill duty cranes, the duty factors for class 3 cranes are as follows

for strength: 1.4 (which takes impact into account)
for wear: 0.6

Hence effective F_t for strength $= 1.4 \times 786 = 1100$ kgf, and for wear $= 0.6 \times 786 = 472$ kgf.

Check for strength: As per Eq. 2.114.

Allowable tangential load (kgf) $= X_b Y S_b m b$

$= 0.32 \times (0.735 \times 1.33 \cos^2 10°) \times 14.8 \times 5 \times 140 = 3149$ for pinion
$= 0.46 \times (0.625 \times 1.33 \times \cos^2 10°) \times 14.8 \times 5 \times 140 = 3860$ for gear

Both these values are greater than 1100 kgf.

Check for wear: As per eq. 2.115

Allowable tangential load (kgf) $= \dfrac{X_c Y_z S_c}{K} \times 25.4 \times b$

$= 0.36 \times (3.13 \times 0.75 \times \sec^2 10°) \times 1.44 \times 25.4 \times 140/13.7 = 1206$ for pinion
$= 0.52 \times (3.13 \times 0.75 \times \sec^2 10°) \times 1.125 \times 25.4 \times 140/13.7 = 1361$ for gear

As both of these values are greater than 472 kgf, the gear pair is safe from strength and wear point of view.

Example 2.13 In a non-reversing type rolling mill drive a gear is designed to run 24 hours per day, transmitting power in the following manner

1500 PS normally
6000 PS for 3 seconds, 600 times per day
8000 PS maximum momentary peak load

all at a constant speed of 40 rpm.

This drive is required to be checked for wear and strength.

Solution: In a problem of this nature where the load is not uniform, it is necessary to calculate first the equivalent running time at a uniform load which have the same effect on the gears.

We use symbol ***M*** to represent power in PS in such a case of varying amounts of power at different phases during operation. These powers are defined as follows

M_1 = The maximum sustained gear load acting for H_1 hours at n_1 rpm
$M_2 M_3, \ldots$ = Smaller loads which act for $H_2 H_3$ hour …
at n, n_3 rpm. . . respectively

Next, the equivalent running time per day at the maximum sustained load and the corresponding speed is calculated. This is given by

$$H_{eq} = H_1 + H_2 \left(\frac{n_2}{n_1}\right)\left(\frac{M_2}{M_1}\right)^3 + H_3 \left(\frac{n_3}{n_1}\right)\left(\frac{M_3}{M_1}\right)^3 + \ldots \tag{2.116}$$

In the example in question, the maximum sustained load is 6000 PS acting for 3 x 600/3600 = 0.5 hours per day. Speed is constant and so $n_2 = n_1 = 40$ rpm.

Hence

$$H_{eq} = 0.5 + (24 - 0.5)\left(\frac{40}{40}\right)\left(\frac{1500}{6000}\right)^3$$

$$= 0.5 + 0.37 = 0.87 \text{ hour per day}$$

The equivalent load is 6000 PS for 0.87 hour per day.

Now, it is to be examined whether the momentary peak load of 8000 PS exceeds the least of the allowable overload capacity. This is done as follows.

The momentary overload is that load, the duration of which is too short to be defined with certainty. It is considered to act for a period of not more than 15s, i.e. 0.004 hour.

The maximum momentary loads are given by

$$\text{for wear} = \frac{M_1 (2 + \sqrt{x_{c\,12}})(\sqrt{x_{c\,12}})}{X_c} \text{ or } 3M_1 \quad \text{whichever is less} \tag{2.117}$$

$$\textbf{for strength} = \frac{M_1 (0.8 + \sqrt{x_{b\,12}})(\sqrt{x_{b\,12}})}{X_b} \text{ or } 2M_1 \quad \textbf{whichever is less} \tag{2.118}$$

Here

X_{c12} = Speed factor for wear at 12 hours running time per day
X_{b12} = Speed factor for strength at 12 hours running time per day

X_c, X_b and M_1 have meanings as before. The values of X_c **and** X_{c12} are to be taken from Fig. 2.52 and those of X_b and X_{b12} from Fig. 2.51

From these figures, we have the following values for the example in question

X_c = 1.2 and X_b = 0.70 for 40 rpm for 0.87 hour per day
X_{c12} = 0.480 and X_{b12} = 0.480 for 40 rpm for 12 hours running time per day

Inserting these values in Eqs 2.117 and 2.118, we have

for wear = 9300 and $3M_1 = 3 \times 6000 = 18000$
for strength = 8850 and $2M_1 = 2 \times 6000 = 12000$

The maximum momentary power (PS), i.e., 8000, does not exceed the least of these allowable values which is 8850.

The gear, therefore, fulfils the conditions it is designed for, and is thus acceptable.

Table 2.18 Basic surface stress and bending stress factors

Material	*Condition (finished gear)*	*Minimum tensile strength kgf/mm²*	*Brinell hardness HB kgf/mm²*	*Basic surface stress factor S_c*	*Basic bending stress factor S_b*
Fabric	—	—	—	0.39	3.16
Malleable Cast Iron					
Whiteheart malleable iron castings, Grade B (IS 2107-1962)	—	28	217 max	0.599	6.9
Blackheart malleable iron castings, Grade B (IS:2108-1962)	—	32	149 max	0.599	7.72
Cast Iron (IS :210-1962) (Iron castings for gears and gear blanks)					
Grade 20	As cast	20	179 min	0.81	4.22
Grade 25	As cast	25	197 min	0.876	5.27
Grade 35	As cast	35	207 min	0.915	8.60
Grade 35	Heat-treated	35	300 min	1.00	8.60
Phosphor Bronze (IS :28-1958)					
Phosphor bronze castings (for gear blanks)	Sand cast	16	60 min	0.436	4.07
Phosphor bronze castings (for gear blanks)	Chill cast	24	70 min	0.50	5.8
Phosphor bronze castings (for gear blanks)	Centrifugally cast	26.77	90	0.69	6.92
Cast Steel (IS : 1030-1962)					
Grade 1	—	55	145	1.125	13.38
Forged Steels (IS :1570-1961)					
Carbon steel					
0.30 per cent carbon steel (C 30)	Normalised	50	143	0.985	11.95
0.30 per cent carbon steel (C 30)	Hardened and tempered	60	152	1.125	14.80
0.40 per cent carbon steel (C 40)	Normalised	58	152	1.125	14.05
0.40 per cent carbon steel (C 40)	Hardened and tempered	60	179	1.44	14.80
0.55 per cent carbon steel (C 55 Mn 75)	Normalised	70	201	1.63	16.90
0.55 per cent carbon steel (C 55 Mn 75)	Hardened and tempered	72	223	1.83	17.60
0.55 per cent carbon steel (C 55 Mn 75)	—	80	248	2.11	19.70
Carbon chromium steel					
0.55 carbon chromium steel (55 Cr 70)	Hardened and tempered	90	225 min	2.105	22.2
0.55 Carbon chromium steel (55 Cr 70)	Hardened and tempered	100	285 min	2.47	24.6
0.40 per cent carbon, 1 percent chromium steel (40 Cr 1)	Hardened and tempered	80	229 min	1.90	19.7

(Contd.)

Table 2.18 (contd.)

Carbon manganese steel					
Carbon manganese steel (27 Mn 2)	Normalised	55	—	—	13.4
Carbon manganese steel (27 Mn 2)	Hardened and tempered	60	170 min	1.41	14.8
Carbon manganese steel (27 Mn 2)	Hardened and tempered	70	201 min	1.69	16.9
Carbon manganese steel (37 Mn 2)	—	70	201 min	1.69	16.9
Carbon manganese steel (37 Mn2)	—	80	229 min	1.90	19.7
Maganese molybdenum steel					
Manganese molybdenum steel (35 Mn 2 Mo 28)	Hardened and tempered	70	201 min	1.69	16.9
Manganese molybdenum steel (35 Mn 2 Mo 28)	Hardened and tempered	80	229 min	1.90	19.7
Manganese molybdenum steel (35 Mn 2 Mo 45)	Hardened and tempered	80	229 min	1.90	19.7
Manganese molybdenum steel (35 Mn 2 Mo 45)	Hardened and tempered	90	255 min	2.105	23.2
Chromium molybdenum steel					
1 per cent chromium molybdenum steel (40 Cr 1 Mo 28)	Hardened and tempered	70	201 min	1.69	16.9
1 per cent chromium molybdenum steel (40 Cr 1 Mo 28)	Hardened and tempered	80	229 min	1.90	19.7
1 per cent chromium molybdenum steel (40 Cr 1 Mo 28)	Hardened and tempered	90	255 min	2.105	23.2
1 per cent chromium molybdenum steel (40 Cr 1 Mo 28)	Hardened and tempered	100	285 min	2.47	24.6
1 per cent chromium molybdenum steel (40 Cr 1 Mo 60)	Hardened and tempered	90	248 min	1.75	23.2
1 per cent chromium molybdenum steel (40 Cr 1 Mo 60)	Hardened and tempered	100	293 min	2.06	24.6
3 per cent chromium molybdenum steel (15 Cr 3 Mo 55 and 25 Cr Mo 55)	Hardened and tempered	90	255 min	2.105	23.2
3 per cent chromium molybdenum steel (15 Cr 3 Mo 55 and 25 Cr 3 Mo 55)	Hardened and tempered	155	144 min	1.16	36.6
Nickel steel					
3 per cent nickel steel (40 Ni 3)	Hardened and tempered	80	229 min	1.90	19.70
3 per cent nickel steel (40 Ni 3)	Hardened and tempered	90	255 min	2.105	23.2
Nickel chromium steel					
$4\frac{1}{4}$ percent nickel chromium steel (30 Ni 4 Cr 1)	Hardened and tempered	154	444 min	3.87	35.9
Nickel chromium steel					
$1\frac{1}{2}$ per cent nickel chromium molybdenum steel (40 Ni 2 Cr 1 Mo 28)	Hardened and tempered	90	255 min	2.1 1	23.2
$1\frac{1}{2}$ percent nickel chromium molybdenum steel (40 Ni 2 Cr 1 Mo 28)	Hardened and tempered	155	444 min	3.87	37.3
$2\frac{1}{2}$ percent nickel chromium molybdenum steel (medium carbon) (31 Ni 3 Cr 65 Mo 55)	Hardened and tempered	90	255 min	2.11	23.2

(Contd.)

Table 21.8 *(Contd.)*

$2\frac{1}{2}$ percent nickel chromium molybdenum steel (medium carbon) (31 Ni 3 Cr 65 Mo 55)	Hardened and tempered	155	444 min	3.87	36.6
$2\frac{1}{2}$ percent nickel chromium molybdenum steel (medium carbon) (31 Ni 3 Cr 65 mo 55)	Hardened and tempered	110	331 min	2.60	26.7
$2\frac{1}{2}$ percent nickel chromium molybdenum steel (high carbon) (40 Ni 3 Cr 65 Mo 55)	Hardened and tempered	100	285 min	2.47	24.6
$2\frac{1}{2}$ percent nickel chromium molybdenum steel (high carbon) (40 Ni 3 Cr 65 mo 55)	Hardened and tempered	155	444 min	3.81	37.3
$2\frac{1}{2}$ percent nickel chromium molybdenum steel (high carbon) (40 Ni 3 Cr 65 Mo 55)	Hardened and tempered	120	341 min	3.82	28.8
Surface Hardened Steels (IS : 1570-1961)					
Carbon steel					
0.4 per cent carbon steel	—	55.12	145 (core) 460 *(case)*	1.97	8.35
0.55 per cent carbon steel	—	70.86	200 (core) 520 *(case)*	2.80	10.55
Carbon chromium steel					
0.55 per cent carbon chromium steel	—	86.61	250 (core) 500 (case)	3.58	12.91
1 per cent chromium steel	—	70.86	500 (case)	2.80	10.55
Nickel steel					
$3\frac{1}{2}$ per cent nickel steel	—	70.86	200 (core) 300 (case)	3.57	12.91
Nickel steel					
$3\frac{1}{2}$ per cent nickel steel	—	86.61	250 (core) 500 (case)	3.58	12.91
$1\frac{1}{2}$ per cent nickel, 1 per cent chromium steel	—	86.61	250 (core)	3.58	12.91
Case Hardened Steel *(IS*:1570-1961)					
Carbon steel					
0.12 to 0.22 per cent carbon steel	—	50.39	650 (case)	7.00	28.00
0.15 per cent carbon steel	—	50.39	140 (core)	7.00	28.00
0.20 per cent carbon steel	—	50.39	140 (core) 640 *(case)*	7.00	28.00
Nickel steel					
3 per cent nickel steel	—	70.86	200 *(case)*	7.17	28.12
$3\frac{1}{2}$ per cent nickel steel	—	70.86	200 (core) 620 *(case)*	7.17	28.12
5 per cent nickel steel	—	86.61	250 (core) 600 *(case)*	7.87	33.07

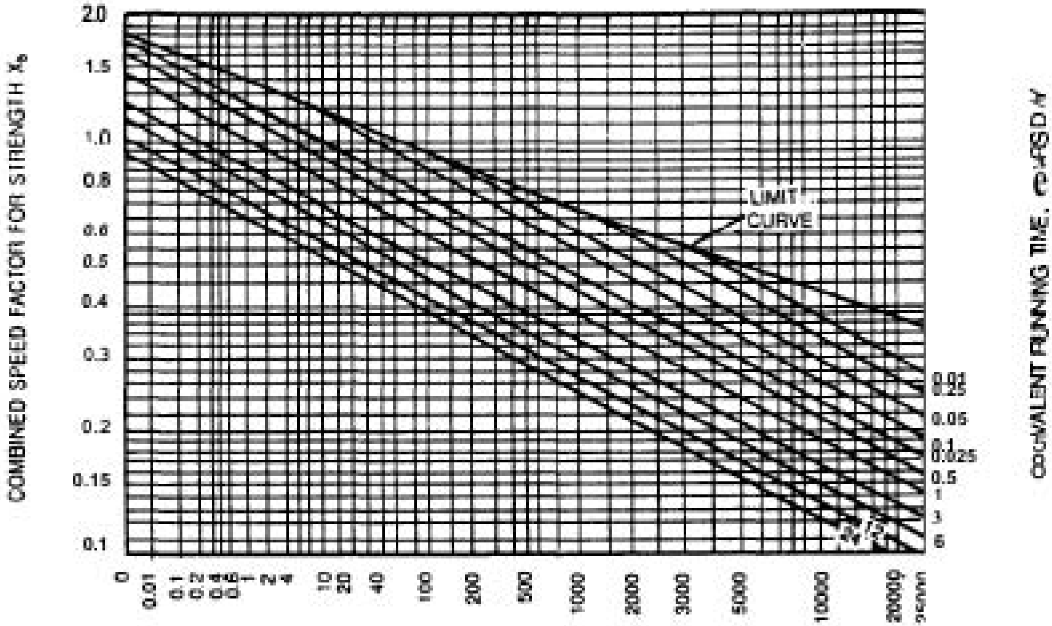


Fig. 2.51 Combined speed and running time factors for spur and helical gears for strength

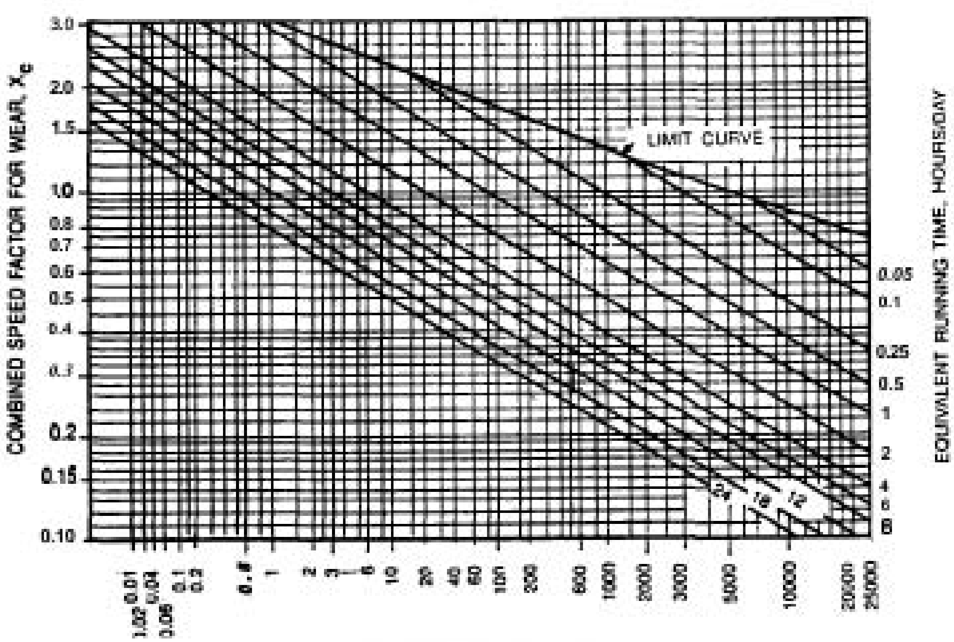


Fig. 2.52 Combined speed and running time factors for spur and helical gears for wear

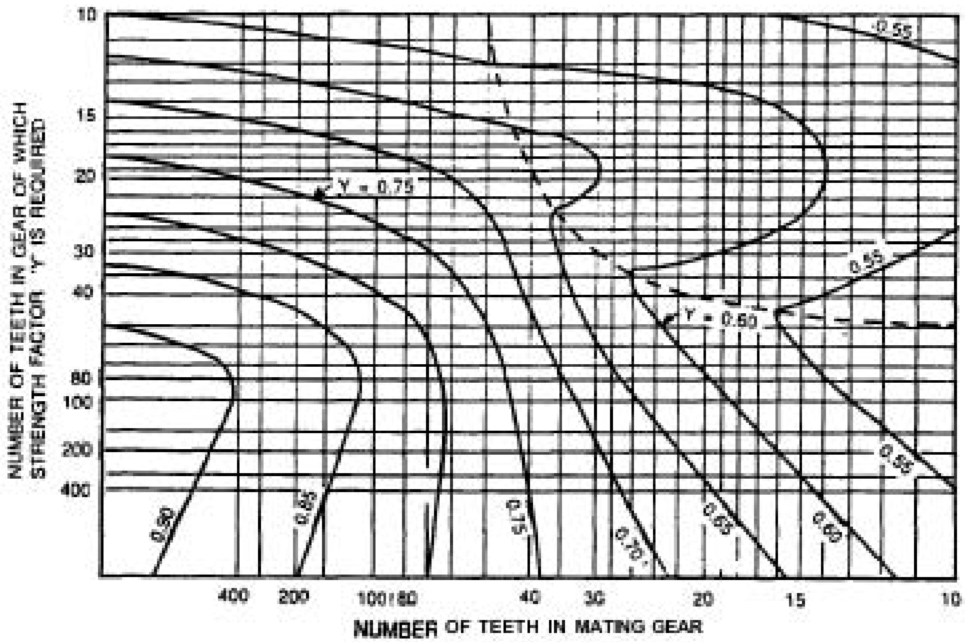


Fig. 2.53 Strength factor for helical gears with 30° helix angle and spur gears —20° normal pressure angle

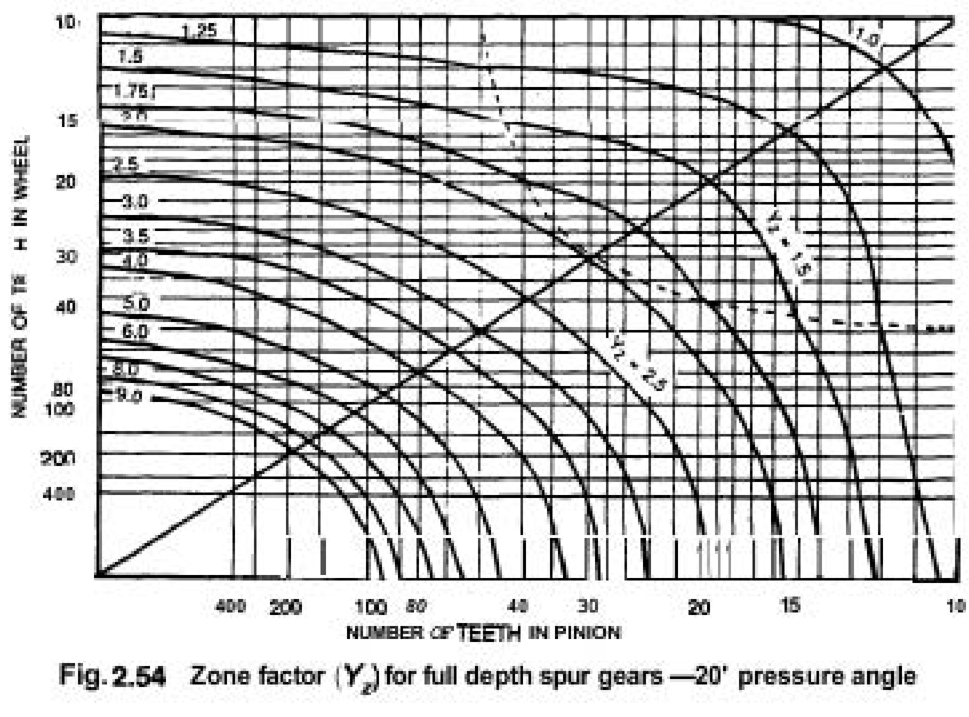


Fig. 2.54 Zone factor (Y_z) for full depth spur gears —20° pressure angle

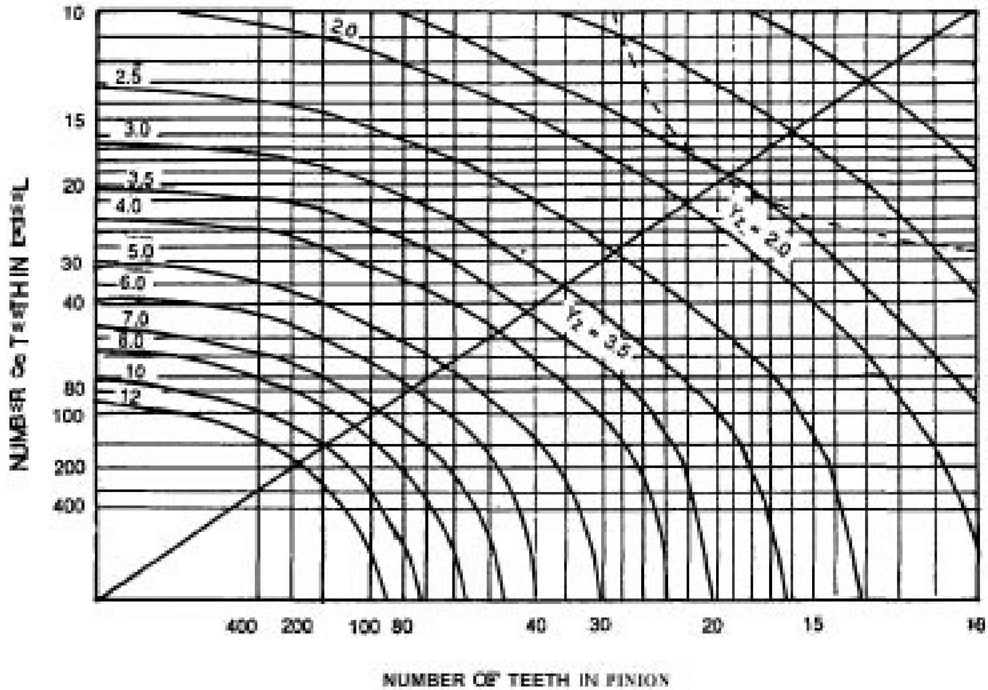


Fig. 2.55 Zone factor (Y_z) for helical gears with 30° helix angle —20° normal pressure angle

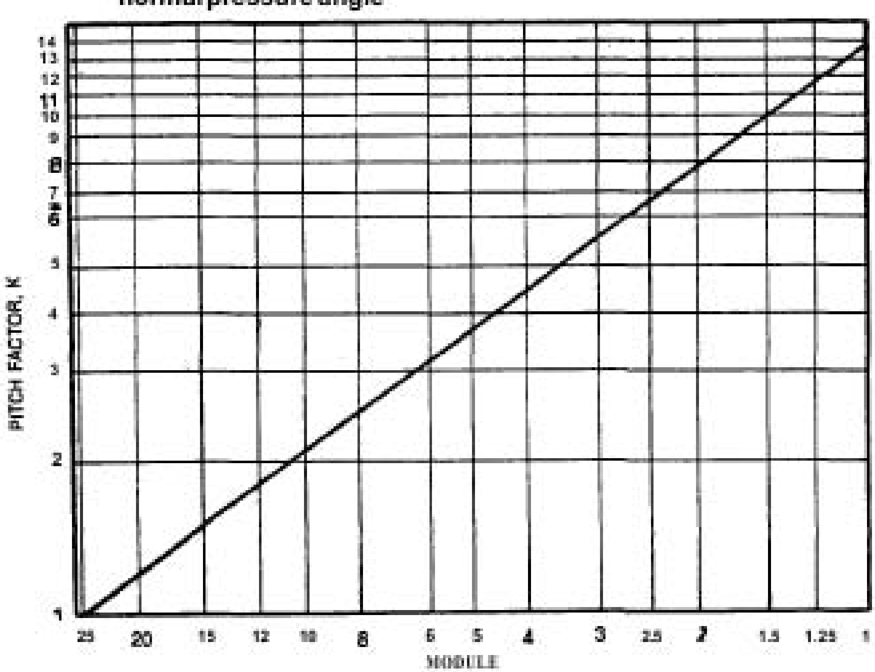


Fig. 2.56 Pitch factor

Other Strength Factors

Apart from the bending stresses and the surface fatigue stresses discussed so far, there are other factors which must be considered while designing a gear. Design parameters will change according to such factors as overloading or shock, errors, high starting torque and acceleration torque. Besides, the dynamic load is of prime consideration in designing a gear set. These aspects have already been discussed in detail in Sec. 2.22.

Another important gear failure factor is scoring (or scuffing). This kind of failure appears in the form of coarse ridges down the teeth surfaces from the tip to the pitch circle. Though opinion differ, scoring is mainly considered as a result of lubrication failure. Momentary local welding of high spots of the tooth flanks takes place which is caused by high surface loads, high sliding velocities, improper surface finish and the unsuitable characteristics of the lubricant applied. Various other causes have also been postulated.

To check a gear against failure by scoring, an empirical formula can be used. This is valid when straight mineral oil is used, that is, no extreme-pressure (EP) lubricant is used.

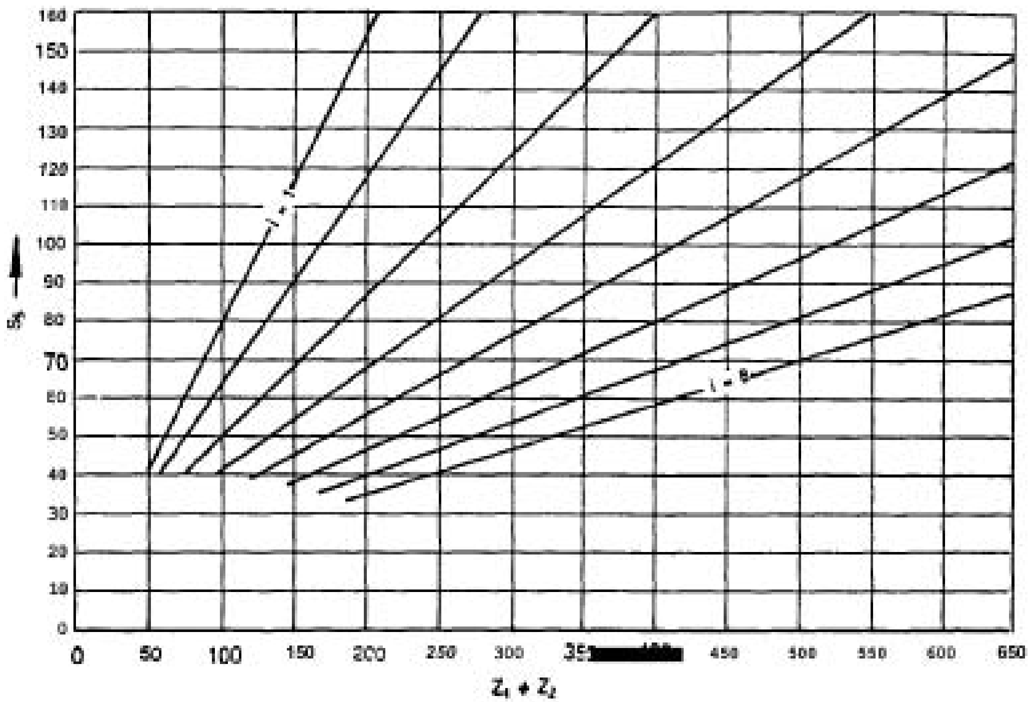


Fig. 2.57 Scoring factor

Based on MAAG Handbook of M/s. MAAG Zahnraeder AG, Zurich, page 129.

Referring to Fig. 2.57, the abscissa represents the sum $z_1 + z_2$. The ordinate represents a quotient given by

$$S_s = F_w \frac{4\sqrt{v}}{\sqrt{a}} \tag{2.119}$$

where,

F_u (N/mm) = Average load per unit length along the width of tooth

v (m/s) = Pitch line velocity

a (mm) = Centre distance

In Fig. 2.57 lines have been drawn corresponding to the transmission ratio i If the value of S_d lies below the line representing i which is relevant to the problem being investigated it may be assumed that scoring will not occur even when the full load is applied to new gears.

Strength Calculation of Non-Metallic Gears

Properties of non-metals as gear meterials have been discussed in Sec. 1.8. Synthetic material like plastic, synthetic rubber, and other materials like raw hide, rubber impregnated laminates and pressed fibres are used in gear drive mainly for their high damping properties and noise-free operation. Such gears are normally paired with metallic gears. Lubrication should be copious. From strength and other points of view, these gears are, comparable to cast iron gears.

The non-metallic gears are obviously not meant for heavy load and tough duty. Their load carrying capacity is limited by beam strength, temperature and permissible sliding. The transmitted load is given by

$$F_t \text{ (N)} = K_1 K_2 b p \tag{2.120}$$

where

K_1 (N/mm²) = Load factor as per Table 2.19; it depends upon the material and the velocity

K_2 = Number of teeth factor as per Table 2.20

b (mm) = Tooth width = 8 to 10m

p (mm) = Circular pitch = π m

Table 2.19 Load factor K_1

Circumferential velocity v	*Materials*			
	Vulcanised fibre	*Raw hide*	*Synthetic material*	*Cast iron*
m/s	K_1 (N/mm²)			
0.5	0.8	1.6	2.5	2.7
1	0.8	1.6	2.3	2.6
2	0.7	1.4	2.2	2.3
4	0.6	1.1	1.7	1.9
6	0.5	1.0	1.3	1.7
8	0.4	0.8	1.1	1.4
10	0.4	0.7	0.95	1.2
12	0.3	0.6	0.85	1.0
15	0.3	0.6	0.70	1.0

The data for cast iron are given in the above table for comparison purposes.

Table 2.20 Number of teeth factor K_2

$z =$	13	15	20	25	30	40	60	100
$K_2 =$	0.7	0.85	1.0	1.08	1.14	1.21	1.27	1.34

The following practical guidelines are of relevance in connection with the design and operation involving non-metallic gears:

1. A non-metallic gear should always be paired with a metallic gear. However, in special operational conditions, e.g., in acidic environments, both the gears can be non-metallic.
2. Dimensions permitting, the larger gear of the pair should normally be the non-metalic one.
3. The width of the non-metallic gear should be somewhat smaller than that of its metallic mating component. It should never exceed the width of the metallic gear.
4. The tooth width, b, should not preferably be more than 10 m.
5. The teeth must lie at right angles to the direction of laminations.
6. The teeth of the mating metallic gear should be clean and polished.
7. Copious lubrication is indispensable for long life and noise-free drive.
8. Before the beginning of first operation, application of a graphite paste on the non-metallic gear is recommended. A small amount of graphite can also be added to the lubricating oil from time to time during the running of the gear pair.
9. These gears can be fitted on the shaft through taper-seat (1:10 to 1:20) or through feather keys.
10. To facilitate better fastening with the shaft, metallic bushes with keys-ways can be press-fitted inside the bore of the non-metallic gears, if other conditions permit.

Example 2.13 a In a spur gear drive, the following data are given: $z = 60$, n = 300 rpm, $\alpha = 20°$, material of gear : synthetic material.
To calculate the appropriate dimensions of the gear to transmit 1.75kw.

Solution : From Eqs 2.52 and 2.120, we have

$$F_t\ (\mathrm{N}) = \frac{1000\ \mathrm{P\ (kw)}}{v\ (\mathrm{m/s})} = K_1\ K_2\ bp$$

$$v = \frac{\pi d n}{60000} = \frac{\pi m z n}{60000}$$

where d is in mm. Taking b = 9 m and $p = \pi m$, we get

$$F_t = \frac{1000 \times \mathrm{P} \times 60000}{\pi\ m z n} = K_1 K_2\ 9 m\, \pi\, \mathrm{m}$$

From Table 2.20, $K_2 = 1.27$. A value of $K_1 = 2.2$ is tentatively selected from Table 2.19. Transposing the above equations, we have

$$m = 3\sqrt{\frac{1000 \times P \times 60000}{9\pi^2 z \mathrm{n} K_1 K_2}} = 3\sqrt{\frac{1000 \times 1.75 \times 60000}{9\pi^2 \times 60 \times 300 \times 2.2 \times 127}} = 2.86,\ m = 3 \text{ mm (selected)}$$

$$d = m\, z = 3 \times 60 = 180\,\mathrm{mm}, \quad v = \frac{\pi \times 180 \times 300}{60000} = 2.83\ \mathrm{m/s}$$

K_1 = 2.0 from Table 2.19 by interpolation, $b = 9m = 9 \times 3 = 27$ mm.

$$F_t = 2.0 \times 1.27 \times 27 \times 3\pi = 646 \text{ N}$$

$$P = F_t \times v/1000 = 646 \times 2.831/1000 = 1.83 \text{ kw} > 1.75 \text{ kw}.$$

Hence, the above non-metallic gear is capable to transmit the required power.

2.26 Acceleration Torque and Motor Characteristics

We discussed the various parameters concerning the acceleration torque in Sec. 2.22, and arrived at the following equation

$$T_a \text{ (Nm)} = \frac{WD^2 n}{375 t}$$

where T_a is the acceleration torque in newton metres, WD^2 is the fly-wheel moment in newton metres squared, n is speed in rpm and t is the acceleration time in seconds. By transposing, we get

$$t \text{ (s)} = \frac{WD^2 n}{375 T_a} \tag{2.121}$$

For proper selection and subsequent operational behaviour of the motor which drives the gear box, the torque curves of the driven machines as well as the torque characteristics of the motor must be known. During acceleration (starting) and deceleration (braking), the difference between these two torques is the measure of the acceleration (or deceleration) torque T_a. A typical motor drive has been shown in Fig. 2.58 a.

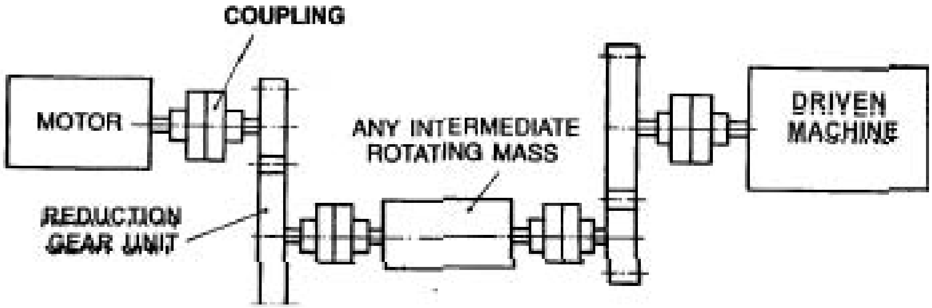
In Fig. 2.58 b (i and ii), the four basic torque curves (representing the load torque) and power curves of the various types of driven machines commonly encountered are given. These are shown as functions of speed, power and torque. The four basic curves for usual applications are elaborated in the table below.

Case	Load torque	Power	Application (driven machine)
1.	Practically constant	Directly proportional to speed	Cranes, reciprocating pumps and compressors, rolling mill conveyors, machine tools etc.
2.	Directly proportional to speed	Proportional to square of speed	Glazing rollers
3.	Proportional to square of speed	Proportional to cube of speed	Centrifugal pumps, blowers, etc.
4.	Inversely proportional to Speed	Constant	Some types of lathes and allied machine tools, coiler, shaving machines variable speed drives. etc.

The above four cases are represented by the curves 1 to 4 in Fig. 2.58 b (i and ii).

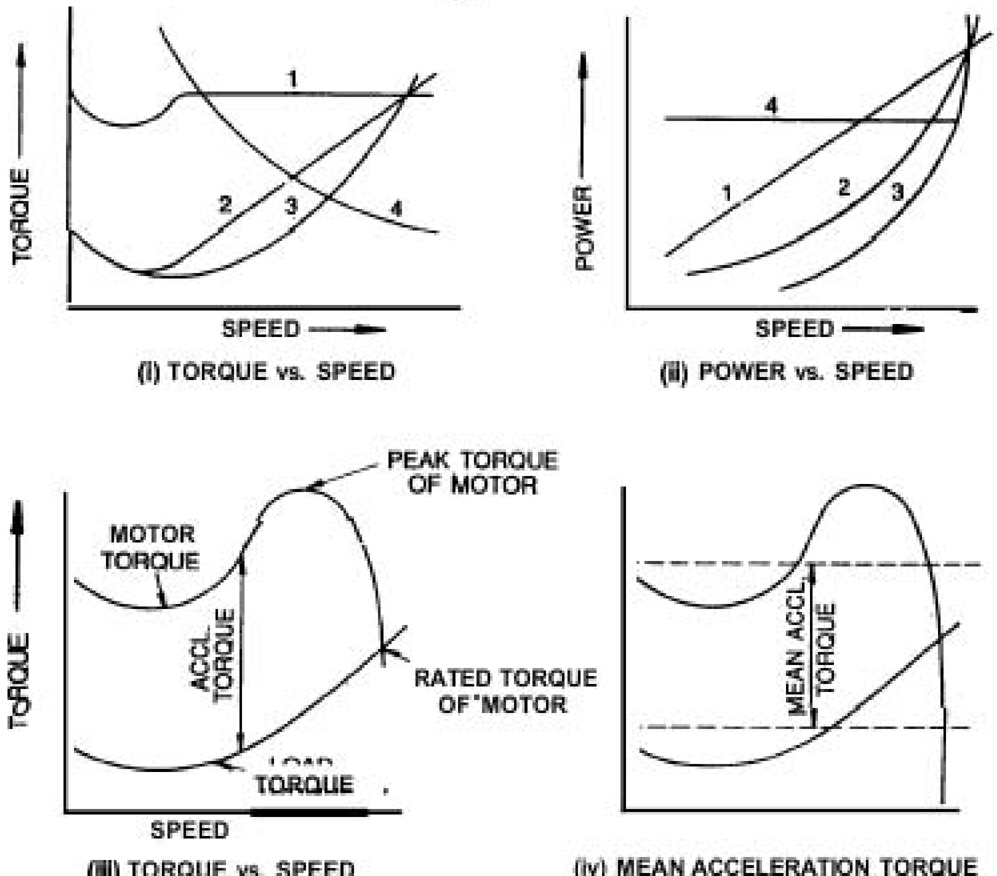
Apart from the mechanical considerations, the acceleration time t is of utmost importance because this is a determining parameter for the selection of the right type of motor for the following reasons-during starting and the period of acceleration, heat is generated due to normal losses. A temperature rise takes place during this time and also during service until the normal operating temperature, relevant to the particular motor and service condition, is attained.

It is, therefore, obvious that since too much heat is detrimental to the windings and other parts of the motor, and since the acceleration time is a contributary factor in heat generation, a limit has to be set as regards the duration of this time factor. However, no hard and fast rule can be established as to this duration since it is a function of so many factor. It may range from a few



A TYPICAL MOTOR DRIVE

(a)



(b)

Fig. 2.58 Characteristics of an electric motor seconds to several seconds. Guiding values are: **2** to 10 seconds for normal duty, **2** to 5 seconds for intermittent duty, and up to **20** seconds for heavy loads such as in the case of cooling tower fans, etc. Catalogues of motor manufacturing companies and codes of practice give the values of the maximum allowable temperature rise for the windings which in turn is determined by the temperature limits of the insulating materials. The usual practice is to assume a reasonable value for the acceleration time after consulting these codes and catalogues and then proceed further to determine the acceleration torque from the formula given. **As** discussed in Sec. **2.22,** this acceleration torque is to be added to the rated torque for eventual determination of the gear-drive data.

It is to be noted that the instantaneous acceleration time is a variable factor as the machine speeds up from rest to the normal running condition. It is truly given by

$$t = \frac{WD^2}{375} \int_{n_1=0}^{n_2} \frac{1}{T_a}\, dn$$

where n_a is the normal running speed.

Equation 2.121 is an approximation where T_a actually represents a mean-constant acceleration torque. In Fig. **2.58** b (iii) the torque characteristics of an asynchronous, squirrel-cage motor, commonly used in industries, are given. Characteristics of other types of motors can be obtained from motor manufacturers catalogues.

For all practical purposes, the peak torque of a motor can be taken to be around **2.2** to **2.5** times its rated torque. It can be seen from Fig. 2.58 b (*iii*) that

the acceleration torque at any instant = motor torque at that instant — load torque of the driven machine at that instant

The mean acceleration torque has been shown in Fig. **2.58** b (*iv*). This can be arrived at by the usual graphical methods or by integrating the area and then dividing this area by the base.

2.27 Quality Grades and Errors of Gears

Since it is practically impossible to produce any engineering component without manufacturing errors, deviations from theoretically perfect dimensions of the component have to be allowed. These deviations are known as tolerances or permissible errors. For economic reasons, the values of these tolerances should be selected as high and wide as possible since unnecessarily precise tolerances make the products costlier. On the other hand, too coarse a tolerance may not serve the purpose. It is, therefore, imperative for the gear designer to acquire the necessary knowledge and experience to enable him to select proper tolerances, keeping in mind the various factors involved, such as service conditions, availability of cutting machines, duration of operating hours possibility of interchangeability if needed, and above all, techno-economics concerning the economic viability of his design.

The conditions which prevail for the determination of the tolerances for a pair of mating gears are quite different from those in case of the usual mating parts in other engineering components, such as the shaft and the mounting with the corresponding bore. Since kinematics are involved, design considerations are more complicated for a pair of mating gears. Only with special machines, manufacturing and testing of gears can be carried out.

On the basis of experience in gear-drive design as well as from the theoretical considerations and research work in the field, experts have laid down guidelines and codes for the permissible values of these tolerances and errors which may be applied in case of gears and gear-boxes so that a reasonably good gear-drive for the purpose for which it is meant can be attained. These guidelines also serve as the yardsticks for acceptance tests by the inspectors, customers and competent authorities.

As in the case of limits, fits and tolerances for ordinary engineering components, a system has been evolved for grades and tolerances of gear toothing. As per the requirements of Draft ISO Recommendation No. 1328, "Accuracy of parallel involute gears" and the relevant Indian Standard Specifications, e.g. IS: 4702, 4725, 4058 and 4059, gears have been classified into 10 quality or accuracy grades — from 3 (high-precision gears) to 12 (coarse-quality, low-speed gears). Qualities 1 and 2 also exist in German practice, but these are mainly for master gears and are produced by extremely sophisticated machines. The quality numbers refer to the degree of precision with which the gear teeth have been made. The quality of a gear depends on the limits of tolerance for pitch, tooth profile and tooth alignment. The quality or grade assigned to any particular gear is the finest grade number selected for any of the above three elements. Normally, for a pair of mating gears, the elements of the components belong to identical accuracy grades, but these may also have different grades as per the agreement between the manufacturer and the user.

For common engineering components, the qualities usually in practice are from 5 to 10. Many factors influence the choice of quality grade, viz. available testing procedures for accuracy, assembly requirements, backlash accuracy requirements, permissible noise level. Another very important consideration is the additional dynamic forces caused by manufacturing errors of toothing (Dynamic forces have been discussed in Sec. 2.22). Obviously, the quality of operation of a gear drive is a direct function of the degree of the quality of toothing: higher the quality grade, smoother the running, and lesser the error, lower the magnitude of the disturbing dynamic forces. A direct relationship exists between the quality of toothing, circumferential velocity, dynamic forces, and other determining factors. This is shown in Table 2.21.

The usual qualities which are used for toothing for different types of application and equipments are given in Table 2.22. These should act as guidelines for the designer for selection of the requisite qualities.

Table 2.21 Relationship of gear quality to other factors

	Manufacturing Process					
Type of gear	*cast, pressed forged*	*Machined with form cutter*	*Milled generated*	*Fine finished*	*Finished by grinding, scraping etc.*	*Type of loading*
	Quality of gear					
	10 to 12	9 to 10	8 to 9	6 to 7	4 to 5	
	Maximum allowable velocity, m/sec					
Spur	0.8	1.2	5	8	15	Normal
Helical	1.0	2.5	15	25	100	Heavy
Herringbone	1.0	2.5	15	25	100	Impact
Straight bevel	0.5	1.0	4	6	—	Normal
Spiral bevel	0.8	2.0	8	12	25	Heavy to Impact

Based on die Tragfaehigkeit der Zahnraeder, Thomas and Charchut, 7th Edition, 1971, table no2, p. 15. Carl Hanser, Verlag, Munich.

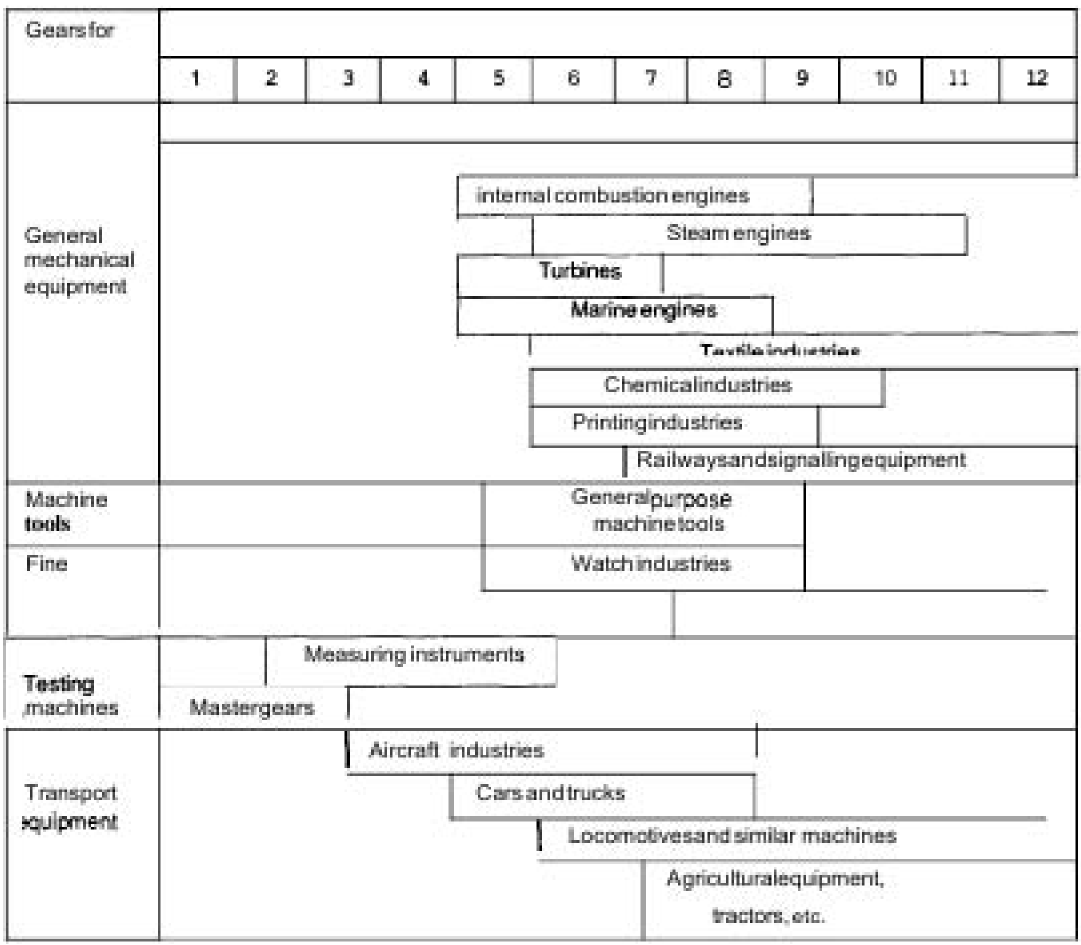
Depending on the quality grades involved, gears have been classified as under by the Indian Standard Specifiations

ISS Number	Quality Grade	Nomenclature
4702	3 and 4	High precision gears
4725	5 and 6	Precision gears
4058	10, 11 and 14	Coarse quality, low speed gears
4059	7, 8 and 9	Medium quality, medium speed gears

Besides the above IS Specifications, IS: 4071 lays down requirements for master gears which are intended for checking other working gears. As stated before, grades 1 and 2, (sometimes also 3) are assigned for master gears as per German specifications and the specifications of some other countries.

While selecting the quality grade, the designer should always keep in mind the cost angle involved. The cost or the time required for manufacturing parts do not vary in a rectilinear way with the quality grades. In practice, they follow power curves with ascending order of fineness of the qualities. For ground gears, for example, the grinding time (and consequently the cost as well) will vary more or less according to the data given below, taking the time for quality grade 8 as unity for the sake of comparison.

Quality	8	7	6	5	4	3	2
Grinding time	1	1.6	2.4	4	6.6	20	33



It has been pointed out before that the criterion of assigning quality grades is the error for pitch, for tooth profile and for tooth alignment. The departures from the theoretically ideal geometrical parameters can be accurately measured by precise instruments, but such measurements are not enough to correctly interpret the motions of running gears, because running qualities will depend upon the sum of errors and are also made complicated by radial and lateral run-outs.

Types of Errors

An error is defined as the value obtained by subtracting the design value of a dimension from its actual value

Errors can be broadly classified into two main categories — the individual errors and the composite errors.

Individual errors involve those errors which are deviations of individual parameters of toothing from their ideal values. Under such headings fall: profile error f_f, adjacent pitch error f_{pt}, base pitch error f_{pb}, tooth alignment error f_β, radial run-out f_r, tooth thickness error f_s, etc. These errors are measured by special measuring instruments. Symbol f denotes errors over single unit of measurement parameters, while F is used when the measurement extends over a certain specified range of several such units and their cumulative effects thereof.

The composite errors of a tooth system denote the total effect on the position and form of the tooth surfaces when a number of individual errors act jointly and simultaneously. This type of error will be elaborated later in this section.

Zndividual errors The different types of the idividual errors are discussed below. Their permissible values are given in tabular form in Table 2.23. As indicated earlier, each type of individual error is measured by special measuring and inspecting equipment.

Profile errors This error is an indication of departures of the actual profile from the ideal involute profile. This departure at any point is measured normal to the involute profile. It is denoted by the symbols f_f and F_f.

Pitch errors This error denotes the departure of the actual spacing of the teeth from the ideal one. The adjacent pitch error f_{pt} is this departure measured on similar flanks of two adjacent teeth. When the measurement is done over a length more than one pitch apart, namely, k number of pitches, it is called the cumulative pitch error F_{pk}. Besides, there is the base pitch error f_{pb}, which is the difference between the actual and the ideal base pitch.

Tooth alignment error This is also known as the error of distortion. When a spur or a helical gear is cut, its tooth traces should follow the ideal path, i.e. parallel to the axis in case of a spur gear and at an angle equal to the helix angle in case of a helical gear. The tooth alignment error is an indication of deviation from this ideal path. This error is usually measured in micrometres (μm) over a given distance on the tooth width of the gear. It is denoted by f_β and F_β.

Radial run-out This is a measure of the eccentricity of the tooth system and is denoted by f_r.

Axial run-out This is a measure of the wobble of the gear, and is measured by placing a dial gauge whose axis is held at a specific distance and parallel to the axis of rotation of the gear.

Tooththicknesserror This is the value obtained by subtracting the design tooth thickness from the actual tooth thickness, measured along the surface of the reference or pitch cylinder. This is denoted by f_s.

Table 2.23 Permissible values of individual and composite errors

(All values of errors in μm)

Gear Quality	Tooth profile error F_f	Adjacent pitch error f_{pt}	Cumulative pitch error F_{pk}
3	3.0 + 0.160,	2.0 + 0.160,	1.6 + 0.63 $\sqrt{L}$
4	4.0 + 0.250,	3.2 + 0.250,	2.5 + 1.00 $\sqrt{L}$
5	5.0 + 0.400,	5.0 + 0.400,	4.0 + 1.60 $\sqrt{L}$
6	6.3 + 0.630,	8.0 + 0.630,	6.0 + 2.50 $\sqrt{L}$
7	8.0 + 1.000,	11.0 + 0.900,	8.0 + 3.55 $\sqrt{L}$
8	10.0 + 1.600,	16.0 + 1.250,	12.0 + 5.00 $\sqrt{L}$
9	16.0 + 2.500,	22.0 + 1.800,	17.0 + 7.10 $\sqrt{L}$
10	25.0 + 4.00 ϕ_f	32.0 + 2.500,	25.0 + 10.00 $\sqrt{L}$
11	40.0 + 6.300,	45.0 + 3.550,	33.0 + 14.00 $\sqrt{L}$
12	63.0 + 10.000,	63.0 + 5.00 ϕ_p	50.0 + 20.00 $\sqrt{L}$

Note: $\phi_f = m + 0.10\sqrt{d}$, $\phi_p = m + 0.25\sqrt{d}$, L (mm) = Arc length *k ir mona sector of k number of pitches.* $L_{max} = \pi d/2$, m (module) *and d (pcd) are in mm.*

Gear quality	Tooth alignment error F_β	Radial run-out f_r
3	2.5 + 0.50 $\sqrt{b}$	7 + 0.560,
4	3.0 + 0.63 $\sqrt{b}$	11 + 0.900,
5	4.0 + 0.80 $\sqrt{b}$	18 + 1.400,
6	5.0 + 1.00 $\sqrt{b}$	28 + 2.240,
7	6.0 + 1.25 $\sqrt{b}$	40 + 3.15 ϕ_p
8	10.0 + 2.00 $\sqrt{b}$	50 + 4.000,
9	16.0 + 3.15 $\sqrt{b}$	63 + 5.000,
10	25.0 + 5.00 $\sqrt{b}$	80 + 6.300,
11	40.0 + 8.00 $\sqrt{b}$	100 + 8.000,
12	63.0 + 12.50 $\sqrt{b}$	125 + 10.00 ϕ_p

Note: b (mm) = Tooth width, b_{max} = 150 *mm*

Gear Quality	Tooth-to-tooth composite error, double flank f''_i	Total composite error, double flank F''_i
3	4.0 + 0.320,	10 + 0.800,
4	6.0 + 0.450,	16 + 1.250,
5	9.0 + 0.560,	25 + 2.000,
6	12.0 + 0.90 ϕ_p	40 + 3.20 ϕ_p
7	16.0 + 1.250,	56 + 4.500,
8	22.0 + 1.800,	71 + 5.600,
9	20.0 + 2.240,	90 + 7.100,
10	36.0 + 2.800,	112 + 9.000,
11	45.0 + 3.550,	140 + 11.20 ϕ_p
12	56.0 + 4.500,	180 + 14.00 ϕ

Base circle error This error denoted by f_g is the difference between the actual and the theoretical dimensions of the base circle diameters.

The commonly used formulae from which the individual errors can be calculated are given in Table 2.23. The relevant formulae for deriving the composite errors are also included in the same table for convenience.

Composite Errors

In a composite error test, the combined effects of a number of errors, acting simultaneously, are revealed. These errors include two or more of the individual errors, such as profile error, pitch error, tooth alignment error, tooth thickness error, radial and axial runout (or wobble), etc. This type of test approximates the action of the gear in question under service conditions. Since it is not practicable in regular production schedule to measure each and every individual error of each and every product-gear, the composite error type of test is resorted to for general workshop use.

The composite error tests are of two types—single-flank composite error test and the double-flank composite error test. Each of these tests are sub-divided into two categories—tooth-to-tooth composite error test and total composite error test.

In both single-flank and double-flank composite error tests, the product gear is rotated through at least one complete revolution in intimate contact with a "master gear" of known accuracy. The single-flank type of test reveals errors in angular transmission whereas the double-flank type of test represents variations in centre distance.

The double-flank tests are more prevalent in practice and therefore it will be discussed in detail. This type of test is also known as the "rolling gear test" or simply "roll test" in workshop vocabulary. In this test the gear to be tested and the master gear are mounted on a variable centre distance fixture which is suitably designed for gear rolling, and the resulting data are measured by a suitable device. This fixture may consist of a spring-loaded movable slide or one gear may be pivoted on a spring-loaded arm as shown in Fig. 2.59(d). The fixture may also be like the one shown in Fig. 2.59(c). In any case the main idea is that the two gears should rotate in tight mesh without backlash. During rotation, there is a variation in the centre distance and the effects of errors in the gear are shown by a dial indicator or by a recording or tracing device.

In the single-flank test the rolling is done on any one flank only. The centre distance is kept constant. The master gear rotates with perfect and constant angular motion. Any deviation in the angular movement of the product gear shows up in the single-flank test. The conditions for single-flank and the double-flank tests are represented in Fig. 2.59 (a and b).

In the double-flank test, the tooth-to-tooth composite error is the error which snows up as flicker on the indicator of the variable centre distance fixture arrangement when the gear rotates from tooth-to-tooth in tight mesh with the master gear. This shows the combined effect of the errors, e.g. profile error, pitch error and the variation in tooth thickness. It is represented by the difference between the highest (peak) and the lowest (trough) adjacent points of the error-line for each pitch or spacing as the two gears roll.

The total composite error is the total variation in centre distance as the work-gear is similarly rotated with the master gear. Besides including the tooth-to-tooth composite error (which in turn includes the errors mentioned above), the total composite error also includes runout and wobble.

In the usual types of gear-testers, the error may be recorded in the form of a cirular trace or a linear trace as shown in Fig. 2.59 (e and f). The result is then compared with the standard or permissible values as given in the Table 2.23 for acceptance or rejection of the gear. In case of an ideally perfect gear, which is virtually impossible to attain no changes in the centre distance will show up on the chart and hence a straight line will result.

The values calculated for the double-flank composite errors are to be modified in case of pressure angles other than 20°. They are to be multiplied by factor K which is given by

$K = \dfrac{\tan 20^\circ}{\tan a}$ in case of uncorrected spur gear with pressure angle equal to a degrees

$K = \dfrac{\tan 20^\circ}{\tan \alpha_w}$ in case of corrected spur gears with working pressure angle equal to α_w degrees

$K = \dfrac{\tan 20^\circ}{\tan \alpha_t}$ in case of uncorrected helical gears with working pressure angle equal to a, degrees

$K = \dfrac{\tan 20^\circ}{\tan a_{tw}}$ in case of corrected helical gears with working pressure angle in the transverse plane equal to α_{tw} degrees

The values of the centre distance tolerances given in Appendix L are also to be multiplied by the above factors.

2.28 Metrology, Inspection and Tolerances of Gears

After production, gears are checked and inspected to ensure correctness of different parameters and smoothness of operation.

Inspection of Gears

Different methods are followed for measurement and checking of gears. Several types of measuring gadgets and instruments are kept in shops because one inspection method for a particular type of gear may not be suitable for another type. For example, spur gears are easy to measure and check over pins or by backlash when meshed with a master gear. Although pin type method of measurement can be used in case of helical gears with an even number of teeth (but with more difficulty than in case of comparable spur gears), measurement becomes extremely difficult in case of helical gears having an odd number of teeth. Besides, the backlash and master gear method of measurement is not economically viable in case of helical gears because of the infinite number of helix angles in use.

Broadly, the different methods employed for measuring and checking of gears are:

(i) checking by means of accurately ground pins or wires of proper diameters,
(ii) checking by vernier calipers or micrometers,
(iii) checking by suitable dial indicator set-ups, and
(iv) checking with the help of optical comparators.

Of all the methods enumerated above, we will later discuss in detail only the second method, i.e. inspection and cheking of gear teeth by Vernier calipers or micrometers, as this is the most widely and conveniently used method in shops.

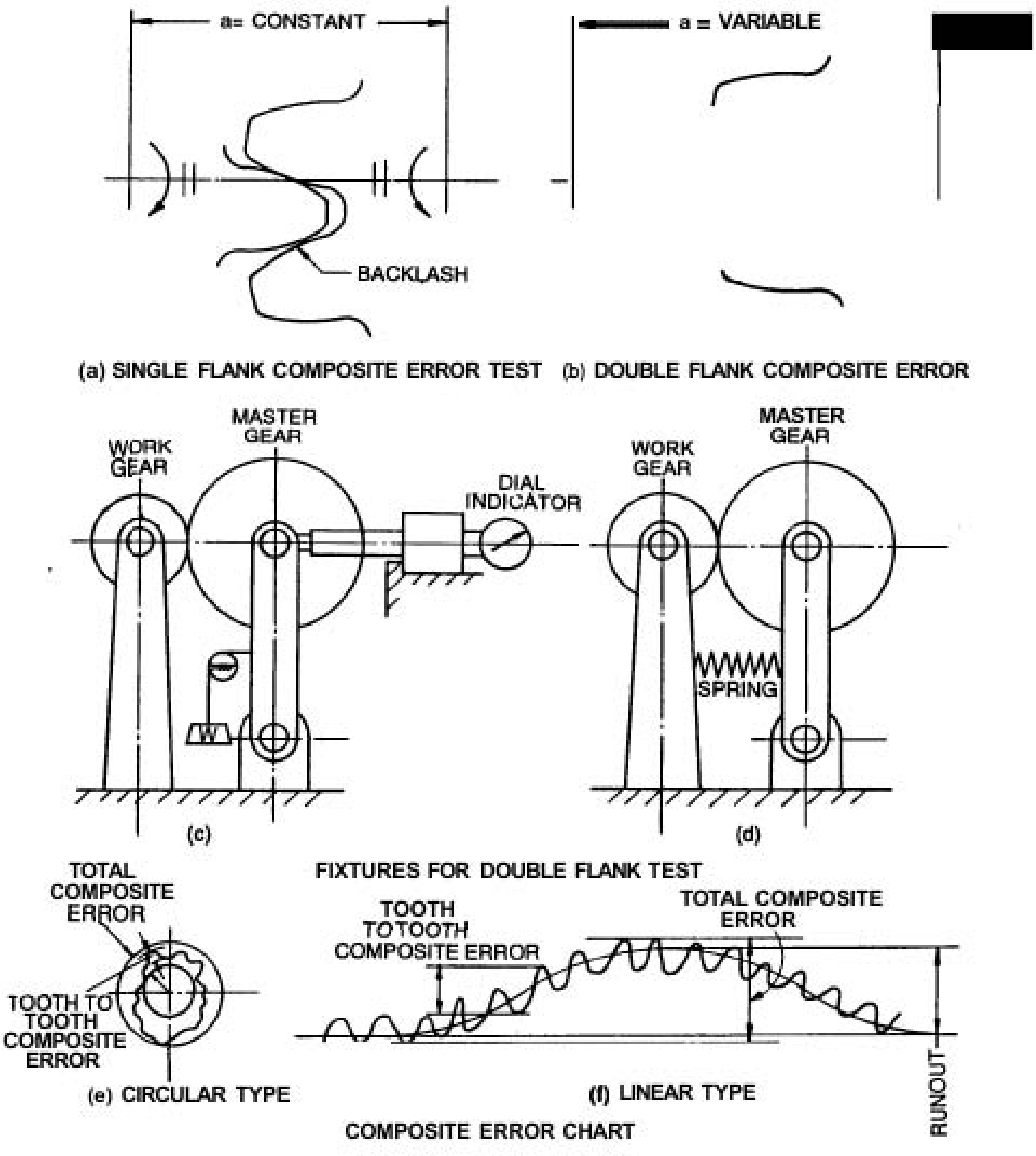


Fig. 2.59 Composite error

The method using suitable pins or wires are also used quite frequently, especially in simple cases. In this method two accurately ground pins of proper diameter are placed in diametrically opposite tooth spaces in cases of gears with an even number of teeth. Measurement over the pins is made with a micrometer. Theoretical values for such measurements are given in relevant tables. The actual measured values are then compared with the theoretical values. If the actual value is greater than the value given in the table, the tooth is thicker than the standard measurement, and if it is less, the tooth is thinner. In case of gears with an odd number of teeth, the pins are to be placed in tooth spaces as nearly opposite as possible. Relevant tables for such cases are also available. For each module or diametral pitch, separate sets of pins or wires of proper diameter must be used. (See Appendix W).

The individual errors and the composite errors have already been discussed in detail in Sec. 2.27. For measuring each type of individual errors, different types of special checking instruments suitable for the relevant types of errors are available. Normally, individual error checkings are required only for gears of the highest class, e.g. marine turbine reduction gearing, instrumentation gearing, etc. and hence these are not carried out in production line gears. Rolling gear tests for composite errors generally suffice the purpose. These tests have been dealt with in Sec. 2.27.

By and large, the most comonly used method for checking of toothing is the measurement of width over a certain specified number of teeth by means of vernier calipers or micrometers with special attachments. This method is known as the "span system of measurement", the "block measurement system", or more commonly as the "base tangent length" measurement system. It is near ideal method and is universally applied because of the fact that it is simple yet accurate, gears can be measured while still in the machine (even in running condition), and no specialised skill is required to obtain an accurate measurement.

For accurate generation to be effective, measurement and checking of toothing are of vital importance. The base tangent length method of measurement system stems from the geometric fact that if a normal is drawn to an involute tooth profile, the normal will lie in a plane which is tangent to the base cylinder. This is clear from the very principles of involumetry. If now two parallel caliper jaws contact the two tooth profiles as shown in Fig. 2.60 (a) the jaws will touch the profiles tangentially. The operator makes the measurement with the calipers with a "feel". It can be easily seen from Fig. 2.60 (a) that if the straight line of fixed length *ab* rolls back and forth over the base circle without slipping, its end points a and *b* will generate the involute curves which are the opposite profiles of the two teeth as shown. It can also be seen that this straight line is also the developed length of the arc confined between the initial points of generation of the two involute curves. In other words

$$W = \text{straight length } ab = \text{Arc length } a'b'$$

From Fig. 2.60 (a) we can also infer that

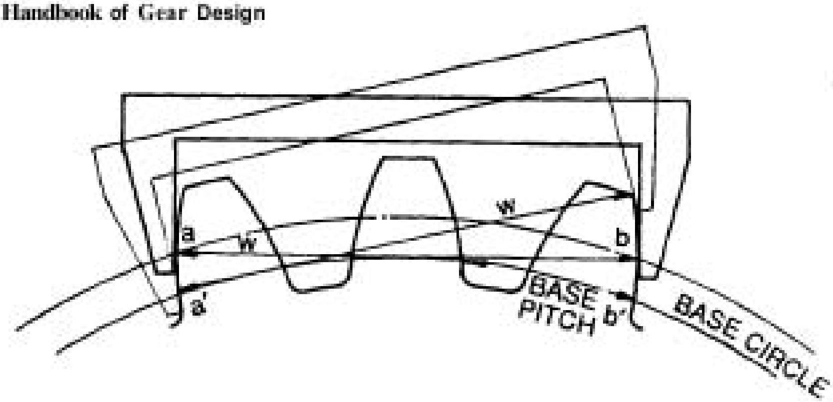
$$W = ab = (2 \times \text{base pitch}) + (1 \times \text{tooth thickness on the base circle})$$

Now base pitch = circular pitch $\times \cos\alpha = \pi m \cos\alpha$ (from Eq. 2.3)

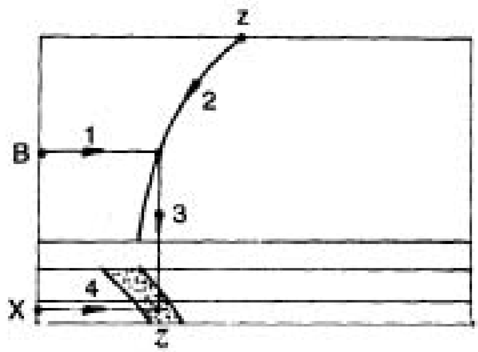
Tooth thickness on the base circle $= d_b \left(\dfrac{\pi m}{2d} + \text{inv}\,\alpha - \text{inv}\,\alpha_b\right)$ (from Eq. 2.8)

where d_b = Base circle diameter = $d \cos\alpha$

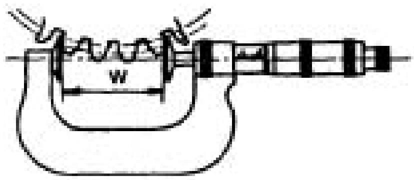
α_b = Pressure angle at the base circle



(a) BASE TANGENT LENGTH



(b) DIRECTION FOR READING Z'



(c) METHOD FOR MEASUREMENT OF BASE TANGENT

Fig. 2.60 Inspection of gears

On the base circle, the pressure angle α_b is zero. Hence

$$\text{Tooth thickness} = d_b\left(\frac{\pi m}{2d} + \text{inv}\,\alpha\right) = d\cos\alpha\left(\frac{\pi m}{2d} + \text{inv}\,\alpha\right)$$

$$= mz\cos\alpha\left(\frac{\pi m}{2mz} + \text{inv}\,\alpha\right) = mz\cos\alpha\left(\frac{\pi}{2z} + \text{inv}\,\alpha\right)$$

If z' be the number of teeth over which the measurement is made, then we have the generalised expression

$$W = [(z'-1) \times \text{base pitch}] + 1 \times \text{tooth thickness on the base circle}$$

$$= \left[(z'-1)\pi m \cos\alpha\right] + \left[mz\cos\alpha'\left(\frac{\pi}{2z} + \text{inv}\,\alpha\right)\right]$$

$$= z'\pi m \cos\alpha - \pi m\cos\alpha + \frac{\pi m \cos\alpha}{2} + mz\cos\alpha\,\text{inv}\alpha$$

$$= m\cos\alpha[(z'-0.5)\pi + z\,\text{inv}\,\alpha] \quad (2.122)$$

For corrected gears, the expression is modified as

$$W = m\cos\alpha[(z'-0.5)\pi + z\,\text{inv}\,\alpha] + 2\,x\,m\sin\alpha \quad (2.123)$$

For helical gears with correction, the value is given by

$$W = m_n\cos\alpha[(z'-0.5)\pi + z\,\text{inv}\,\alpha_t] + 2\,x\,m_n\sin\alpha \quad (2.124)$$

Instruments by which the base tangent length measurements can be made are: Vernier calipers, gear-tooth calipers, micrometers having suitable fixtures on the anvils, and other measuring devices (Fig. 2.60 c).

There are, however, certain limitations of this system of checking. If the helix angle is high and the face width of the gear is narrow, the ordinary calipers or micrometers which are not fitted with special attachments cannot be used if the calculated gear teeth over which the measurement is to be made are many in number. Moreover, certain errors such as helical-lead error, total index error, profile error, and some other errors may also affect the accuracy of the measured values to some extent. But, by and large, this type of measurement is good enough for all practical purposes. Reference charts, figures and tables have been drawn up to alleviate the tedium of calculation and the possibility of errors inherent therein. Here Chart 2.1 and Tables 2.24 and **2.25** have been given for quick reference.

The data are valid for 20° pressure angle gears.

To find the desired value of the span measurement, the following formula is used

$$W = m[K_1 + K_2 z + 2\,x\sin\alpha] \text{ in mm} \quad (2.125)$$

The value of z' is first ascertained from Chart 2.1, after taking cognizance of the relevant parameters, viz. z, β, and x. Figure 2.60 b gives the direction for reading off the value of z'.

The value of K_1 is found from Table 2.24. The value of K_2 is a function of the helix angle and is given in Table 2.25 for the helix angle range 5° to 22° which is the range commonly encountered. For spur gears ($\beta = 0°$), $K_2 = 0.014006$.

Example 2.14 *Given:* $z = 50$, $m = 6$, $\alpha = 20°$, $\beta = 8°\,20'$, $x = +0.5$.
To find the value of W.

Solution: From Chart 2.1, $z' = 7$, from Table 2.24 $K_1 = 19.18885$, from Table 2.25

$K_2 = 0.014437$, $\sin 20° = 0.34202$.

$\therefore W = 6[19.18885 + (0.014437 \times 50) + (2 \times 0.5 \times 0.34202)]$

$= 121.516$ mm

Another way of ensuring the dimensional accuracy is to check the chordal thickness of individual tooth at a pre-determined height. These measurements can be carried out by the gear-tooth calipers as shown in Fig. 2.61. The relevant equations are given on page 174.

Table 2.24 Values of K_1

z'	K_1	z'	K_1	z'	K_1
1	1.47607	21	60.51869	41	119.56132
2	4.42820	22	63.47082	42	122.51345
3	7.38033	23	66.42296	43	125.46558
4	10.33246	24	69.37509	44	128.41772
5	13.28459	25	72.32722	45	131.36984
6	16.23672	26	75.27935	46	134.3219
7	19.18885	27	78.23148	47	137.2741
8	22.14098	28	81.18361	48	140.22624
9	25.09312	29	84.13574	49	143.17837
10	28.04525	30	87.08788	50	146.13050
11	30.99738	31	90.04001	51	149.08263
12	33.94951	32	92.99214	52	152.03477
13	36.90164	33	95.94427	53	154.98690
14	39.85377	34	98.89640	54	157.93902
15	42.80590	35	101.84853	55	160.98116
16	45.75804	36	104.80066	56	163.84329
17	48.71017	37	107.75280		
18	51.66230	38	110.70493		
19	54.61443	39	113.65706		
20	57.56656	40	116.60919		

Based on Catalogue No.956.800/00.01 of M/s MAAG Zahnraeder AG, Zurich, page 10.

Table 2.25 Values of K_2 vis-a-vis helix angle

Helix angle β	K_2					
	Degrees					
Min	5°	6°	7°	8°	9°	10°
0′	.014159	.014227	.014308	.014402	.014510	.014631
1′	.014160	.014228	.014309	.014404	.014512	.014633
2′	.014161	.014229	.014311	.014406	.014514	.014635
3′	.014162	.014231	.014312	.014407	.014515	.014638
4′	.014163	.014232	.014314	.014409	.014517	.014640
5′	.014164	.014233	.014315	.014411	.014519	.014642
6′	.014165	.014235	.014317	.014412	.014521	.014644
7′	.014166	.014236	.014318	.014414	.014523	.014646
8′	.014167	.014237	.014320	.014416	.014525	.014648
9′	.014168	.014238	.014321	.014417	.014527	.014651
10′	.014169	.014240	.014323	.014419	.014529	.014653
11′	.014170	.014241	.014324	.014421	.014531	.014655
12′	.014171	.014242	.014326	.014423	.014533	.014657
13′	.014173	.014243	.014327	.014424	.014535	.014659
14′	.014174	.014245	.014329	.014426	.014537	.014662
15′	.014175	.014246	.014330	.014428	.014539	.014664
16′	.014176	.014247	.014332	.014430	.014541	.014666
17′	.014177	.014249	.014333	.014431	.014543	.014668
18′	.014178	.014250	.014335	.014433	.014545	.014670
19′	.014179	.014251	.014336	.014435	.014547	.014673
20′	.014180	.014253	.014338	.014437	.014549	.014675
21′	.014181	.014254	.014339	.014438	.014551	.014677

(Contd)

Table 2.25 (Contd.)

Helix angle β	K_s					
	Digrees					
Min	5°	6°	7°	8°	9°	10°
22'	.014182	.014255	.014341	.014440	.014553	.014679
23'	.014183	.014257	.014343	.014442	.014555	.014681
24'	.014185	.014258	.014344	.014444	.014557	.014684
25'	.014186	.014259	.014346	.014445	.014559	.014686
26'	.014187	.014260	.014347	.014447	.014561	.014688
27'	.014188	.014262	.014349	.014449	.014563	.014690
28'	.014189	.014263	.014350	.014451	.014565	.014693
29'	.014190	.014265	.014352	.014452	.014567	.014695
30'	.014191	.014266	.014353	.014454	.014569	.014697
31'	.014192	.014267	.014355	.014456	.014571	.014699
32'	.014194	.014269	.014357	.014458	.014573	.014702
33'	.014195	.014270	.014358	.014460	.014575	.014704
34'	.014196	.014271	.014360	.014461	.014577	.014706
35'	.014197	.014273	.014361	.014463	.014579	.014709
36'	.014198	.014274	.014363	.014465	.014581	.014711
37'	.014199	.014275	.014364	.014467	.014583	.014713
38'	.014201	.014277	.014366	.014469	.014585	.014715
39'	.014202	.014278	.014368	.014471	.014587	.014718
40'	.014203	.014280	.014369	.014472	.014589	.014720
41'	.014204	.014281	.014371	.014474	.014591	.014722
42'	.014205	.014282	.014373	.014476	.014593	.014725
43'	.014206	.014284	.014374	.014478	.014595	.014727
44'	.014208	.014285	.014376	.014480	.014597	.014729
45'	.014209	.014287	.014377	.014482	.014600	.014732
46'	.014210	.014288	.014379	.014483	.014602	.014734
47'	.014211	.014289	.014381	.014485	.014604	.014736
48'	.014212	.014291	.014382	.014487	.014606	.014739
49'	.014214	.014292	.014384	.014489	.014608	.014741
50'	.014215	.014294	.014386	.014491	.014610	.014743
51'	.014216	.014295	.014387	.014493	.014612	.014746
52'	.014217	.014296	.014389	.014495	.014614	.014748
53'	.014218	.014298	.014390	.014496	.014616	.014750
54'	.014220	.014299	.014392	.014498	.014618	.014753
55'	.014221	.014301	.014394	.014500	.014621	.014755
56'	.014222	.014302	.014395	.014502	.014623	.014757
57'	.014223	.014304	.014397	.014504	.014625	.014760
58'	.014225	.014305	.014399	.014506	.014627	.014762
59'	.014226	.014307	.014400	.014508	.014629	.014764
	uegrees					
Min	11°	12°	13°	14°	15°	16°
0'	.014767	.014917	.015082	.015264	.015461	.015676
1'	.014769	.014920	.015085	.015267	.015465	.015679
2'	.014772	.014922	.015088	.015270	.015468	.015683
3'	.014774	.014925	.015091	.015273	.015471	.015687
4'	.014776	.014928	.015094	.015276	.015475	.015691
5'	.014779	.014930	.015097	.015279	.015470	.015694
6'	.014781	.014933	.015100	.015283	.015482	.015698
7'	.014784	.014936	.015103	.015286	.015485	.015702

(Contd)

Table 2.25 *(Contd)*

Min II"	Degrees 11°	12°	130°	14°	15°	16°
8'	.014786	.014938	.015106	.015289	.015489	.015706*
9'	.014788	.014941	.015109	.015292	.015492	.015709
10'	.014791	.014944	.015112	.015295	.015496	.015713
11'	.014793	.014946	.015115	.015299	.015499	.015717
12'	.014796	.014949	.015117	.015302	.015503	.015721
13'	.014798	.014952	.015120	.015305	.015506	.015724
14'	.014801	.014954	.015123	.015308	.015510	.015728
15'	.014803	.014957	.014126	.015311	.015513	.015732
16'	.014805	.014960	.015129	.015315	.015517	.015736
17'	.014808	.014962	.015132	.015318	.015520	.015740
18'	.014810	.014965	.015135	.015321	.015524	.015743
19'	.014813	.014968	.015138	.015324	.015527	.015747
20'	.014815	.014970	.015141	.015328	.015531	.015751
21'	.014818	.014973	.015144	.015331	.015534	.015755
22'	.014820	.014976	.015147	.015334	.015538	.015759
23'	.014823	.014979	.015150	.015337	.015541	.015763
24'	.014825	014981	.015153	.015341	.015545	.015766
25'	.014828	.014984	.015156	.015344	.015548	.015770
26'	.014830	.014987	.015159	.015347	.015552	.015774
27'	.014833	.014990	.015162	.015350	.015556	.015778
28'	.014835	.014992	.015165	.015354	.015559	.015782
29'	.014838	.014995	.015168	.015357	.015563	.015786
30'	.014840	.014998	.015171	.015360	.015566	.015790
31'	.014843	.015001	.015174	.015364	.015570	.015793
32'	.014845	.015003	.015177	.015367	.015573	.015797
33'	.014848	.015006	.015180	.015370	.015577	.015801
34'	.014850	.015009	.015183	.015374	.015581	.015805
35'	.014853	.015012	.015186	.015377	.015584	.015809
36'	.014855	.015014	.015189	.015380	.015588	.015813
37'	.014858	.015017	.015192	.015383	.015591	.015817
38'	.014860	.015020	.015195	.015387	.015595	.015821
39'	.014863	.015023	.015198	.015390	.015599	.015825
40'	.014865	.015026	.015201	.015393	.015602	.015828
41'	.014868	.015028	.015205	.015397	.015606	.015832
42'	.014870	.015031	.015208	.015400	.015609	.015836
43'	.014873	.015034	.015211	.015403	.015613	.015840
44'	.014876	.015037	.015214	.015407	.015617	.015844
45'	.014878	.015040	.015217	.015410	.015620	.015848
46'	.014881	.015043	.015220	015414	.015624	.015852
47'	.014883	.015045	.015223	.015417	.015628	.015856
48'	.014886	.015048	.015226	.015420	.015631	.015860
49'	.014888	.015051	.015229	.015424	.015635	.015864
50'	.014891	.015054	.015232	.015427	.015639	.015868
51'	.014894	.015057	.015235	.015430	.015642	015872
52'	.014896	.015060	.015239	.015434	.015646	.015876
53'	.014899	.015062	.015242	.015437	.015650	.015880
54'	.014901	.015065	.015245	.015441	.015653	.015884

(Contd)

Table 2.25 *(Contd)*

55′	.014904	.015068	.015248	.015444	.015657	.015888
56′	.014907	.015071	.015251	.015447	.015661	.015892
57′	.014909	.015074	.015254	.015451	.015665	.015896
58′	.014912	.015077	.015257	.015454	.015668	.015900
59′	.014914	.015080	.015260	.015458	.015672	.015904

	Degrees					
Min	17°	18°	19°	20°	21°	22°
0′	.015908	.016159	.016429	.016720	.017033	.017368
1′	.015912	.016163	.016434	.016725	.017038	.017374
2′	.015916	.016168	.016439	.016730	.017044	.017380
3′	.015920	.016172	.016443	.016735	.017049	.017386
4′	.015924	.016176	.016448	.016740	.017055	.017392
5′	.015928	.016181	.016453	.016746	.017060	.017397
6′	.015932	.016185	.016457	.016751	.017065	.017403
7′	.015936	.016189	.016462	.016756	.017071	.017409
8′	.015940	.016194	.016467	.016761	.017076	.017415
9′	.015944	.016198	.016472	.016766	.017082	.017421
10′	.015948	.016203	.016476	.016771	.017087	.017427
11′	.015953	.016207	.016481	.016776	.017093	.017433
12′	.015957	.016211	.016486	.016781	.017098	.017438
13′	.015961	.016216	.016491	.016786	.017104	.017444
14′	.015965	.016220	.016495	.016791	.017109	.017450
15′	.015969	.016225	.016500	.016796	.017115	.017456
16′	.015973	.016229	.016505	.016802	.017120	.017462
17′	.015977	.016233	.016510	.016807	.017126	.017468
18′	.015981	.016238	.016514	.016812	.017131	.017474
19′	.015985	.016242	.016519	.016817	.017137	.017480
20′	.015989	.016247	.016524	.016822	.017142	.017486
21′	.015994	.016251	.016529	.016827	.017148	.017491
22′	.015998	.016256	.016534	.016832	.017153	.017497
23′	.016002	.016260	.016538	.016838	.017159	.017503
24′	.016006	.016265	.016543	.016843	.017164	.017509
25′	.016010	.016269	.016548	0.16848	.017170	.017515
26′	.016014	.016274	.016553	.016853	.017175	.017521
27′	.016019	.016278	.016558	.016858	.017181	.017527
28′	.016023	.016283	.016563	.016863	.017187	.017533
29′	.016027	.016287	.016567	.016869	.017192	.017539
30′	.016031	.016292	.016572	.016874	.017198	.017545
31′	.016035	.016296	.016577	.016879	.017203	.017551
32′	.016039	.016301	.016582	.016884	.017209	.017557
33′	.016044	.016305	.016587	.016890	.017215	.017563
34′	.016048	.016310	.016592	.016895	.017220	.017569
35′	.016052	.016314	.016597	.016900	.017226	.017575
36′	.016056	.016319	.016601	.016905	.017231	.017581

(Contd)

Table 2.25 *(Contd)*

Min	Degrees 17°	18°	19°	20°	21°	22°
37'	.016060	.016323	.016606	.016910	.017237	.017587
38'	.016065	.016328	.016611	.016916	.017243	.017593
39'	.016069	.016332	.016616	.016921	.017248	.017599
40'	.016073	.016337	.016621	.016926	.017254	.017605
41'	.016077	.016342	.016626	.016932	.017260	.017611
42'	.016082	.016346	.016631	.016937	.017265	.017618
43'	.016086	.016351	.016636	.016942	.017271	.017624
44'	.016090	.016355	.016641	.016947	.017277	.017630
45'	.016094	.016360	.016646	.016953	.017282	.017636
46'	.016099	.016364	.016651	.016958	.017288	.017642
47'	.016103	.016369	.016655	.016963	.017294	.017648
48'	.016107	.016374	.016660	.016969	.017299	.017654
49'	.016111	.016378	.016665	.016974	.017305	.017660
50'	.016116	.016383	.016670	.016979	.017311	.017666
51'	.016120	.016387	.016675	.016985	.017317	.017673
52'	.016124	.016392	.016680	.016990	.017322	.017679
53'	.016129	.016397	.016685	.016995	.017328	.017685
54'	.016133	.016401	.016690	.017001	.017334	.017691
55'	.016137	.016406	.016695	.017006	.017340	.017697
56'	.016142	.016411	.016700	.017011	.017345	.017703
57'	.016146	.016415	.016705	.017017	.017351	.017709
58'	.016150	.016420	.016710	.017022	.017357	.017716
59'	.016155	.016425	.016715	.017028	.017363	.017722

Based on Catalogue No. 956. 800/00.01 of M/s MAAG ZahnraederAG, Zurich, Switzerland, page no. 14.31.

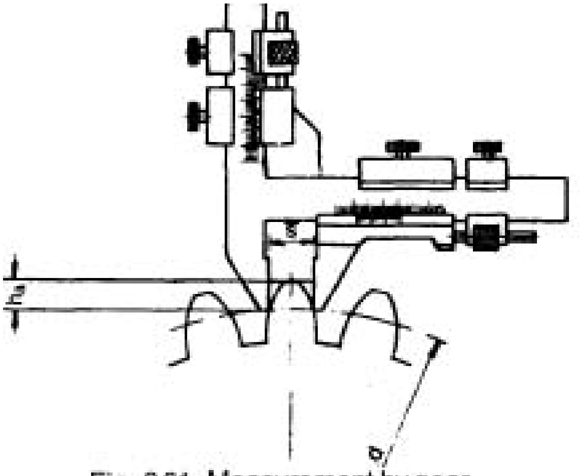


Fig. 2.61 Measurement by gear tooth calipers

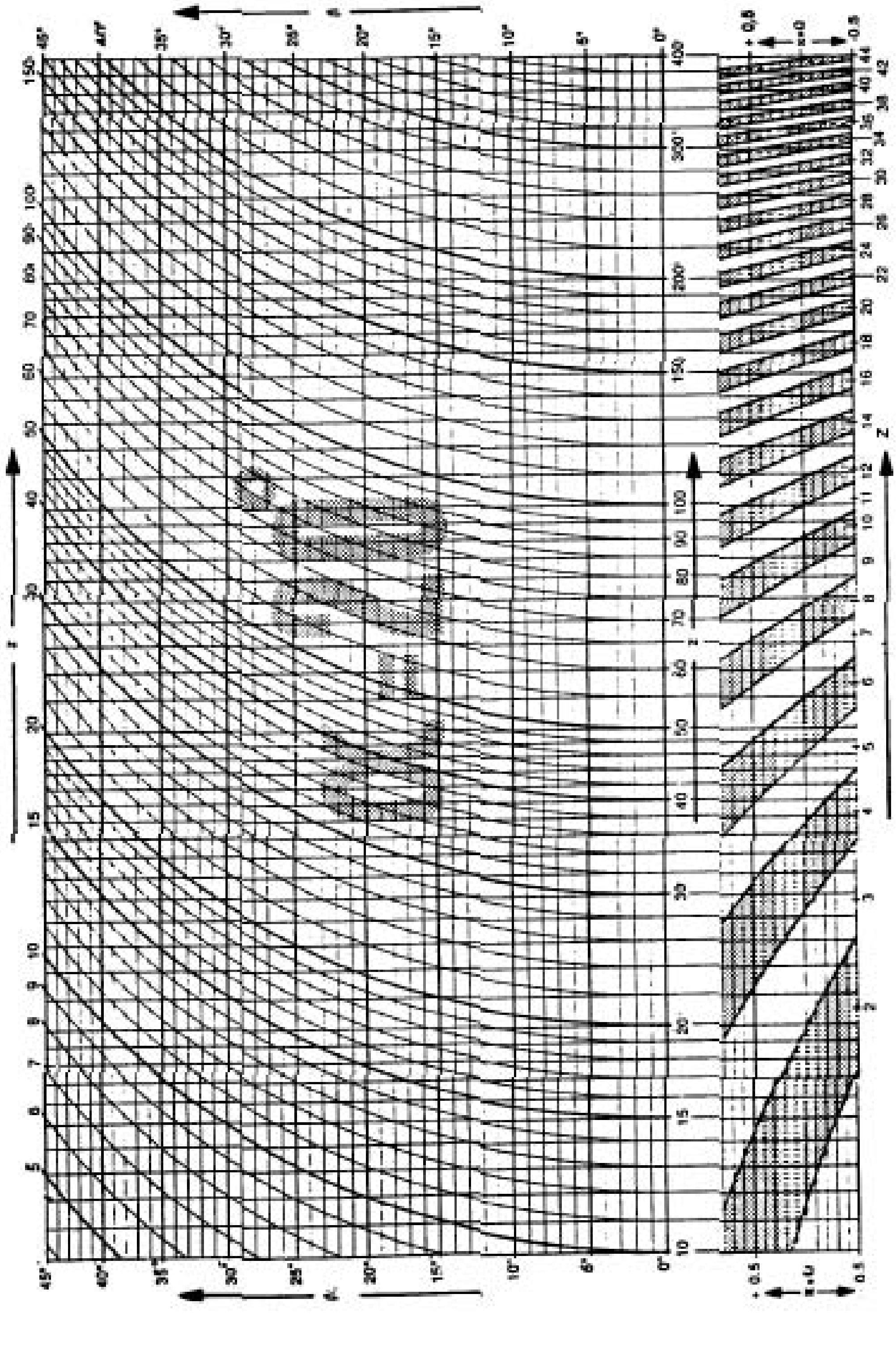


Chart 2.1 Values of number of teeth spanned

Based on Catalogue No. 956.800/00.01 of M/s MAAG Zahnräder AG, Zurich, page 7.

For a spur gear with correction, the chordal tooth thickness is given by

$$\bar{S} = d\sin\frac{\psi}{2} \qquad \text{where } \psi = \frac{S}{r} \qquad \text{radians}$$

$$\hat{S} = \frac{p}{2} + 2xm\tan\alpha = \text{Circular tooth thickness}$$

Since $p = \pi m$ and $d = mz$, we have

$$\bar{S} = mz\sin\left[\left(\frac{\pi m}{2} + 2xm\tan 20^\circ\right)/mz\right] (\psi \text{ in radians}) \tag{2.126}$$

$$= mz\sin\left[\frac{\frac{\pi}{2} + 2x\tan 20^\circ}{z} \cdot \frac{180^\circ}{\pi}\right] \quad (\text{when } \psi \text{ is in degrees}) \tag{2.127}$$

Chordal height $$\bar{h}_a = m(1+x) + \frac{d}{2}\left(1 - \cos\frac{\hat{S}}{d}\right) \tag{2.128}$$

Equation for $\bar{h}_a$ is applicable when addendum correction or topping is not considered. When this is to be taken into account, then the value of $\bar{h}_a$ is to be reduced by the amount y m. (see Eq. **2.36**). Putting $x = 0$ in the above equations, we get the relevant values for normal, uncorrected, spur gears. These values can also be obtained from Table **2.26.** This table gives the values for module = 1. For any other module, the values from the table against z are to be multiplied by the module in question.

Example 2.15 : $z = 34$, $m = 22$. To find $\bar{S}$ and $\bar{h}_a$

Solution : $\bar{S}$ = **1.57024** (for z = **34**) x **22** = **34.5453** mm

$\bar{h}$ = **1.01815** x **22** = **22.3993 mm**

Tolerances of Gears

After ascertaining the tooth distance W and the tooth thickness $\bar{S}$, the values are to be "toleranced". Alongwith the tolerances on the centre distance, the tolerances on W or $\bar{S}$ determine the final backlash between the meshing gears in the mounted conditon. This has been explained in Sec. **2.8** dealing with backlash. Tolerances on W and $\bar{S}$ are given in Appendices K and J respectively.

The values of the given tolerances are valid for transverse section. For spur gears, the normal and the transverse sections are the same and hence the values can be directly used. For helical gears, however, the values are to be multiplied as shown in Eqs **2.129** and **2.130** to get the values in the normal section

$$A_{sn} = A_{st}\cos\beta \tag{2.129}$$

$$A_{wn} = A_{wt}\cos\beta \tag{2.130}$$

where A_{sn} = Tooth thickness tolerance in the normal section

A_{st} = Tooth thickness tolerance in the transverse section as given in Appendix J

A_{wn} = Tooth distance tolerance in the normal section

A_{wt} = Tooth distance tolerance in the transverse section as given in Appendix K

The tolerances on tooth thickness A_s and on tooth distance A_w are related to each other as shown in the following equation

$$A_w = A_s \cos a \qquad (2.131)$$

Table 2.26 Chordal tooth thickness and height

z	$\bar{S}/m$	$\bar{h}_a/m$	z	$\bar{S}/m$	$\bar{h}_a/m$	z	$\bar{S}/m$	$\bar{h}_a/m$
10	1.56433	1.06160	40	1.57039	1.01541	70	1.57066	1.00880
11	1.56546	1.05590	41	1.57041	1.01502	71	1.57066	1.00868
12	1.56631	1.05135	42	1.57043	1.01468	72	1.57066	1.00856
13	1.56698	1.04740	43	1.57045	1.01430	73	1.57067	1.00845
14	1.56750	1.04405	44	1.57046	1.01393	74	1.57067	1.00834
15	1.56793	1.04110	45	1.57048	1.01360	75	1.57067	1.00824
16	1.56827	1.03850	46	1.57049	1.01331	76	1.57068	1.00813
17	1.56856	1.03625	47	1.57050	1.01302	77	1.57068	1.00803
18	1.56880	1.03425	48	1.57051	1.01275	78	1.57069	1.00793
19	1.56900	1.03235	49	1.57052	1.01249	79	1.57069	1.00783
20	1.56918	1.03073	50	1.57053	1.01227	80	1.57069	1.00774
21	1.56933	1.02920	51	1.57054	1.01202	81	1.57069	1.00764
22	1.56946	1.02795	52	1.57055	1.01181	82	1.57069	1.00755
23	1.56957	1.02675	53	1.57056	1.01162	83	1.57070	1.00746
24	1.56967	1.02560	54	1.57057	1.01141	84	1.57070	1.00737
25	1.56976	1.02461	55	1.57058	1.01121	85	1.57070	1.00729
26	1.56984	1.02371	56	1.57059	1.01105	86	1.57070	1.00720
27	1.56991	1.02275	57	1.57059	1.01081	87	1.57071	1.00712
28	1.56997	1.02210	58	1.57060	1.01061	88	1.57071	1.00703
29	1.57003	1.02131	59	1.57061	1.01042	89	1.57071	1.00696
30	1.57008	1.02065	60	1.57061	1.01025	90	1.57071	1.00687
31	1.57012	1.01996	61	1.57062	1.01008	91	1.57071	1.00679
32	1.57016	1.01930	62	1.57062	1.00991	92	1.57071	1.00672
33	1.57020	1.01871	63	1.57063	1.00975	93	1.57072	1.00665
34	1.57024	1.01815	64	1.57063	1.00959	94	1.57072	1.00657
35	1.57027	1.01762	65	1.57064	1.00945	95	1.57072	1.00650
36	1.57029	1.01710	66	1.57064	1.00931	96	1.57072	1.00643
37	1.57032	1.01665	67	1.57065	1.00918	97	1.57072	1.00637
38	1.57035	1.01620	68	1.57065	1.00905	98	1.57072	1.00630
39	1.57037	1.01577	69	1.57065	1.00892	99	1.57073	1.00623

For any pressure angle α_0, other than 20°; the requisite value of A_{w0} relevant for that angle can be found as follows

$$A_{w0} = A_w \frac{\cos \alpha_0}{\cos 20°} \qquad \text{r2.131,}$$

Example 2.16 *Given:* $A_{st} = -40$ and $-60\ \mu$m. To find the corresponding values of A_{wn} for pressure angle 15° and helix angle 10°.

Solution: $A_{wt} = A_s \cos a = -40 \times \cos 20° = -37.6\ \mu$m

$A_{wn} = A_{wt} \cos \beta = -37.6 \times \cos 10° = -37\ \mu$m

$A_{wn0} = A_{wn} \cos \alpha_0 / \cos 20° = -37 \times \cos 15°/\cos 20° = -38\ \mu$m $= -0.038$mm

Similarly, the corresponding values of A_{wn} for A_s = −60 μm can be calculated.

The proper choice of the tooth thickness tolerances or tooth distance tolerances as well as the tolerances on the centre distance (discussed later) will depend upon the discretion of the designer. The designer determines the tolerances mainly from experience after considering many factors, viz. backlash required, method of manufacture of gear, material, speed, type of lubrication, permissible stresses, magnitude and type of load, duty factors and other relevant parameters. As the Appendix on tolerance indicates, the tolerance values are functions of quality grade, module, pcd, and the zone position designated by a letter. The zone position starts with letter h having zero as one of the limits of tolerance and then goes on to letter *a* having maximum deviation from the datum, i.e. zero value as can be seen from the Appendices on tooth thickness and tooth distance tolerances. This has been diagramatically represented in Fig. 2.62. There are further zone positions beyond a, but these are not included in the tolerance table because they are normally not much used. The Appendices contain values relevant only for usual qualities, modules and other factors of general, practical use. Zone h is used for gear drives which run practically without backlash. Zone g is valid for gear drives with very small backlash. Zones *f* to a along with centre distance tolerances J and *K* provide backlash in all cases.

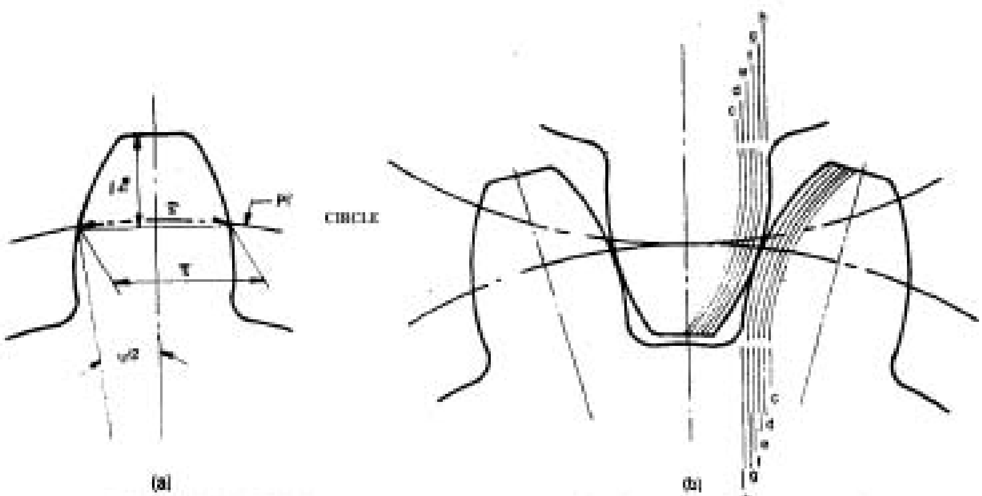


Fig. 2.62 Thickness measurement and tolerance of gear tooth

Sometimes the zonal deviations may encompass more than one position as shown in Example 2.17.

Example 2.17 *Given:* $m = 3$, pcd = 45, tolerance and zone = 9 db

To find the tolerance values of *W*

Solution: From Appendix *K*, we have

	Tolerance values of *d*	Tolerance values of *b*
Upper limit A_{wU}	−0.085	−0.169
Lower limit A_{wL}	−0.127	−0.211

In each individual case, the field of tolerance is 42 μm (127 − 85 = 42 and 211 − 169 = 42). If the tolerance is given as "db", it means that a tolerance field of 0.042 mm can float between the two extreme limits that is, −0.085 mm and −0.211 mm. The actual tolerance can then have any value between these two extremes, provided the difference is maintained at 42 μm. For example, it can have values − 0.100 mm and − 0.142 mm, and so on. The usual practice is to have greater flexibility in backlash values, as in the case of change gears, variable or elevated thermal operating conditions, etc. This also minimises the quantity of rejects during manufacture.

If, for instance, the quality and zone are 7 e, then the final values of W (see Example 2.14) are given by

$$W = 121.516^{-0.050}_{-0.084} \quad \text{(mm)}$$

It should be noted here that the tolerance value, as given in Appendix K, were multiplied by the value of cos β, i.e. cos 8° 20′, to arrive at the above values since the span measurement is done in normal section as explained earlier.

Centre distance tolerances are given in Appendix L. There are two zone positions, J and K. Values in zone K are twice as large as those in zone J. As mentioned in Sec. 2.27, the given values of the centre distance tolerances are to be modified by multiplying them by the factor K as detailed at the end of that section.

As emphasised earlier, no cut and dried method can be given to select the proper tolerances for any type of gear, this being entirely the discretion of the designer who draws from his knowledge and experience to arrive at certain values of the tolerances concerned. Table 2.27 serves as a guide for the proper selection of tolerances.

Table 2.27 Selection of quality and tolerance zone

Gear Quality	7	8,9	10 and above
Scope of application	Fast running gears, gears for turbines, screw compressors, and similar machines	Normal running gears, gears for general purpose machines, cranes and presses	Coarse gears, hand-operated gear boxes, slow running gears
Centre distance tolerance	7 J	8 J or 9 J	10 K and above
Tooth thickness tolerance	7 c 7 d 7 cd	8 b or 9 b 8 c or 9 c 8 bc or 9 bc	10 a and above 10 b and above 10 ab and above

It must be kept in mind that these tolerances are the parameters which determine the backlash in the mounted condition of gearing, and hence, they should be selected judiciously. Gear blank tolerances are given in Appendix I. The common linear dimensional tolerances are given in Appendix 5. The dimensional tolerances as well as the IT values can also be obtained from IS: 919 or from any standard book on fits and tolerances.

2.29 Efficiency of Spur Gear Drive

In any gear drive, the efficiency of the system is given by

$$\text{Efficiency } (\eta) = \frac{\text{Output power}}{\text{Input power}}$$

There is a power loss in the system due to the sliding action of toothing. Besides this, there are power losses at the bearings, loss due to the churning of lubericating oil and other losses, so that the overall efficiency becomes still lesser than the efficiency calculated on the basis of loss due to sliding only. The following values are typical for the spur gear drive system

η = **92** to **94%** for unmachined teeth

= **96%** for smooth and lubricated teeth

= **98** to **99%** for very carefully machined teeth with hydrodynamic lubrication between teeth surfaces

With the ideal kind of design, construction and bearing systems coupled with very good lubrication and gears of high quality, it is not difficult to attain an efficiency as high as **98** to **99%**. For automotive drives in general, it is around **97%**.

If the sliding velocity is high, it leads to considerable power loss due to friction and the efficiency may fall to around 85%.

In this section, efficiency considering the toothing aspects only will be discussed. That is, loss due to bearings, lubricants, etc. will not be taken into account.

Neglecting friction, torques T_1 and T_2 of the two meshing gears have the following relation

$$\frac{T_1}{T_2} = \frac{O_1P}{O_2P} = \frac{r_1}{r_2} \qquad \text{(see Fig. 1.11)}$$

where O_1 and O_2 are the centres and P is the pitch point, dividing the line of centres O_1O_2 into pitch circle radii r_1 and r_2. Due to the effect of the frictional force, however, the actual pressure line cuts the line of centres O_1O_2 somewhat away from the pitch point P. This point P' always lies between P and the centre O_2 of the driven gear. Considering friction, we have the relation

$$\frac{T_1}{T_2} = \frac{O_1P'}{O_2P'}$$

The instantaneous efficiency at any point of contact other than the pitch point (where the motion is one of pure rolling) is given by

$$\eta_{inst} = \frac{O_2P'}{O_1P'}\Big/\frac{O_2P}{O_1P} = \frac{O_2P'}{O_1P'} \times \frac{r_1}{r_2}$$

Since P' always lies between O_2 and P, $O_2P' < r_2$ and $r_1 < O_1P'$.

Therefore, η_{inst} is always less then 100%. Only when P' coincides with P, i.e. when the net effect of the relative sliding velocity is zero, η_{inst} is 100% because at that particular instant the power loss due to sliding is zero.

Since there is elastic deformation of teeth under load (see Sec. **2.22** on dynamic loads), determination of tooth pressure at individual contact points is a statically indeterminate problem. Moreover, tooth load remains the same only when the contact ratio is a whole number and when the actual tooth profile exactly conforms to the theoretical one. Besides, in normal cases where the contact ratio is not a whole number, meshing of two pairs of teeth during part of the course of action also prevent the calculation from being precise. For all practical purposes, however, the following treatment should suffice.

$$\eta = 1 - \frac{\mu}{\cos\alpha}\left(\frac{1}{r_2} + \frac{1}{r_2}\right)\frac{1}{2}\left[\frac{(AP)^2 + (BP)^2}{AP + BP}\right]$$

Here, AP and BP are the segments of the length of contact **AB**, as shown in Fig. **2.12.** Points

A and B are the extreme points where the sliding effect is maximum.

Taking module = 1 and inserting factor f, given by

$$f = \frac{1}{\cos\alpha}\left[\frac{(AP)^2+(BP)^2}{AP+BP}\right]$$

in the above formula for efficiency, we get the simplified expression

$$\eta = 1 - f\mu\left(\frac{1}{z_1} \pm \frac{1}{z_2}\right)$$

The above relation is same as Eq. 2.11 in Sec. 2.6, and the definitions and values given in that section are also the same.

2.30 Engineering Drawing of Gears

A manufacturing or shop drawing of a gear should mainly consist of

(i) a dimensioned drawing of the gear as an engineering component, and

(ii) a table containing the relevant gear data which should preferably accompany the gear drawing, both be placed side by side.

Besides the above, it should contain the usual information on various aspects such as heat-treatment, machining, material specifications, hardness, etc. which is commonly given in an engineering drawing.

The drawing of the body of the gear should be executed just like that of any other engineering item that is, it should contain the relevant views, sections, etc. to represent the solid body of the gear in orthographic projections.

There is a great diversity in the manner in which the relevant gear data are presented. In present-day practice, although some kind of standardised data table has been recommended by different Standards institutions or gear manufacturers associations, many individual gear manufacturers and engineering firms follow their own practices. Some general guidelines, however, are given below.

Broadly, the table should contain the undermentioned three categories of data.

1. Basic data and specifications These relate to the number of teeth, module or diametral pitch, pressure angle, helix angle and hand of helix in case of helical gears, tip circle and pitch circle diameters, tooth form, amount of correction in case of corrected gears, etc. In case of bevel gears worm and worm-wheels, and other types of gearing, the relevant data are to be inserted. Besides, the transverse module and transverse pressure angle in case of helical gears, the standard whole depth of tooth, addendum, amount of topping, etc. may also be given. The value of the tip circle and the pitch circle diameters are sometimes given both on the body of the gear drawing as linear dimensions as well as in the gear data table.

2. Inspection data These are also used in different stages of gear manufacture besides serving inspection purposes. These data include permissible values of different types or errors, quality and zone of tolerance, base tangent length measurement with tolerance, chordal thickness and height of tooth at the pitch circle, etc. Details of some allowable errors, e.g. F''_i, F_p, F_f, may also be given. Normally, the permissible value of the double-flank total composite error F''_i should suffice for all practical purposes.

3. Engineering reference data These are auxiliary data which are useful for the gear-drive as a whole unit and also for mounting purposes. These include the number of teeth of the mating gear, its part number in the drawing, centre distance with tolerance, etc.

As examples of engineering drawing of gears, Drawings 2.1 and 2.2 are given. The numerical values and data given in those drawings refer to the example of a spare part drawing-set of a gear drive given in Sec. 8.9. Drawings of gears which are designed from first principles will also be represented in the same manner. Simplified versions of drawings of different types of gears are illustrated in Fig. 2.63 which are mostly used in assembly drawings.

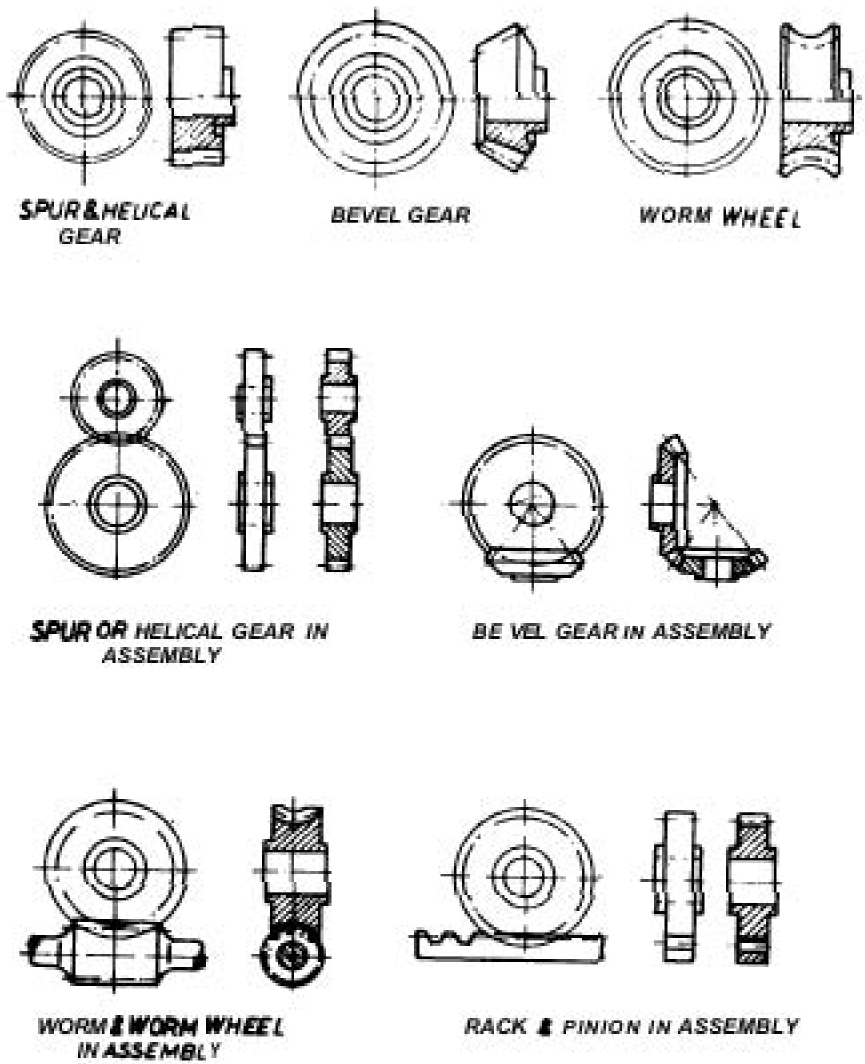
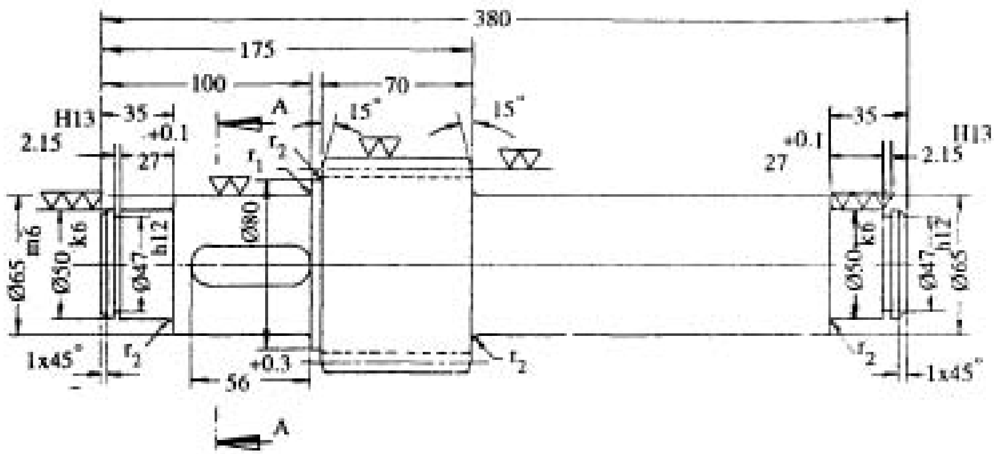
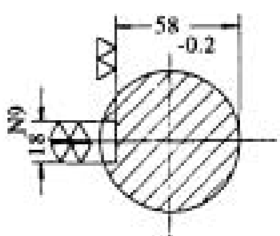


Fig. 2.63 Conventional representation of gear and gear assemblies in engineering drawings





SECTION A A

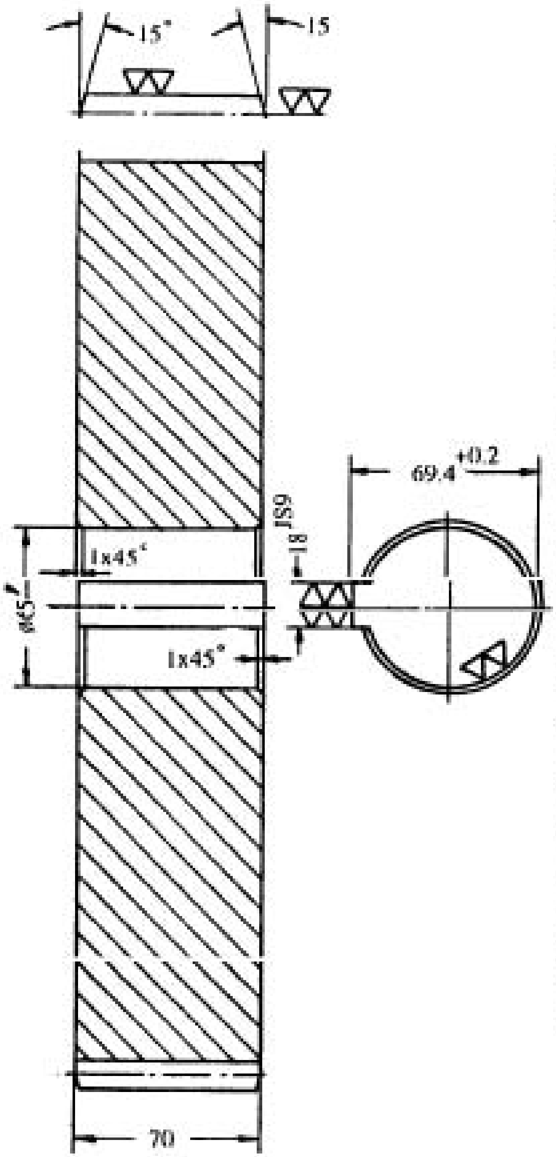
NOTE:

1. HARDNESS TO BE HB 2550-2860N/mm² GEAR TOOTH SURFACE SHOULD BE CASE HARDENED TO HRC 50-55 N/mm² UPTO A DEPTH OF 1mm.
2. RADIAL RUNOUT OF TIP CYLINDER, RADIAL & AXIAL RUNOUT OF REF. SURFACE WITH RESPECT TO BORE SHOULD NOT BE MORE THAN 0.018 mm.
3. THE OVALITY & TAPER OF FITTING DIAMETERS SHOULD BE WITHIN HALF OF THEIR RESPECTIVE DIAMETRAL TOLERANCE.
4. FOR REPLACEMENT, BOTH THE GEARS OF THE MATING PAIR SHOULD BE CHANGED.

SL. NO.	DESCRIPTION		DATA
1	NO. OF TEETH	z	18
2	MODULE	m	5
3	OUTSIDE DIAMETER	d_a	102.42_{h8}
4	PITCH CIRCLE DIAMETER	d	90.88
5	PRESSURE ANGLE	α	20°
6	HELIX ANGLE	β	8°
7	DIRECTION OF HELIX		L.H.
8	PROFILE CORRECTION	x.m	+0.8
9	TOOTH FORM		AS PER IS: 2535
10	QUALITY OF TOLERANCE		7dS"
11	NO. OF TEETH TO BE MEASURED	z'	3
12	TOOTH DISTANCE	w	$38.745^{-0.053}_{-0.078}$
13	TOTAL COMPOSITE ERROR, DOUBLE FLANK	F_i''	0.083
14	NO. OF TEETH IN MATING GEAR		73
15	DRG. NO. OF MATING GEAR		DRG. NO. 2
16	CENTRE DISTANCE IN HOUSING WITH TOLERANCE	a	231.5±0.034

∇ (∇∇, ∇∇∇)

Drawing 2.1



GEARDATA

SL. NO.	DESCRIPTION		DATA
1	NO. OF TEETH	z	73
2	MODULE	m	5
3	OUTSIDE DIAMETER	d_a	380.52
4	PITCH CIRCLE DIAMETER		368.58
5	PRESSURE ANGLE	a	20°
6	HELIX ANGLE	β	8°
7	DIRECTION OF HELIX		R.H
8	PROFILE CORRECTION	x.m	+1.0
9	TOOTH FORM		AS PER IS:2535
10	QUALITY OF TOLERANCE		7dS"
11	NO. OF TEETH TO BE MEASURED	z'	
12	TOOTH DISTANCE	w	
13	TOTAL COMPOSITE ERROR DOUBLE FLANK	F i	
14	NO. OF TOOTH IN MATING GEAR		18
15	DRG. NO. OF MATING GEAR		DRG.NO.1
16	CENTRE DISTANCE IN HOUSING WITH TOLERANCE	a	231.5 ± 0.034

MATERIAL : 45C8

(∇, ∇∇)

NOTE:

HARDNESS H7 TO BE HB 2115-2550 N/mm². GEAR TOOTH SURFACE SHOULD BE CASE HARDENED TO HRC 50-55 N/mm² UP TO A DEPTH OF 1mm.

SCALE 1:2.5

Drawing 2.2

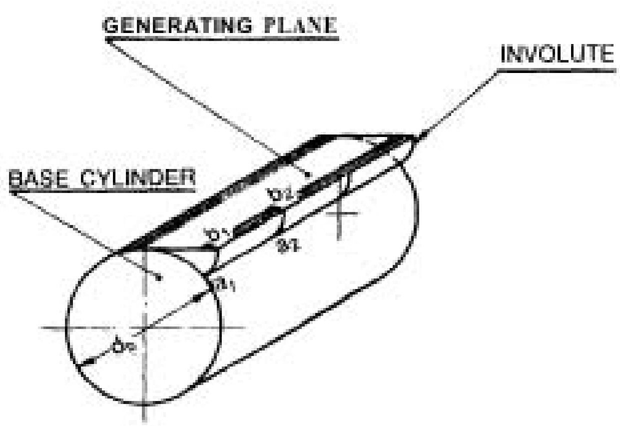
3
Helical Gears

3.1 Geometry of Helical Gears

Like spur gears, the helical gears are employed to transmit motion between parallel shafts. These gears can also be used for transmitting motion between non-parallel, non-intersecting shafts. In the former case, the gears are called parallel helical gears, while in the latter case, they are termed as crossed helical gears. Crossed helical gears are discussed in detail in Sec. 3.15. In Secs 3.1-3.14 only parallel helical gears will be discussed.

Figure 3.1(a) shows a plane which rolls on a base cylinder. The edge of this plane is a straight line parallel to the axis of the cylinder. When this plane rolls or unwraps from the cylinder, each point on the edge traces an involute, namely, curves a_1b_1, a_2b_2, etc. Since points b_1, b_2, etc. are equidistant from their original positions, i.e. a_1, a_2, etc. all the involute curves thus generated are identical in every respect. In other words, the surface of an involute spur gear tooth is thereby created by the edge of the unwrapping plane.

Now consider Fig. 3.1(b) where the edge of the plane is inclined to the axis. When this inclined edge coincides with the base cylinder, the line is in the form of a helix, a_1a_x. As the generating



(a) SPUR

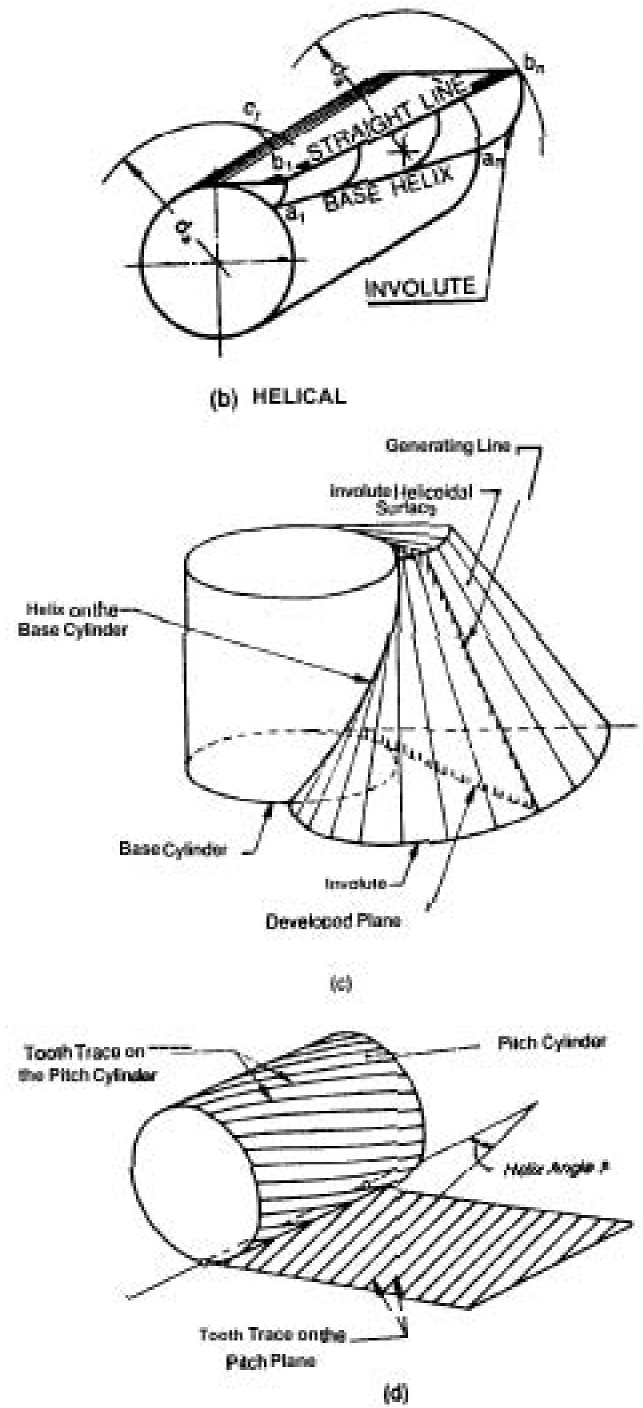


Fig. 3.1 Generation of gear tooth

plane unwraps and becomes a taut, horizontal plane, the edge becomes a straight line. The points, as before, trace out involute curves, viz. a_1b_1 .. a_nb_n, etc. The curved surface bounded by points a_1–b_1–b_n–a_n is an involute helicoid and it forms the surface of the tooth of a helical gear.

A pictorial view of generated helicoidal surface is shown in Fig. 3.1(c). In Fig. 3.1(d), a pictorial view of the pitch cylinder of a helical gear has been shown in partly unwrapped condition. One can easily discern the helical orientation of the tooth traces on the body of the pitch cylinder, which become straight lines on the developed pitch plane.

It is obvious from the figure that curves a_1b_1 and a_nb_n are unequal in length because of the initial inclination of the edge of the generating plane. Curve a_1b_1 is now extended to c_1, so that $a_1b_1c_1 = a_nb_n$. Point c_1 and b_n will now be equidistant radially from the central axis of the base cylinder. In other words, they will lie on circles of the same diameter, d_a. These are the two tip circles of the helical gear at the two extreme ends of the cylinder, covering the width of the gear. Note that there exists an angular phase difference between points c_1 and b_n when referred to the cylinder axis. This represents the twist of the helical gear tooth which is quite apparent in Fig. 3.2. Because of this twist along ths width of the helical gear, the top land is not parallel to the bottom land as can be seen from the figure. This is not so in case of a spur gear where all the lines are parallel to the axis of the gear. Therefore, if the helical gear is thought of as consisting of an infinite number of concentric hollow cylinders, placed one inside another so that the resulting body is a solid, then if any individual cylinder is considered, the path of the tooth-trace on the surface of that cylinder is a helix along the whole length covering the tooth width. But, because of the twist, the inclinations of the helices of different cylinders with reference to the gear axis are different. This results in different helix angles at different cylinders which will be discussed in Sec. 3.2.

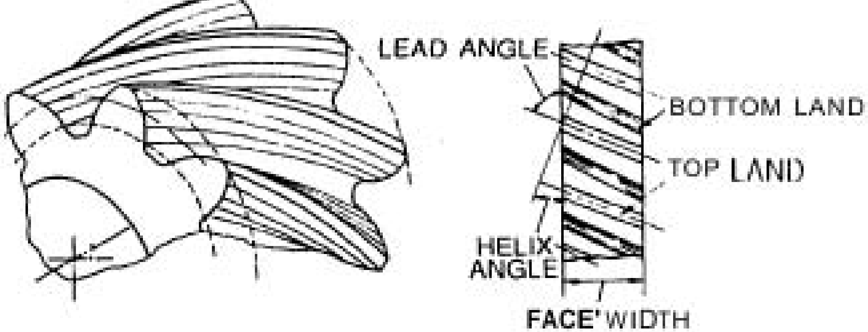


Fig. 3.2 Shape of helical gear tooth

Based on Practical Gear Design, Dudley, 1954 edition, fig. 1-13, p. 19. McGraw-Hill Book Co. Inc., New York.

3.2 Helical Gear Terminology and Relations

Because spur gears are easier to design and manufacture, engineers usually prefer these gears when power is transmitted between parallel shafts. There are, however, some design considerations like greater contact ratio, greater strength, and some operational requirements, such as, noiselessness, smoother engagement of meshing of teeth, for which the use of helical gears is preferred.

When a pair of parallel helical gears mesh, the following conditions must be satisfied for proper running of the set:

(*i*) The gears must have helix angles of equal value;
(*ii*) The gear teeth of each member must have the same module, and
(*iii*) The gear teeth of each member must have opposite helices, that is, one gear must have right-handed helical teeth while the other must have left-handed ones.

One ofthe fundamental differences between spur andhelical gear asfar asthe courseof action is concerned is that the contact of the two spur gears in mesh takes place always in a line extending all along the surface ofthe tooth, this line being always parallel to the axis ofthe gear. In a helical gear, the initial contact is a point which gradually changes into a line as engagement proceeds. Unlike spur gear, this line ofcontact is a diagonal across the face and flank ofthe tooth. This aspect and its implication will be discussed in detail in Secs 3.4 and 3.5.

The basic parameters of spur and helical gears have been shown in Fig. 2.3. Figure 2.3 (c) represents the developed pitch cylinder of a helical gear. The definitions of basic nomenclature and geartooth terminology are the same in spur and helical gears, and for these the reader should refer to Sec. 2.2. The special features which characterise and differentiate a helical gear from a spur as well as their inter-relations will be discussed in this section.

The following special parameters of a helical gear are defined here (Fig. 2.3c).

Helix angle This is the angle which the tooth trace of a helical gear in the pitch plane makes with the gear axis. This inclination to the gear axis varies along the involute as it originates from the base cylinder and develops outwards, that is, away from the base cylinder. It should be carefully noted that when the term "helix angle" is used in connection with a helical gear, it means the helix angle at the pitch cylinder only and is usually denoted by β without any subscript. With other cylinders, the general expression is given by

$$\tan\beta_c = \tan\beta \times \frac{d_c}{d} \qquad (3.1)$$

Where β_c = The helix angle of any cylinder c
d_c = The diameter of the above cylinder
β = The helix angle at the pitch cylinder and
d = The pitch circle diameter of the helical gear

Lead angle The lead angle is the complimentary angle to the helix angle, and is given by

$$\gamma = 90^\circ - \beta \qquad (3.2)$$

In case of a helical gear, two types of sections are consideredthe normal section and the transverse section. These sections have been shown in Fig. 3.3. (a)with respect to the reference profile. The normal section is taken by passing a plane at right angles to the tooth trace of the top-most tooth and through the pitch point P on the pitch cylinder as shown. If the plane is passed at right angles to the axis of the gear, the transverse section is obtained. The reader should note that normal section ofthe pitch cylinder is an ellipse while the transverse section is a circle. The relevant parameters and the relations thereof are explained below.

Module Two kinds of modules are differentiated in helical gears — the normal module and the transverse module. They bear the following relation to each other

$$m_t = \frac{m_n}{\cos\beta} \qquad (3.3)$$

where m_t = The transverse module

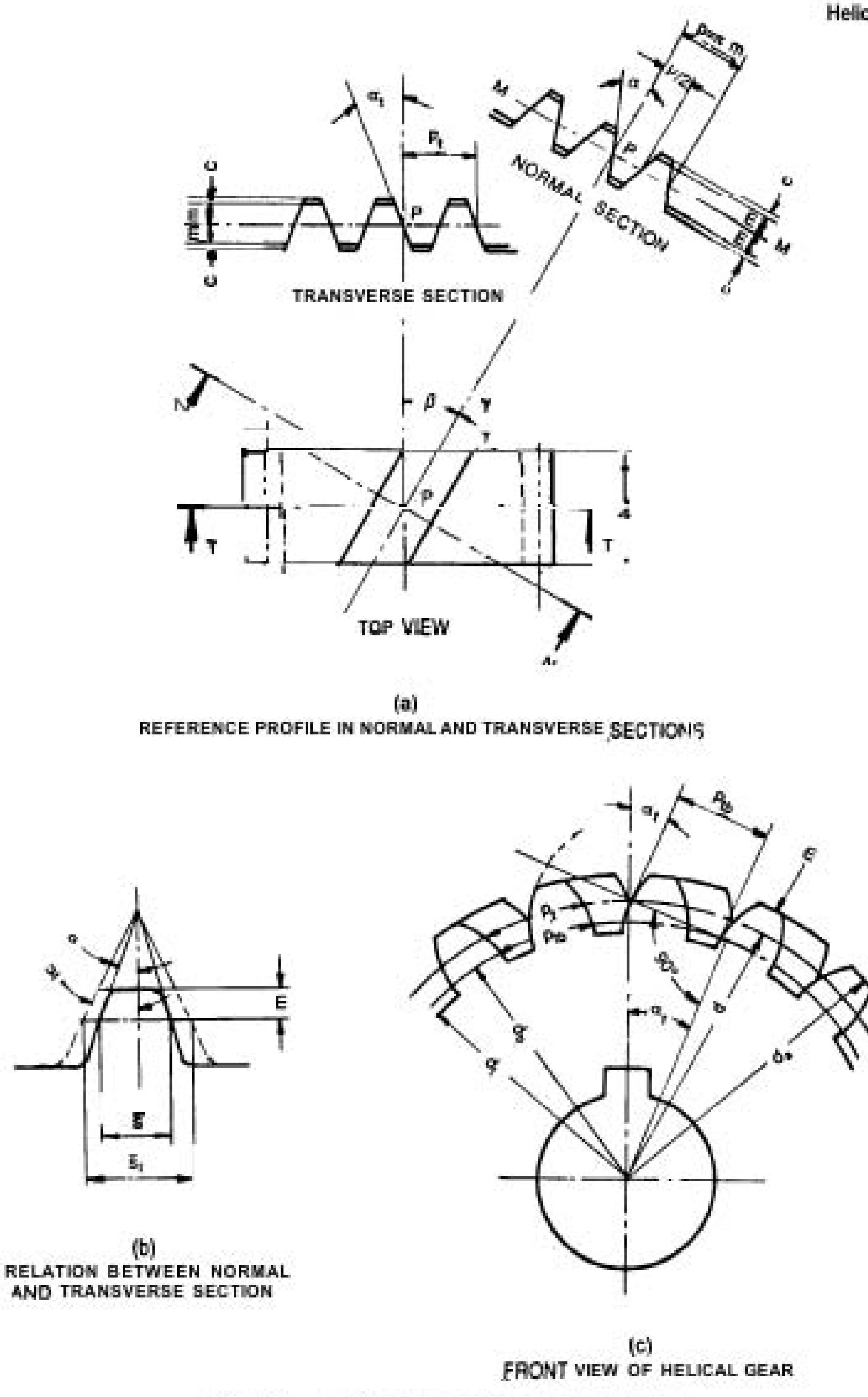


Fig. 3.3. Helical gear-views and sections

m_n = The normal module. (The normal module has sometimes been written as simply m, that is, without the subscript n in this book.)

Circular pitch Normal circular pitch $p (\text{or } p_n) = \pi m$ (3.4)

Transverse circular pitch $p_t = \pi m_t$ (3.5)

Axial pitch $$p_x = \pi m_x = \pi m_t \tan \gamma = \pi m_t \cot \beta = \frac{\pi m}{\sin \beta} = \frac{\pi m}{\cos \gamma} = \frac{H}{z} \quad (3.6)$$

where H = Lead, and z = Number of teeth

Base pitch Normal base pitch $p_{bn} = p_n \cos \alpha$ (3.7)

Transverse base pitch $p_{bt} = p_t \cos \alpha_t$ (3.8)

Pressure angle In case of a helical gear, it is important to differentiate between the normal pressure angle α (or α_n), which is the pressure angle in a plane perpendicular to the tooth trace (normal section), and the transverse pressure angle α_t, which is the pressure angle in a plane perpendicular to the gear axis (transverse section). The two pressure angles are related thus:

$$\tan \alpha = \tan \alpha_t \cos \beta \quad (3.9)$$

The normal pressure angle α is 20° for standard tooth profile. The above relation can be easily arrived at from Fig. 3.9 (Sec. 3.12), which gives a pictorial view of the system. Besides, like corrected spur gears, there are also working pressure angles α_w and α_{tw} which will be used in connection with corrected helical gears in Sec. 3.8.

Face advance It can be defined as the distance on the pitch circle through which a helical tooth (or a spiral tooth, in case of a spiral bevel gear) moves from the initial position at which the contact begins at one end of the tooth curve to the final position at the other end across the face width when the contact ceases.

The face advance of a helical gear is created due to the helical orientation of the course of the tooth along its length and this off-set is given by

$$FA = b \times \tan \beta \quad (3.10)$$

The above relation can be established from Fig. 2.3(c), where b = width of the gear. Obviously, the face advance of a spur gear is zero. In a parallel helical gear, width b is made larger with a view to making the face advance greater than the circular pitch corresponding to a given helix angle. This ensures continuous contact in the axial plane as the gears rotate. The ratio—face advance to circular pitch (in transverse section)—can be considered as a contact ratio (termed "face contact ratio"), and along with the regular contact ratio as explained in the case of spur gears in Sec. 2.7, the total contact ratio will consist of the sum of these two values. This is obviously greater than that of the spur gears. Contact ratio for helical gears is discussed in Sec. 3.6.

For the sake of safety, it is customary to increase the limiting value of the tooth-width by at least 15% (the limiting condition being FA = p), so that

$$b > \frac{1.15\, p}{\tan \beta}$$

Lead   It is the axial advance of the helix in one complete turn.

Pitch circle diameter   In a helical gear, because of the helix angle, the pitch circle diameter and other diameters are greater than those of the corresponding spur gear. It has been pointed out before that the transverse section only is circular, the normal one being elliptical. In Fig. 3.3(c) a frontal view of a helical gear is shown. This is the same as a transverse section. It is to be emphasised here that irrespective of the section, the pitch cylinder remain the same, whose diameter is given by

$$d = \frac{zm}{\cos\beta} = zm_t = zm\sec\beta \tag{3.11}$$

Tip circle diameter   In a spur gear, the tip circle is given by

$$d_a = zm + 2m$$

In a helical gear, it is given by

$$d_a = zm_t + 2m \tag{3.12}$$

Many readers may wonder why it is not given by $d_a = zm_t + 2m_t$.

The reason is quite apparent from Fig. 3.3(c). It a is circle only in the transverse section and, therefore, to reach the tip circle one has to add an amount equal to m on both sides of d radially from the centre, so that

$$d_a = d + 2m = \frac{zm}{\cos\beta} + 2m = zm_t + 2m$$

In Fig. 3.3(b), comparative, superimposed pictures of the basic rack tooth in the normal and transverse sections have been shown. Note the difference in tooth thicknesses in the two sections and note also that the height of the tooth remain the same. The two thickness are related thus

$$\bar{S} \text{ (or } \bar{S}_n) = \bar{S}_t \cos\beta \qquad \text{(see Fig. 2.3)} \tag{3.13}$$

From Fig. 3.3(b), we can arrive at the following relation

$$\frac{\bar{S}_t}{2} \times \frac{1}{\tan\alpha_t} = \frac{\bar{S}}{2} \times \frac{1}{\tan\alpha}$$

But $\bar{S} = \bar{S}_t \cos\beta$

$$\therefore \frac{\bar{S}_t}{2} \times \frac{1}{\tan\alpha_t} = \frac{\bar{S}_t\cos\beta}{2\tan\alpha}$$

or

$$\tan\alpha = \tan\alpha_t\cos\beta$$

The basic relations of a helical gear pair in mesh are given in Table 3.1. The gears are normal ones conforming to the standard basic rack.

The tooth thicknesses on any cylinder can be found in a similar way as explained in Sec. 2.3. Tooth thickness in transverse plane is given by

Table 3.1 Dimensions for a standard helical gear-set

Description	Pinion	Gear
Number of teeth	z_1	z_2
Pitch circle diameter	$d_1 = z_1 m/\cos\beta = z_1 m_t$	$d_2 = z_2 m/\cos\beta = z_2 m_t$
Tip circle diameter	$d_{a1} = d_1 + 2m$	$d_{a2} = d_2 + 2m$
Root circle diameter	$d_{f1} = d_1 - 2 \times 1.25\,m$	$d_{f2} = d_2 - 2 \times 1.25\,m$
Base circle diameter	$d_{b1} = d_1 \cos\alpha_t$	$d_{b2} = d_2 \cos\alpha_t$
Tooth thickness on pitch circle (measured in circular arc)	Normal section: $S_n = e_n = p_n/2 = \pi m/2$ Transverse section: $S_t = e_t = p_t/2 = \pi m_t/2$	
Centre distance	$a_0 = (d_1 + d_2)/2 = (m/\cos\beta) \times (z_1 + z_2)/2$	

$$S_{tc} = d_c(S_t/d + \text{inv}\,\alpha_t - \text{inv}\,\alpha_{tc}) \quad (3.14)$$

Here, subscript c denotes any cylinder and other subscripts have the usual connotations. The pressure angles are related to each other by the equation

$$\cos\alpha_{tc} = \cos\alpha_t \frac{d}{d_c} \quad (3.15)$$

In the normal plane, the tooth thickness is given by

$$S_{nc} = S_{tc} \cdot \cos\beta_c = S_{tc}\, d/\sqrt{d^2 + d_c^2 \tan^2\beta} \quad (3.16)$$

where β_c is the helix angle at the tip circle d_c of the cylinder.

In this connection, the reader is advised to refer to Eqs 2.7 and 2.8 given in Sec. 2.3.

3.3 Equivalent Spur Gear and Virtual Number of Teeth

While studying a helical gear, the concept of an equivalent spur gear is very useful in as much as it renders the helical gear into a spur gear from the calculation point of view, which in turn makes it possible to approximately calculate strength and other data of a helical gear as if it were a spur gear. This simplifies calculation procedures and the relevant spur gear formulae can be used (with modifications) as the helical gear is analogous to a spur gear.

If a section $N-N$ in Fig. 3.4 is cut through a plane which is normal to the tooth in a helical gear, the resultant sectional plane on the pitch cylinder will be a that of an ellipse of minor axis d and major axis $d/\cos\beta$, where d is the pitch diameter of the helical gear and β the helix angle. On the periphery of this ellipse, no two teeth profiles are quite alike. However, the shape of the tooth at P, i.e. at the top of the minor axis of the normal section, can be approximately taken to be that of a spur gear situated on the periphery of a circle whose radius R is equal to the radius of curvature of the ellipse at P. The centre of this circle is at 0.

The parameters of this ellipse are given by

$$\text{semi-major axis} = a = \frac{1}{2}\frac{d}{\cos\beta} = \frac{r}{\cos\beta}$$

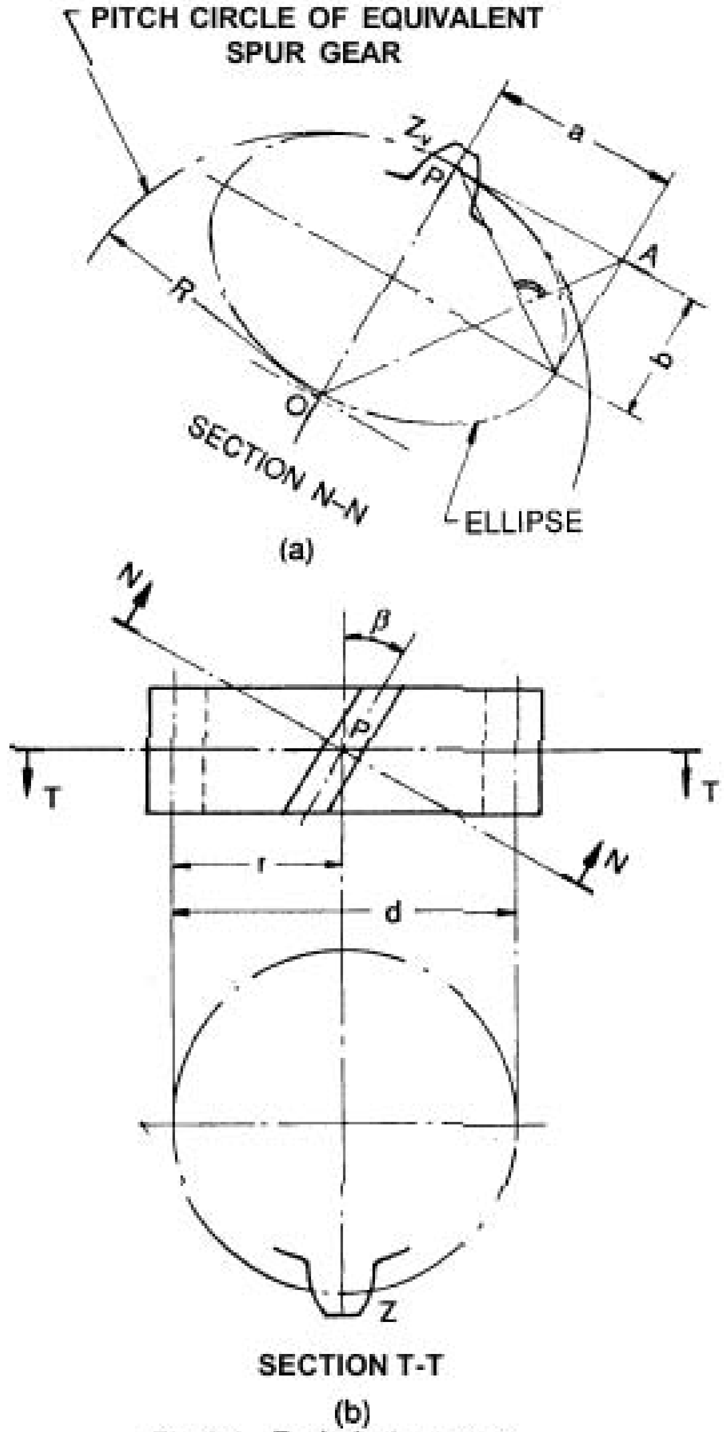


Fig. 3.4 Equivalent spur gear

$$\text{semi-minor axis} = b = d/2 = r$$

To arrive at an expression for R, we proceed as follows. A rectangle is drawn with sides a and b, and a diagonal is drawn as shown in Fig. 3.4(a). From point A of the rectangle, a perpendicular is dropped on this diagonal. This perpendicular is extended to meet the centre line coinciding with the minor axis at O. From similarity of triangles, we have

$$\frac{R}{a} = \frac{a}{b} \text{ or } R = \frac{a^2}{b} \text{ or } R = \frac{r^2}{\cos^2\beta} \times \frac{1}{r} = \frac{r}{\cos^2\beta} \text{ or } D = 2R = \frac{d}{\cos^2\beta}$$

A spur gear having a pitch radius of R is called an "equivalent spur gear" because, for all practical purposes, the tooth profiles on such a spur gear correspond to the tooth profile of the helical gear in question at P. Such a spur gear will have properties similar to those of the helical gear as designed on the normal section. The number of teeth of such a spur gear is given by

$$\text{Virtual number of teeth } (z_v) = \frac{\text{Circumference of the equivalent spur gear}}{\text{Circular pitch}}$$

$$= \frac{2\pi R}{\pi m} = \frac{2}{m} \times \frac{d}{2\cos^2\beta} = \frac{1}{m\cos^2\beta} \times \frac{m_t}{\cos\beta} = \frac{z}{\cos^2\beta\cos\beta}$$

Therefore

$$z_v = \frac{z}{\cos^3\beta} \tag{3.17}$$

Hence, the virtual number of teeth of a helical gear is the number of teeth which can be generated on the surface of a cylinder having a pitch radius equal to the radius of curvature of the point at the tip of the minor axis of an ellipse obtained by taking a section through the helical gear in a plane which is normal to the tooth at that point.

The above mathematical treatment is an approximation which is sufficiently accurate for all practical requirements. To obtain the exact value, the section is passed along the course of the helix, resulting in a helicoidal surface and not in a simple, two dimensional plane. The exact equation is

$$z_v = z/\cos^2\beta_b \cos\beta, \text{ where } \beta_b = \text{Helix angle at the base circle}$$

For cutting spur and helical gears in a workshop using form cutters, a set of 8 cutters valid for each pitch or module, is used. The details about this cutter series are given in a tabular form in Sec. 8.5.

To select the proper cutter number for cutting helical gears, the value of z_v—(and not z)—is to be inserted in the column marked: "Number of teeth of gear to be cut".

To find z_v, equation 3.17 is allowable only up to around $\beta = 20°$. For bigger helix angles, as in the case of crossed helical gears (see Sec. 3.15), Eq. 3.17 yields too inaccurate result which is not permissible. The accurate expression, which is normally used in workshops and for which ready-made charts are usually available to avoid time consuming calculations, is given below,

$$z_v = z\frac{\tan\alpha_t - \alpha_t\dfrac{\pi}{180°}}{\tan\alpha - \alpha\dfrac{\pi}{180°}} \quad (\alpha \text{ and } \alpha_t \text{ in degrees}).$$

$$= z \frac{\tan \alpha_t - \alpha_t}{\tan \alpha - \alpha} \qquad (\alpha \text{ and } \alpha_t \text{ in radian})$$

$$= z \frac{\text{inv } \alpha_t}{\text{inv } \alpha}$$

The normal pressure angle, α, which is usually 20°, is related to the transverse pressure angle, as per the Eq. 3.9. For involute functions, see Appendix H.

3.4 Characteristics of Helical Gears

Helical gears are analogous to a set of stepped gears which consist of a number of identical spur gears so arranged that the teeth of each individual member are slightly out of phase relative to each other. In such an arrangement, there is an overlap during successive engagement of teeth, that is, when two teeth are in mesh at the pitch line, other mating pairs of teeth are in different phases of contact including approach and recess contacts. A helical gear construction is approximated if a composite body is made up of an infinite number of such stepped gears, each of which is a lamination of infinitesimal thickness, placed side by side successively with a slight phase difference. This has been illustrated in Fig. 3.5.

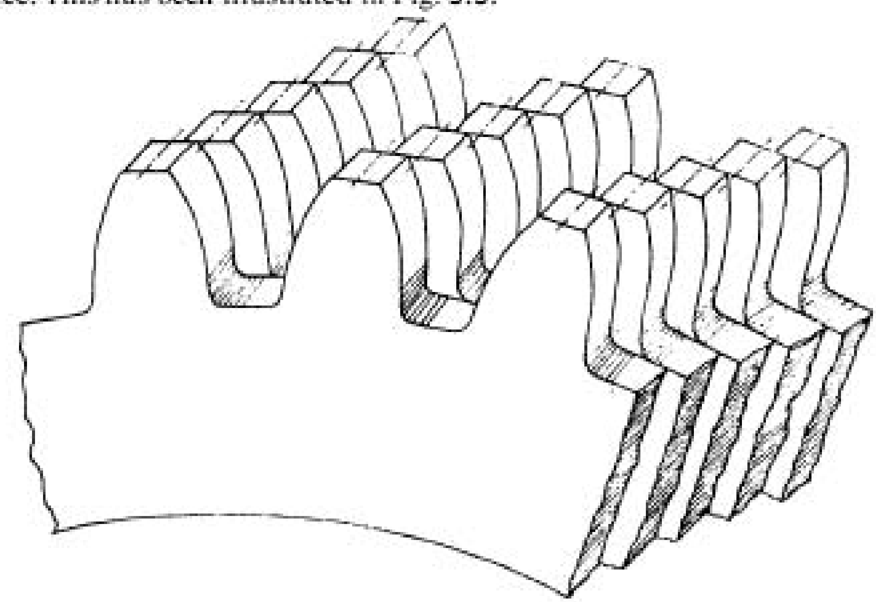


Fig. 3.5 Formation of helical gear

During load transmission in a helical gear pair in mesh, the leading end of the tooth comes in contact first and the trailing end last. Thus the tooth picks up load gradually. In contrast, during load transmission in spur gear, the entire face width (theoretically, at least) makes contact at the same instant, and the contact line between the two mating gears is parallel to the axes of the gear shafts. Thus, the contact takes place over the whole length of the tooth along the width simultaneously. In case of a helical gear pair, however, this contact line across the tooth's surface is diagonal, beginning from a point high on the face, i.e. near the tooth top, at one end to some point low on the flank, i.e. near the tooth root, at the other end of the tooth. The contact, therefore, progresses gradually along the whole range of the tooth width, covering the tooth face and flank. The result of such an engagement is smoother operation as compared to spur gears, greater load carrying capacity and practically noiseless running. The quieter operation of helical gears is, therefore, due to their favourable meshing conditions of the gear pair.

It is obvious that unlike a pair of mating spur gears, the load is never concentrated wholly at a particular position of a mating helical gear pair, as the contact starts at one end of a tooth and extends continuously across the tooth to the other end. Besides, other pairs of teeth are also in different phases of contact simultaneously during this period.

Hence, quality of manufacture, lubrication, type of loading, and other factor remaining the same, helical gears have a much smoother and quieter running and operational characteristics than comparable spur gears. The helical gears offer considerable advantages for high speed and heavy duty gear drives. They are, therefore, preferred for such drives.

A meshing helical gear pair has greater contact ratio than a corresponding spur gear pair. The frictional forces, which are generated due to the mutual sliding of teeth before and after the pitch point, are considerably reduced. Due to the greater total contact ratio and lesser magnitude of frictional forces, the detrimental effects are comparatively smaller in case of reversible drive and impact-type service.

It has been estimated that one consequence of the sloping contact line in a meshing pair of helical gears is that the maximum bending moment on a helical gear tooth is only a little greater than half that on the same size of spur gear tooth, assuming the load to be the same in each case. It follows, therefore, that from the strength point of view, a helical gear tooth has a greater load-transmitting capacity than a spur gear tooth for the same pitch and face width. Besides, in case of a mating pair of helical gears, there is always more than one pair of teeth in mesh at any instant. This is not always true in case of spur gears in mesh.

By proper selection of the helix angle, the common length of contact of teeth can be considerably increased. This leads to lesser specific loading on tooth surfaces which in turn facilitates retention of a load-carrying lubricant film.

The basic rack for the spur and the helical gear is the same. Both can therefore, be cut or generated by the same tool or cutter, except in certain special cases. For helical gear pair which have parallel axes, the magnitudes of the helix angles are the same. The angles have opposite hands in case of external gear drives and have the same hands in case of internal gear drives. For crossed helical gears, the relations are different, as can be seen in Sec. 3.15. As will be discussed later, a helical gear drive generates axial thrust forces. These can be taken care of by proper bearing mountings or by using a double-helical gear drive. Such a drive is described in Sec. 3.16. The axial thrust in case of a double-helical or herringbone gear drive is nullified by the in-built directionally-opposite hands of teeth of each of the components of the mating pair.

The hand of helix of a helical gear is defined as follows: If a helical gear is held in front of an observer in such a way that its circular face is in a vertical plane and is parallel to the observer with the gear axis in horizontal position and at right angles to the observer, then if the tooth twists from left towards right as it recedes away from the observer, i.e. it twists in a clockwise manner, such a tooth is said to have a right-handed helix. If the tooth twists away from right towards left when a gear is held in a similar manner, then it is left-handed helical gear (see Fig. 3.7).

3.5 Nature of Tooth Engagement in Helical Gear Drive

Unlike tooth engagement characteristics in a pair of meshing spur gears, the action is gradual and smooth in a helical gear drive. As pointed out in Sec. 2.5, contact in a pair of spur gears in mesh takes place along a line throughout the width of the teeth and this line is parallel to the axes of the gears. The contact begins suddenly across the whole tooth width and it ceases also abruptly.

In a helical gear drive, the contact begins at the tooth end and as the rotation progresses, the contact point moves along the whole tooth width till it reaches the other end. This results in a gradual, even tooth action and load distribution. Unlike spur gear drive, the contact line runs diagonal from one end to the other end of the helical teeth. Besides, in a helical gear drive, more than one pair of teeth are always in mesh. This, and other characteristics, like shorter lever arm, allow the helical drive to have considerably more load carrying capacity.

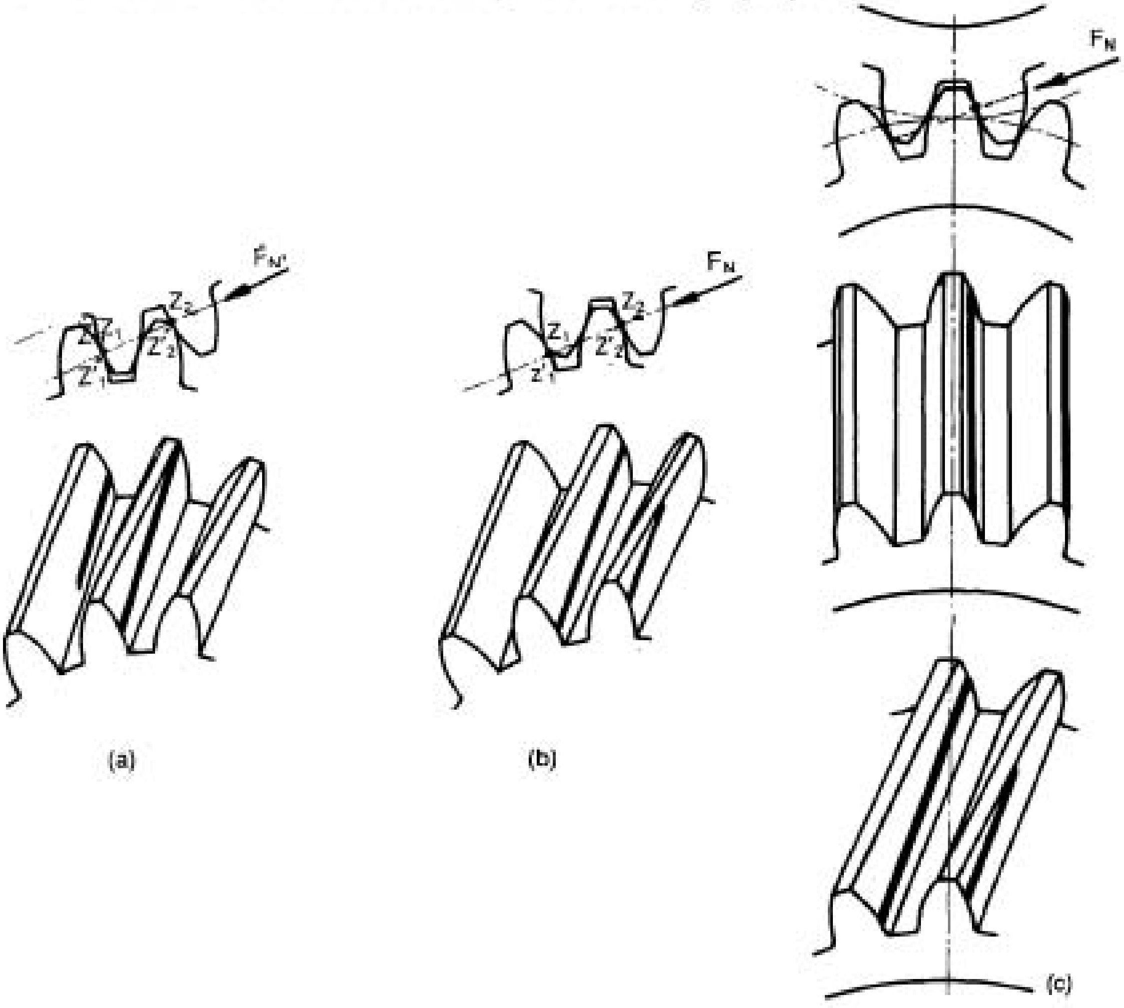


Fig. 3.6 Nature of tooth engagement in helical gear drive

In Fig. 3.6 (a) the initial contact conditions of a mating pair of helical gears have been shown. Figure 3.6(b) depicts the out-going pair $z_1 z'_1$ as it leaves the contact. The contact conditions of spur and helical gear drives have been illustrated in Fig. 3.6(c) for comparison.

3.6 Contact Ratio of Helical Gears

The contact ratio of spur gears has been discussed in Sec. 2.7. The derivation of the contact ratio and its implications have been dealt with in detail. The contact ratio of a pair of helical gears in mesh can be found in a similar way. It has been mentioned in Sec. 3.2 that due to the effect of the face advance in a helical gear, an extra amount of contact ratio is created. This face advance is due to the helical orientation of the tooth along the length of the tooth, covering the width of the gear. The contact ratio due to face advance is called the face contact ratio and is given by

$$CR_{FA} = \frac{\text{Face advance}}{\text{Transverse circular pitch}} - \frac{b \tan \beta}{p_t} \tag{3.18}$$

Now

$$p_t = \frac{\pi m}{\cos\beta} \qquad \therefore CR_{FA} = \frac{b \tan \beta}{\pi m} \cos\beta = \frac{b \sin \beta}{\pi m} \tag{3.19}$$

The face contact ratio is also known as the axial contact ratio and the overlap ratio. Due to this, the total contact ratio in case of a helical gear is greater than that of a spur gear.

The transverse contact ratio in case of a pair of helical gears in mesh can be found in a similar manner as in the case of spur gears. It is given by

$$CR_T = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a \sin \alpha_t}{p_{bt}} \tag{3.20}$$

For corrected helical gears, the corrected values of r_{a1}, r_{a2} and the centre distance a are to be inserted. Moreover, α_t will be replaced by the working pressure angle in the transverse section α_{tw}.

The total contact ratio (or, simply, the contact ratio) of the helical gearing is a summation of the above two contact ratios. Therefore

$$CR = CR_T + CR_{FA}$$

$$= \frac{b \tan \beta}{p_t} + \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a \sin \alpha_t}{p_{bt}} \tag{3.21}$$

Since the circular pitch and the base pitch in the transverse section are related by the expression

Transverse base pitch p_{bt} = Transverse circular pitch $p_t \times \cos \alpha_t$

Eq. 3.21 can be written as

$$CR = \frac{b \sin\beta \cos \alpha_t + \cos\beta\left[\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a \sin \alpha_t\right]}{\pi m \cos \alpha_t} \tag{3.22}$$

3.7 Backlash in Helical Gears

For the determination of backlash in drives comprising helical gears, the same considerations which are applicable in case of spur gears are valid. These have been detailed in Sec. 2.8.

The expressions for the two kinds of tolerances, namely, the normal backlash j_n and the torsional backlash j_t are given below

$$j_n(\min) = -(A_{SeU1} + A_{SeU2}) \cos a \cos\beta + 2A_{aL} \sin\alpha_{wn} \quad (3.23)$$

$$j_n(\max) = -(A_{SeL1} + A_{SeL2}) \cos a \cos\beta + 2A_{aU} \sin\alpha_{wn} \quad (3.24)$$

$$j_t(\min) = -(A_{SeU1} + A_{SeU2}) + 2A_{aL} \tan a_{wt} \quad (3.25)$$

$$j_t(\max) = -(A_{SeL1} + A_{SeL2}) + 2A_{aU} \tan\alpha_{wt} \quad (3.26)$$

Here, symbols α_{wn} and α_{wt} stand for the working pressure angles in the normal and the transverse planes respectively. As in the case of spur gears, all the tolerances are to be entered in the above formulae with the proper algebraic signs (+ or −) they carry.

As in the case of spur gears, the backlash values are to be read in conjunction with Secs 2.27 and 2.28.

3.8 Correction in Helical Gearing

The correction aspect of the gearing and its ramifications have been discussed in detail in Chap. 2. Like spur gears, helical gears are also corrected when needed. The same reasons for which the spur gears are corrected are valid for helical gears also. The minimum number of teeth to avoid undercutting in case of a helical gear is a function of the helix angle. This has been shown in Fig. 2.25(c). For helical gears with standard tooth profile, the minimum number of teeth is given by

$$z_{min} = \frac{2\cos\beta}{\sin^2 a_t} \quad (3.27)$$

By trigonometrical transposition and allowing a marginal amount of undercutting as before in the case of spur gears, we have

$$z_{min} = 14\cos^3\beta \quad (3.28)$$

To avoid undercutting through profile correction, the correction factor is given by

$$x = \frac{14 - z_v}{17} \quad (3.29)$$

where z_v = The virtual number of teeth as per Sec. 3.3.

As in the case of spur gears, the magnitude of the positive correction factor is limited by peaking [see Fig. 2.25(a) putting z_v instead of z in Fig. 2.25(a)]. Where large correction factor is involved, it is advisable to check the tooth thickness at the top land which should not be normally below 0.25m to 0.4 m. Referring to Sec. 2.13, the helical gearing can also be classified into two categories of corrected gearing systems —$_v$-gearing and S-gearing.

Table 3.2 Dimensions for S_0-gearing

Description	*Pinion*	*Gear*
Number of teeth	z_1	?
Pitch circle diameter	$d_1 = z_1 m / \cos\beta$	$d_2 = z_2 m / \cos\beta$
Tip circle diameter	$d_{a1} = d_1 + 2m(1 + x_1)$	$d_{a2} = d_2 + 2m(1 - x_1)$
Root circle diameter	$d_{f1} = d_1 - 2 \times 1.25m + 2x_1 m$	$d_{f2} = d_2 - 2 \times 1.25m - 2x_1 m$
Tooth thickness on pitch circle (normal section)	$S_{n1} = m\left(\frac{\pi}{2} + 2x_1 \tan\alpha\right)$	$S_{n2} = m\left(\frac{\pi}{2} - 2x_1 \tan\alpha\right)$
Centre distance	$a_0 = (d_1 + d_2)/2 = (m/\cos\beta)(z_1 + z_2)/2$	

Table 3.3 Dimensions for S gearing

Description	*Pinion*	*Gear*
Number of teeth	z_1	z_2
Pitch circle diameter	$d_1 = \frac{z_1 m}{\cos\beta}$	$d_2 = \frac{z_2 m}{\cos\beta}$
Tip circle diameter (with topping)	$d_{a1} = 2(a + m - x_2 m) - d_2$	$d_{a2} = d_2(a + m - x_1 m) - d_1$
Tip circle diameter (without topping)	$d_{a1} = d_1 + 2m + 2x_1 m$	$d_{a2} = d_2 + 2m + 2x_2 m$
Root circle diameter	$d_{f1} = d_1 - 2 \times 1.25m + 2x_1 m$	$d_{f2} = d_2 - 2 \times 1.25m + 2x_2 m$
Tooth thickness on pitch circle (normal section)	$S_{n1} = m\left(\frac{\pi}{2} + 2x_1 \tan\alpha\right)$	$S_{n2} = m\left(\frac{\pi}{2} + 2x_2 \tan\alpha\right)$
Tooth thickness on pitch circle (transverse section)	$S_{t1} = m_t\left(\frac{\pi}{2} + 2x_1 \tan\alpha\right)$	$S_{t2} = m_t\left(\frac{\pi}{2} + 2x_2 \tan\alpha\right)$
Topping	$ym = a_0 + (x_1 + x_2)m - a$	
Standard centre distance	$a_0 = \frac{d_1 + d_2}{2} = \frac{m}{\cos\beta}\,\frac{z_1 + z_2}{2}$	
Actual centre distance (after pushing)	$a = a_0 \frac{\cos\alpha_t}{\cos\alpha_{tw}} = \frac{d_{w1} + d_{w2}}{2}$	
Working pressure angle	$\text{inv}\,\alpha_{tw} = 2\frac{x_1 + x_2}{z_1 + z_2}\tan\alpha + \text{inv}\,\alpha_t$	
Top clearance	$c = a - \frac{d_{a1} + d_{f2}}{2} = a - \frac{d_{a2} + d_{f1}}{2}$	
Sum of profile correction factors	$x_1 + x_2 = (z_1 + z_2)\frac{\text{inv}\,\alpha_{tw} - \text{inv}\,\alpha_t}{2\tan\alpha}$	
Working circle diameter	$d_{w1} = d_1 \frac{\cos\alpha_t}{\cos\alpha_{tw}}$	$d_{w2} = d_2 \frac{\cos\alpha_t}{\cos\alpha_{tw}}$
Transverse pressure angle	$\tan\alpha_t = \tan\alpha/\cos\beta$	

Since the reasons for correction, characteristics of corrected gearing and other aspects are the same as in the case of spur gears which have been enumerated in Sec. 2.13, they are not repeated here. The relevant formulae will obviously differ to some extent due to the helix angle involved. In the Tables which follow, these formulae are summarised which are valid for helical gears comprising S_0-and S-gearing. For distribution of correction factors, see Sec. **2.14.**

3.9 Internal Helical Gears

The internal gear drives in case of spur gears have been discussed in Sec. **2.15.** The observations made in that section are also applicable to internal drives having helical gears. In an internal gear drive, both the pinion and the gear have the same amount of helix angle, and the helices are of the same hand. In Tables **3.4** and **3.5** the relevant relations between the different gear parameters are given for uncorrected and S_0-corrected gearings respectively. Table **3.6** gives these data for S_{plus}-corrected gears. In each case, the gears have standard 20° full-depth teeth, conforming to the basic rack given in Sec. **2.1.**

Table 3.4 Dimensions of internal helical gear drive for uncorrected gears

Description	Pinion	Internal gear
Number of teeth	z_1	z_2
Pitch circle diameter	$d_1 = \dfrac{z_1 m_n}{\cos\beta} = z_1 m_t$	$d_2 = \dfrac{z_2 m_n}{\cos\beta} = z_2 m_t$
Tip circle diameter	$d_{a1} = d_1 + 2m_n$	$d_{a2} = d_2 - 2m_n$
Root circle diameter	$d_{f1} = d_1 - 2 \times 1.25m$	$d_{f2} = d_2 + 2 \times 1.25\,m$
Centre distance	$a = \dfrac{d_2 - d_1}{2} = \dfrac{m_n}{\cos\beta}\,\dfrac{z_2 - z_1}{2}$	
Tooth thickness on pitch circle (normal section)	$s_{n1} = S_{n2} = \dfrac{p_n}{2} = \dfrac{\pi m_n}{2}$	

Table 3.5 Dimensions of internal helical gear drive for S_0-corrected gears

Description	Pinion	Internal gear
Pitch circle diameter	$d_1 = \dfrac{z_1 m_n}{\cos\beta} = z_1 m_t$	$d_2 = \dfrac{z_2 m_n}{\cos\beta} = z_2 m_t$
Tip circle diameter	$d_{a1} = d_1 + 2m_n + 2x_1 m_n$	$d_{a2} = d_2 - 2m_n + 2x_1 m_n$
Root circle diameter	$d_{f1} = d_1 - 2 \times 1.25m_n + 2x_1 m_n$	$d_{f2} = d_2 + 2 \times 1.25\,m_n + 2x_1 m_n$
Centre distance	$a = \dfrac{d_2 - d_1}{2} = \dfrac{m_n}{\cos\beta}\,\dfrac{z_2 - z_1}{2}$	
Tooth thickness on pitch circle (normal section)	$S_{n1} = m_n\left(\dfrac{\pi}{2} + 2x_1 \tan\alpha\right)$	$S_{n2} = m_n\left(\dfrac{\pi}{2} - 2x_2 \tan\alpha\right)$

Table 3.6 Dimensions of internal helical gear drive for S_{plus}-corrected gears

Description	*Pinion*	*Internal gear*
Pitch circle diameter	$d_1 = \frac{z_1 m_n}{\cos\beta} = z_1 m_t$	$d_2 = \frac{z_2 m_n}{\cos\beta} = z_2 m_t$
Tip circle diameter	$d_{a1} = d_2 - 2(a - m_n - x_2 m_n)$	$d_{a2} = d_1 + 2(a - m_n + x_1 m_n)$
Root circle diameter	$d_{f1} = d_1 - 2(1.25 - x_1)m_n$	$d_{f2} = d_2 + 2(1.25 + x_2)m_n$
Centre distance	$a = \frac{m_n}{\cos\beta}\frac{z_2 - z_1}{2}\frac{\cos\alpha_t}{\cos\alpha_w} = \frac{d_{w2} - d_{w1}}{2}$	
Transverse pressure angle	$\tan\alpha_t = \frac{\tan\alpha}{\cos\beta}$	
Working pressure angle	$\text{inv}\,\alpha_w = 2\frac{x_2 - x_1}{z_2 - z_1}\tan\alpha + \text{inv}\,\alpha_t$	
Working circle diameter	$d_{w1} = d_1\frac{\cos\alpha_t}{\cos\alpha_w}$	$d_{w2} = d_2\frac{\cos\alpha_t}{\cos\alpha_w}$
Tooth thickness on pitch circle (normal section)	$S_{n1} = m_n\left(\frac{\pi}{2} + 2x_1\tan\alpha\right)$	$S_{n2} = m_n\left(\frac{\pi}{2} - 2x_2\tan\alpha\right)$
Tooth thickness on pitch circle (transverse section)	$S_{t1} = m_t\left(\frac{\pi}{2} + 2x_1\tan\alpha\right)$	$S_{t2} = m_t\left(\frac{\pi}{2} - 2x_2\tan\alpha\right)$
Top clearance	$c = \frac{d_{f2} - d_{a1}}{2} - a = \frac{d_{a2} - d_{f1}}{2} - a$	

3.10 Design Criteria for Helical Gears

Because of simplicity of design and ease of manufacture, most designers prefer to use spur gears for transmitting power between parallel shafts. There are, however, operational requirements which warrant drive by helical gears. The advantages of helical gears over spur gears have been already elaborated in earlier sections of this chapter. Briefly, a helical gear drive can carry heavier loads, run at higher speeds and has smoother, quieter service.

In general, the guidelines given in Sec. 2.16 on spur gears are also valid for helical gears. Table 2.11 for minimum number of pinion teeth is also applicable here, but z_1 is to be replaced by the virtual number of teeth, z_v.

Table 2.12 showing λ values is also applicable for helical gears. Here, the minimum module denotes the model in the normal section, $m_{n\,min}$. The relevant relations are

$$m_{n\,min} = \frac{b}{\lambda} \tag{3.30}$$

$$b_{max} = \lambda\, m_n \tag{3.31}$$

Helix angle

The helix angle is obviously an important criterion of design of helical gears as the gear dimensions, centre distance of drive, thrust forces and other parameters depend on its magnitude and orientation. Hence, it should be judiciously chosen.

For normal application, the value of helix angle should lie in the range shown as follows

$$\text{Helix angle } (\beta) = 8^\circ \text{ to } 20^\circ \quad (3.32)$$

The helix angle should not cross 30° to avoid a large resultant axial thrust. On the lower side, it should not be below 8° as otherwise the advantages offered by helical gearing become marginal. It is desirable to have the helix angle as a whole number as it simplifies machine setting for cutting gears as well as finishing processes.

3.11 Thrust Characteristics of Helical Gears

When the tooth force acting on the surface of a helical gear is resolved, one of the components is the thrust which acts along the axis of the gear. A complete **force analysis for helical gears is dealt** with in Sec. 3.12. In this section, only the thrust component and the effects thereof are discussed.

Determination of the magnitude and the direction of the thrust forces is a fundamental criterion of helical gear design. The directions of helix, i.e. left hand or right hand, of the members

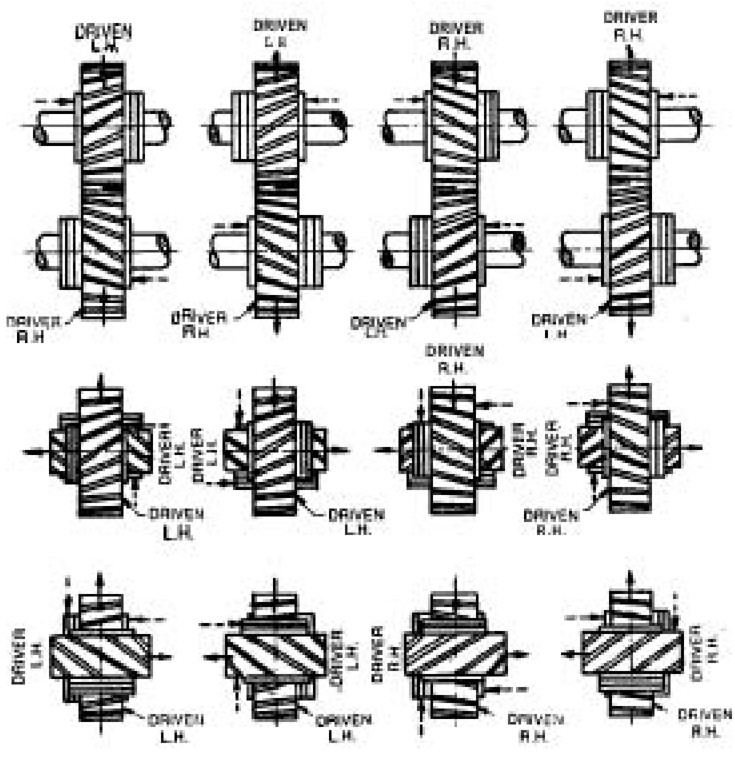


Fig. 3.7 Thrust diagrams for helical gears

Direction of thrust depends upon direction of rotation, relative position of driver and driven gear, and direction of helix.

Direction of rotation ⟶

Direction of thrust : - - - - - - - - - - - - - ⟶

Direction of helix : R.H. (right hand), L.H. (Left hand)

comprising the gear set will be fixed only after the direction of thrust has been determined and the position of the driver *vis-a-vis* the driven gear is known.

The direction of thrust in a helical gear drive is a function of several factors. These are: the direction of helix, the relative positions of the two components of the gearing, and the direction of rotation of the individual gears. If, for some reason, it is considered desirable to change the condition of thrust, it can be brought about by changing any of the three factors mentioned above, that is, the thrust can be made to be effective in the opposite direction if any one of these alterations are carried out. As far as the helix angle is concerned, both its magnitude and direction play a vital role in the determination of the axial force. Since this axial force or thrust is created due to the helical orientation of the teeth, thereby altering the direction of the main tooth force F_N, the helix angle should be chosen carefully. For single helical gears running on parallel shafts, it is prudent to confine the helix angle within 20° in order to avoid excessive end thrust. In exceptional cases, where the detrimental effects produced by the axial force is meticulously taken care of, the helix angle may go up to 30°. But for normal applications, it should not exceed 20°.

The proper selection of the bearings holding the shafts on which the helical gears are mounted will depend upon the amount of the axial thrust. The thrust diagrams for helical gears in normal applications are shown in Fig. 3.7. The direction of thrust can be conveniently determined from the figure.

After ascertaining the magnitude of the thrust force from force analysis and the direction from Fig. 3.7, the selection of the bearing which is appropriate for the purpose can be made after consulting catalogues and manuals of standard anti-friction bearing manufacturing companies.

3.12 Force Analysis for Helical Gears

Force analysis for helical gears can be made in similar manner as in the case of spur gears discussed in Sec. 2.18. Here, because of the helix angle, an additional force component is produced. This appears as an axial force with the resulting axial thrust on the bearings as explained in Sec. 3.11

In helical gears tooth force F_N acts normal to the tooth surface at an angle equal to the pressure angle α Fig. 3.8. This tooth force is resolved into three components which act at right angles to one another. The interrelations of these components can be easily established from Fig. 3.9 which shows a three dimensional representation of the force pattern. The magnitudes of these forces are given by

$$\text{Circumferential force } F_t = \frac{2000\,T}{d} \tag{3.33}$$

$$\text{Axial force } F_a = F_t \tan\beta \tag{3.34}$$

$$\text{Radial force } F_r = F_t \tan \alpha_t = \frac{F_t \tan \alpha}{\cos\beta} \tag{3.35}$$

where α and α_t are the pressure angles in the normal section and transverse section respectively, β is the helix angle, d is the pitch circle diameter in mm, and T is the driving torque in Nm. All the forces are expressed in newtons.

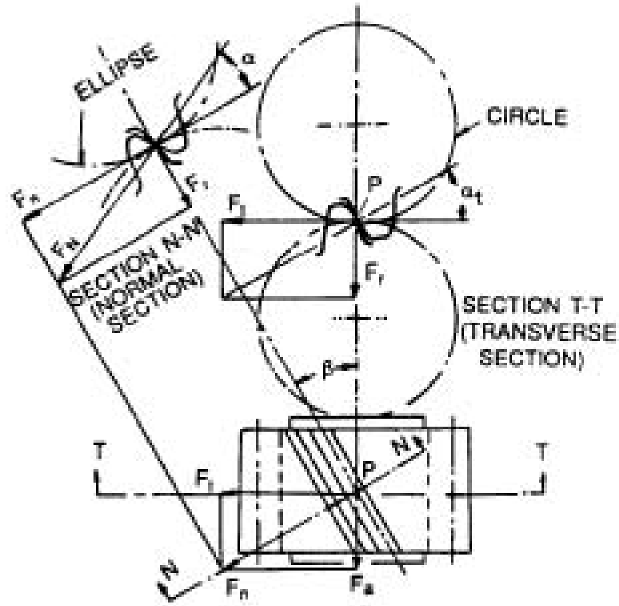


Fig. 3.8 Helical gear tooth forces

Based on Die Tragfaehigkeit der Zahnraeder, Thomas and Charchut, 7th Edition, 1971 Fig. no. 43, p. 100. Carl Hanser Verlag, Munich.

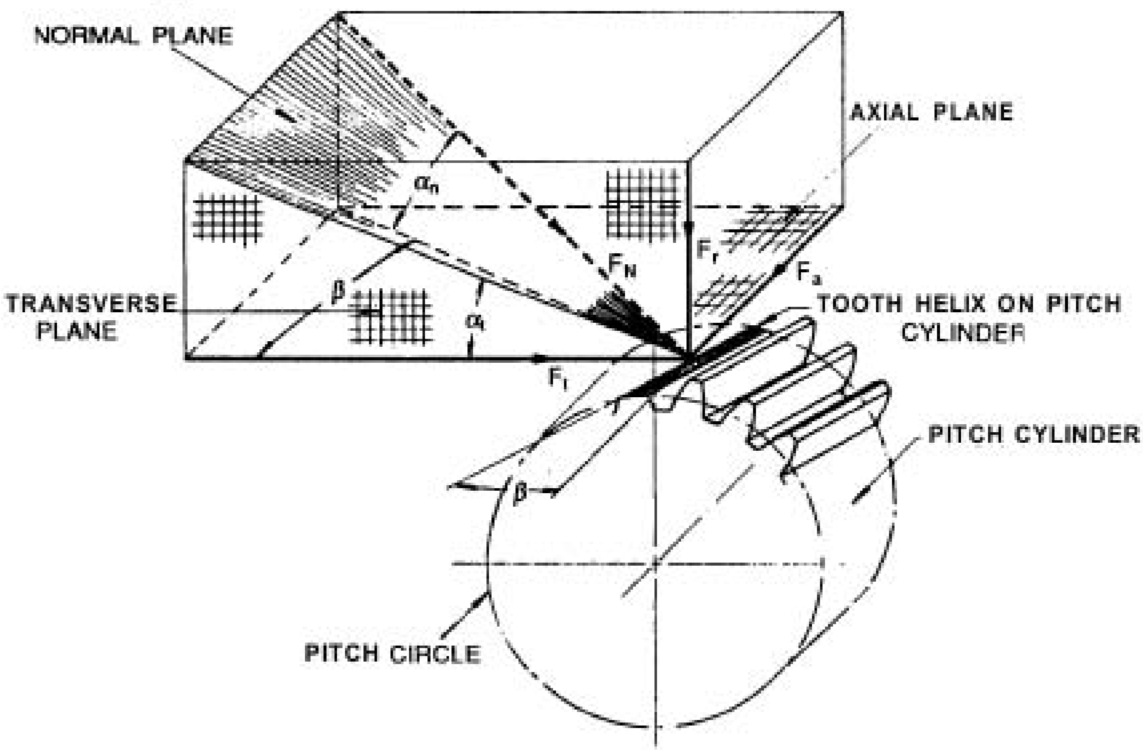


Fig. 3.9 Pictorial view of helical gear tooth forces

In cases where the end thrust of a helical gear set is objectionable for any particular reason or this axial force creates problems for the bearings, a double-helical or herringbone gear set is used. Such a gear is in effect a combination of two similar helical gears, having the same amount of helix angle but of opposite hands, placed side by side, cut on the same gear blank and mounted on the same shaft.

When two such composite units of herringbone gears, mounted on two shafts, mesh with each other, the end thrusts produced are counter-balanced so that the resulting axial force is zero. Usually for relief of cutting tool, a suitable groove is provided on the blank between the right-hand and left-hand helical halves, but the teeth can be made to be cut continuous, i.e. without the groove, by certain processes of gear cutting methods.

Since the limitations imposed by the end thrusts are eliminated in case of herringbone gears, these gears are often cut with high helix angles, namely, 30° or 45°.

Herringbone gears are treated in detail in Sec. 3.16.

3.13 Helical Gear Bearing Loads

The force on the tooth in spur gears is resolved into tangential and radial or separating components as explained in Sec. 2.18. In helical gears, apart from the above two forces, axial forces are also created due to the inclination of teeth as explained in Sec. 3.12. The axial forces on the teeth produce couples which in turn produce additional radial forces and a thrust load on each shaft carrying the mating gears. The radial forces due to these couples are equal in magnitude but opposite in direction in their effect of the two bearings on which each shaft is mounted [Fig. 3.10(a)].

For better understanding of the topic, it is recommended that the reader should once again go through Sec. 2.19 in which the spur gear bearing loads have been dealt with. The bearing loads on helical gears are calculated in the same manner as in the case of spur gears, except that in this case the additional radial forces referred to above are also taken into consideration while computing the resultant load on the bearings. The following forces act on the bearings.

Normal force F_N is resolved into the following components (Sec. 3.12):

$$\text{Tangential force } F_t$$

$$\text{Radial force } F_r = F_t \tan \alpha_t = \frac{F_t \tan \alpha_n}{\cos \beta}$$

$$\text{Axial force } F_a = F_t \tan \beta$$

The cases of gear mountings — straddle mounted and overhung — are discussed here [Fig. 3.10(b)]

Straddle Mounted Bearings

The resultant forces acting on bearing ***BI*** and ***BII*** are given by

$$F_{BI} = \sqrt{(F_{BIt})^2 + (F_{BIr} - F_{BIra})^2} \tag{3.36}$$

$$F_{BII} = \sqrt{(F_{BIIt})^2 + (F_{BIIr} + F_{BIIr_a})^2} \tag{3.37}$$

The respective magnitudes of the different force components are given by

$$F_{BIt} = \frac{F_t L_{II}}{L}, \qquad F_{BIr} = \frac{F_r L_{II}}{L}$$

$$F_{BIIt} = \frac{F_t L_I}{L} \qquad F_{BIIr} = \frac{F_r L_I}{L}$$

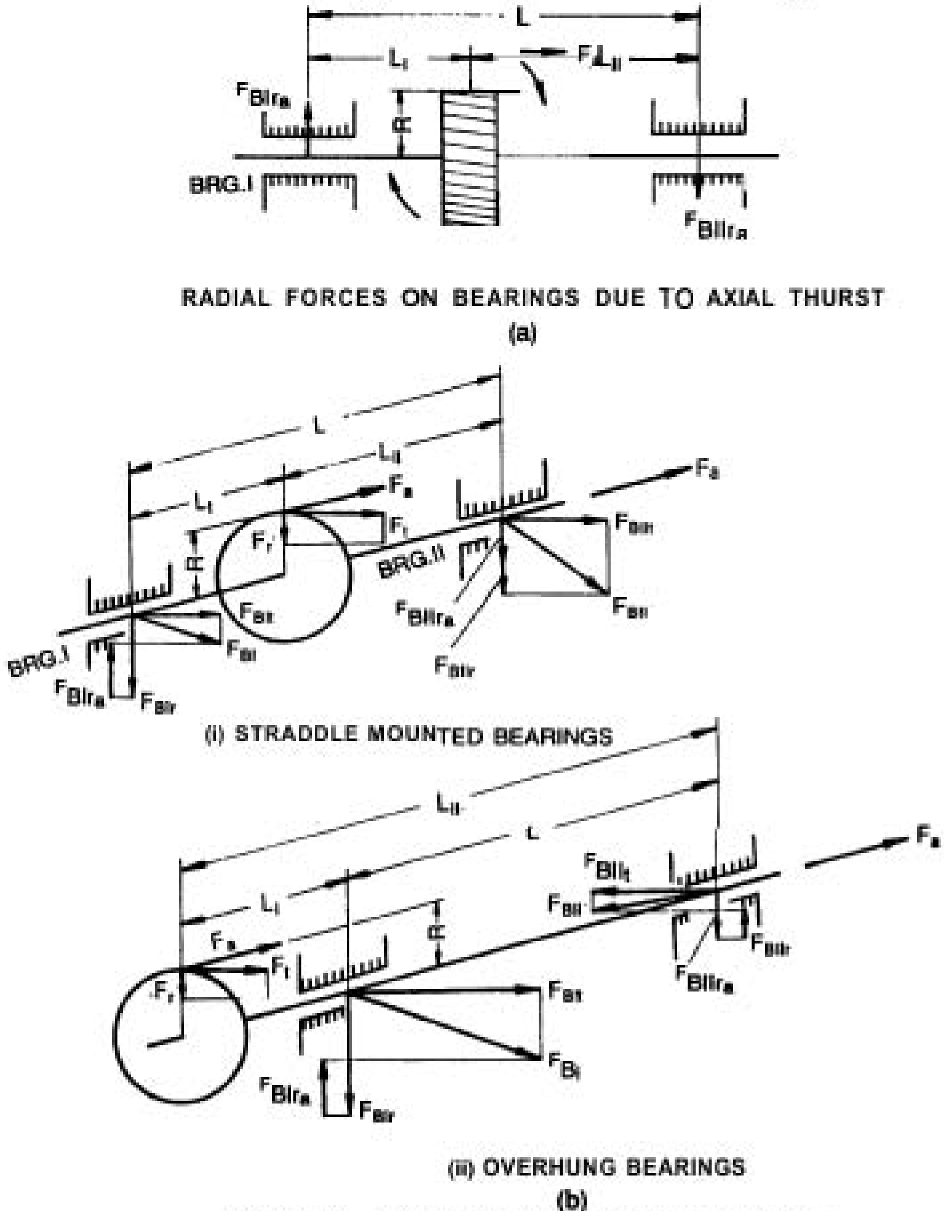


Fig. 3.10 Distribution of gear forces on bearings

Based on Die Tragfaehigkeit der Zahnraeder, Thomas and Charchut, 7th Edition, 1971, Fig. no. 44, p.101. Carl Hanser, Verlag, Munich.

And the additional radial components due to the couple are given by

$$F_{BIIr_a} = F_{BIr_a} = \frac{F_a R}{L} \text{ (in absolute value)}$$

The above value of the additional radial force is obtained by equalising the couples

$$F_a \times R = F_{BIr_a} L \qquad \text{or} = F_{BIIr_a} L$$

Overhung Bearings

Forces are the same as in the case of straddle mounted bearings except the following

$$F_{BII} = \sqrt{(F_{BIIt})^2 + (F_{BIIr_a} - F_{BIIr})^2}$$

As in the case of spur gears, the following relation as regards distances is applicable in case of helical gear system also

$$L_{(min)} = 2.5 \times \text{pitch diameter of gear} \geq 2.5\ L_1$$

Selection of Bearings

If the force analysis is known, proper anti-friction bearing can be selected after consulting the catalogues of standard bearing manufacturers. This is illustrated in the following example.

Example 3.1 In a helical gear drive, the pinion is straddle mounted in the middle of a shaft of 25 mm diameter. The following data are given: pitch circle diameter of pinion = 80 mm, power = 10 kW, speed = 750 rpm, helix angle = 9°. Select a suitable anti-friction bearing for a life of 20,000 hours. The bearings are to be 300 mm apart.

Solution:

$$\text{Velocity } v = \frac{d(\text{mm})\ n\ (\text{rpm})}{19100} = \frac{80 \times 750}{19100} = 3.14 \text{ m/s}$$

Tangential force

$$F_t = \frac{1000 \times \text{Power(kW)}}{v\ (\text{m/sec})} = 1000 \times 1013.14 = 3184\text{N}$$

Since the pinion is mounted in the middle, we have

$$L_I = L_{II} = L\,!2,\ F_{BIt} = F_{BIIt} = \frac{F_t L/2}{L} = 318412 = 1592\text{N}$$

Radial force

$$F_r = \frac{F_t \tan\alpha}{\cos\beta} = \frac{3184 \tan 20°}{\cos 90°} = 1173\text{N}$$

Axial force

$$F_a = 3184 \tan 9° = 504\text{N}$$

$$F_{BIr} = F_{BIIr} = \frac{F_r L/2}{L} = 1173/2 = 586\text{N}$$

$$F_{BIra} = F_{BIIra} = \frac{F_a R}{L} = \frac{504 \times 40}{300} = 67\text{N}$$

Using Eq. **3.37**, which gives the greater of the resulting forces on the two bearings, we have

$$F_{BII} = \sqrt{(F_{BIII})^2 + (F_{BII'} + F_{BII_a})^2} = \sqrt{1592^2 + (586 + 67)^2} = 1720\text{ N}$$

From the catalogues of standard bearing manufacturers and also from the relevant books, we have the following relation.

$$L_h = \frac{10^6}{60n}\left(\frac{C}{P}\right)^k \tag{3.38}$$

where

L_h = Life of bearing in operating hours
C = Basic dynamic load rating of bearing as given in the catalogues (N)
P = Equivalent dynamic load on the bearing as calculated from given data (**N**)
k = An exponent which is **3** for ball bearings
n = Speed (rpm)

The equivalent dynamic load (P) is defined as that hypothetical load which, if applied, will have the same effect on bearing life as the actual loads on the bearing when the bearing carries both radial and axial loads simultaneously. The general equation is

$$P = XF_r + YF_a \tag{3.39}$$

The symbols and the subscripts in the above equation are used universally. Here, F_r stands for the resultant of all the radial forces acting on the bearing (i.e. F_{BI} or F_{BII}), F_a is the axial force, and X and Y are factors to be taken from the bearing catalogues. Referring to an SKF catalogue, we tentatively select ball bearing No. **6205** for a shaft diameter of **25** mm. The following data are taken from the catalogue: C = basic dynamic load rating = 10,800N, and C_0 = basic static load rating = **6950** N. Factors X and Y will depend upon factor **e** which is a function of the quotient F_a/C_0. In this example, F_a/C_0 = 504/1720 = **0.07**. Corresponding to this, the value of **e** is **0.27.** Also, F_a/F_r = 504/1720 = **0.293.**

The values of X and Y will depend on whether the value of F_a/F_r is greater or lesser than the value of **e**. In this case

$$(F_a/F_r =)\ 0.293 > 0.27\ (= e)$$

From catalogue X = **0.56** and Y = **1.6**

$\therefore$ P = **0.56 x 1720 + 1.6 x 504 = 1770 N**. From **Eq. 3.38,** we have

$$L_h = \frac{10^6}{60 \times 750} \times \left(\frac{10800}{1770}\right)^3 = 5048 \text{ hours}$$

Since this is much less than the required life of **20,000** hours, ball bearing No. **6206 is** inadequate and is, therefore, rejected. We try the next higher size (No. **6305**) the relevant values of which are

C = **17300 N**, and C_0 = **11400 N** $\quad\therefore F_a/C_0$ = 504/11400 = **0.044**

The next higher value of F_a/C_0 as given in the catalogue is 0.07 which leads to a value **0.27** for **e**. Since F_a/F_r = 0.293 is greater than **e** in this case also, we have

X = **0.56** and Y = **1.6**

Therefore $P = 0.56 \times 1720 + 1.6 \times 504 = 1770\text{N}$

$$L_h = \frac{10^6}{60 \times 750} \times \left(\frac{17300}{1770}\right)^3 = 20749 \text{ hours}$$

Since this value is greater than the stipulated bearing life of *20,000* hours, ball bearing No. 6305 is finally selected.

Bending moments

For the determination of shaft dimensions and other relevant design criteria it is necessary to know the pattern of the bending mo'ment and also its maximum values. We will discuss here the case of one gear straddle-mounted on a shaft which meshes with another gear mounted on another shaft.

As the forces are effective in different planes, it is convenient to find first the bending moments separately in two mutually perpendicular planes. As shown in Fig. 3.11, forces F_a and F_r are effective in $x-z$ plane, while in $y-z$ plane, only the tangential force F_t is effective. The individual bending moments are then added vectorially to determine the resultant bending moment. The two maximum moments are given by

$$B_{max\,1} = F_{BI} L_I$$

$$B_{max\,2} = F_{BII} L_{II}$$

The greater of the two is used for the determination of shaft diameter. Besides bending moment, the shaft is subjected to the axial thrust (F_a) and the torsional moment or torque T. For shaft calculation, the effect of the axial thrust can be neglected, and the calculation can be carried out on the basis of bending and torsion only. Shaft calculations have been dealt with in Sec. 2.21.

3.14 Strength Calculation for Helical Gears

While describing the nature of tooth engagement in case of spur and helical gears, it has been pointed out that at any stage of tooth action there are in general more teeth in engagement in case of helical gears than in case of spur gears, other factors remaining the same for both the types. The load carrying capacity of a helical gear pair, therefore, is greater than that of a spur gear drive.

In Sec. 2.25 the methods of strength calculation for spur gears have been discussed in detail. The same considerations and the equations — with the relevant modification to take care of the helical aspect — an be used for helical gears. We can write the following equations as applied to the helical gears

$$\sigma_b = \frac{F_t}{bm_n} q_k\, q_e \leq \sigma_{bp} (\text{N/mm}^2) \quad (3.40)$$

$$F_t = \frac{2000\ T_1\ (\text{Nm})}{d_1\ (\text{mm})} = 2000 \times 9550 \times \frac{P_1\ (\text{kW})}{n_1\ (\text{rpm})} \times \frac{\cos\beta}{z_1 m_n}\ (\text{N}) \quad (3.41$$

Factor q_k can be read off from Fig. 2.49 corresponding to z_v instead of z as in spur gears. For initial calculation of module, we assume the following average values

$q_{k1} = 2.2$ and $q_\varepsilon = 0.9$

$$m_n = \sqrt[3]{\frac{4000\, T_1 \cos^2\beta}{z_1^2 \left(\dfrac{b}{d_1}\right) \sigma_{bp}}} \tag{3.42}$$

Factor q_ε takes care of the unequal load distribution on helical teeth during the course of action. However, the same remarks about the load calculation of spur gears are valid here **also**, and we can take value of q_ε as 1 to amve at **a** safe, though conservative, result. We get similar expressions for contact stress as in the case of spur gears. Thus

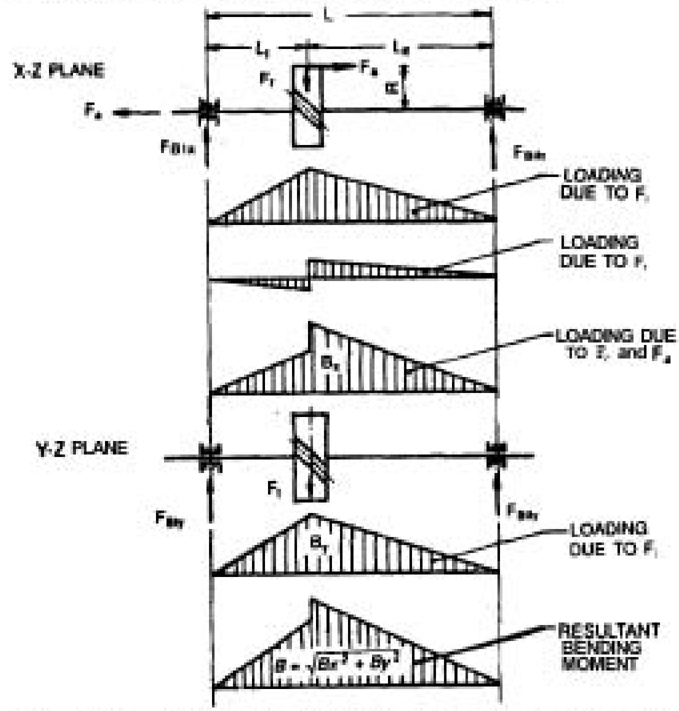


Fig. 3.11 Bending moment diagram of helical gear drive

$$p_{cp} = y_m\, y_p\, y_L \sqrt{\frac{2000\, T_1}{\left(\dfrac{b}{d_1}\right) d_1^3}\, \frac{u + 1}{\mu}} \tag{3.43}$$

$$m_n = \frac{\cos\beta}{z_1} \sqrt[3]{\frac{2000 T_1\, (y_m y_p y_L)^2}{\left(\dfrac{b}{d_1}\right) p_{cp}^2}\, \frac{u + 1}{u}} \tag{3.44}$$

Factor y_L is the tooth length factor and it **takes** care of the different magnitude of load on the teeth during action. Its value can be taken as 1 for all practical purposes. The allowable **stresses** are determined **as** in the case of **spur** gears. The pitch point factor is

$$y_p = \sqrt{\frac{\cos \beta_b}{\cos^2 \alpha_t \tan \alpha_{wt}}} \tag{3.45}$$

The value of y_p can be taken as 1.6 for initial calculations. The expression for the base helix angle (β_b) can be easily established as

$$\tan \beta_b = \tan \beta \cos \alpha_t \tag{3.46}$$

3.15 Crossed Helical Gears

Crossed helical gears belong to that category of helical gears where the respective shafts on which they are mounted are not parallel and their axes do not intersect. Both members of the gear pair are involute helicoids cut on cylindrical blanks. These gears are sometimes referred to as "spiral gears", but to avoid confusing these with spiral bevel gears, the term "crossed helical gears" will be used in this book.

Essentially, these are non-enveloping worm gears (see Fig. 1.9). They connect skew shafts. If the shaft angle is Σ and the helix angles of the two component gears are β_1 and β_2, the basic relationship which holds good is given by

$$\Sigma = \beta_1 \pm \beta_2 \tag{3.47}$$

Here, the convention regarding the algebraic sign which should prefix a helix angle should be maintained—a right-hand helix angle is regarded as positive, and a left-hand helix angle as negative. In normal applications, the shaft angle Σ is 90°. The magnitudes of the helix angles of the mating components need not be the same. Till assembled in mesh, there is no difference between a crossed helical and an ordinary helical gear, the manufacturing processes also being the same.

While in a pair of conventional, ordinary helical gear pair, the helix angles are equal in magnitude and opposite in sense, this need not be so in case of crossed helical gears. The hands of a mating crossed helical gears are usually the same, but by manipulating the shaft angle properly, gears of opposite hands can be meshed together. There are, therefore, four possible combinations

1. RH driver meshing with RH driven
2. RH driver meshing with LH driven
3. LH driver meshing with LH driven
4. LH driver meshing with RH driven

When in mesh, the crossed helical gear teeth have point contact between the mating pair. This develops into a line contact due to wear after some time. This is the reason why this type of gear pair is meant for only small loads, for example in instrumentation, distributor drive of automobile engines and other similar applications. They are not recommended for power transmission. However, with increasing wear, the line contact becomes a band contact, thereby enhancing the load carrying capacity appreciably. In a crossed helical gear system, the reduction ratio (i) is normally limited to a maximum value of 5 : 1. For higher reduction ratios involving systems having similar orientation of axes, worm and worm-wheel reduction units are used.

The bearings which support the shafts on which the crossed helical gears are mounted have to withstand both thrust and radial loads. The pattern of these thrust loads has been discussed in Sec. 3.11. It is relevant to reiterate here that the direction of the thrust produced depends on the direction of helix, the relative positions of the gears, and the direction of rotation.

One of the main advantages of a crossed helical gear system is its reduced sensitivity towards changes in centre distance, shaft angle and alignment, and axial position. With ample face width and backlash, these gear-sets can withstand small changes in centre distance and shaft angle without any deterimental effect in the overall accuracy of transmitted motion. **Also** either member can take endwise shifting, thus facilitating easy mounting. Close accuracies in centre distance and shaft alignment are, therefore, not imperative.

In short, mounting ease, insensitivity to axial movement of either member, invulnerability to small changes in shaft angle **or** centre distance, and low cost are the main advantages of a crossed helical gear drive.

Since unlike ordinary, parallel axes helical gears, the crossed helical gear pairs need not have equal helix angles for individual members, the pitch diameters of such gears are not in proportion to their tooth ratio. It may so happen that the gear having the larger number of teeth may be the smaller one in diameter. Thus, the terms "pinion" and "gear" become arbitrary. Some authors prefer to ascribe "pinion" as that member which has the smaller number of teeth. Others prefer to describe them as "driver" and "driven".

Since different helix angles are used for the two gears, the transverse module (m_t) is not the same. Hence, in specifying the size, the normal module (m_n) is always used. Recalling Eq. 3.11, the pitch or reference diameter of a helical gear is given by

$$d = \frac{m_n z}{\cos\beta} = m_n z \sec\beta$$

For crossed helical gears, these diameters are given by

$$d_1 = \frac{m_n z_1}{\cos\beta_1} = m_{t1} z_1 \tag{3.48}$$

$$d_2 = \frac{m_n z_2}{\cos\beta_2} = m_{t2} z_2 \tag{3.49}$$

Adding $2\,m_n$ to d_1 and d_2 gives the respective outside diameters as in the case of standard helical gears.

By assigning appropriate values, it is quite easy to observe from the above equations that the gear having the larger number of teeth may be the smaller one in diameter of the pair in certain cases. Also, the angular velocity ratio can be obtained only by using the ratio of the tooth numbers and not by using the ratio of the pitch diameters as these are not directly proportional to the number of teeth.

For crossed helical gears, the gear tooth proportions have not been standardised. The following guidelines can be given for the designer.

1. A contact ratio in the normal section of at least 2 gives the best results.
2. The designer should aim at having equal helix angles because this results in minimum sliding velocity.
3. With unequal helix angles having the same hand of helix, the gear with the larger angle should be used as the driver.
4. In order to obtain high contact ratio, a low normal pressure angle should be used. Also, deep tooth depth should be used.

The standard centre distance is given by

$$a = \frac{m_n}{2}\left(\frac{z_1}{\cos\beta_1} + \frac{z_2}{\cos\beta_2}\right) \tag{3.50}$$

Also, the reduction ratio is given by

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2\cos\beta_2}{m_n} \Big/ \frac{d_1\cos\beta_1}{m_n} = \frac{d_2\cos\beta_2}{d_1\cos\beta_1} \tag{3.51}$$

Putting the diameter ratio $D_R = d_2/d_1$, we have

$$a = \frac{m_n}{2}\left(\frac{1}{\cos\beta_1} + \frac{i}{\cos\beta_2}\right)z_1 = \frac{z_1 m_n}{2}(\sec\beta_1 + i\sec\beta_2) \tag{3.52}$$

The following relation can be arrived at by trigonometrical transposition

$$\tan\beta_1 = \left(\frac{i}{D_R} - \cos\Sigma\right)\frac{1}{\sin\Sigma} \tag{3.53}$$

When the shaft angle $\Sigma = 90°$ then $\tan\beta_1 = i/D_R$

To find the condition of minimum centre distance when the shaft angle is 90°, we proceed as follows

Since $\beta_2 = 90° - \beta_1$, $\sec\beta_2 = \sec(90° - \beta_1) = \operatorname{cosec}\beta_1$

By inserting in Eq. 3.52, we have

$$a = \frac{z_1 m_n}{2}(\sec\beta_1 + i\operatorname{cosec}\beta_1) \tag{3.54}$$

Differentiating the centre distance a with respect to β_1, we have

$$\frac{da}{d\beta_1} = \frac{z_1 m_n}{2}(\sec\beta_1\tan\beta_1 - i\operatorname{cosec}\beta_1\cot\beta_1)$$

Setting $da/d\beta_1 = 0$, we get the conditions for the minimum value of a. Thus

$$\sec\beta_1\tan\beta_1 = i\operatorname{cosec}\beta_1\cot\beta_1, \text{ or } \sin^3\beta_1/\cos^3\beta_1 = i \qquad \therefore \tan\beta = i^{1/3} \tag{3.55}$$

Also,

$$D_R = i/\tan\beta_1 = i/i^{1/3} = i^{2/3} \tag{3.56}$$

Inserting the relevant values in Eq. 3.54, we get the minimum value of the centre distance

$$a_{min} = \frac{z_1 m_n}{2}\left[1 + i^{2/3}\right]^{3/2} \tag{3.57}$$

In case of non-parallel shafts connected by gears, the crossed helical gear system affords the possibility of altering the space requirements (if such need arises) without changing the centre distance, module, gear ratio or shaft angle as can be seen from Example 3.2. If necessary, it is also possible to calculate the relevant gear data for conversion to a different gear ratio from the original one, while keeping the centre distance constant.

Example *3.2: Given:* shaft angle = 90°, gear ratio = 1 : 1, $\beta_1 = \beta_2 = 45°$, $m_n = 2.5$,

$$z_1 = z_2 = z = 60$$

The above identical gears are to be so changed that the driven gear (No. 2) has a pitch diameter of around 150 mm in the new arrangement.

Solution: $d_1 = m_n z_1/\cos\beta_1 = m_n z_1/\sin\beta_2, d_2 = m_n z_2/\cos\beta_2$

$$a = (d_1+d_2)/2 = \frac{1}{2} \times 2d = d = 2.5\times60/\cos 45^\circ = 212.132 \text{ mm}$$

$$a = \frac{1}{2}(d_1+d_2) = \frac{1}{2}\left(\frac{m_n z_1}{\sin\beta_2} + \frac{m_n z_2}{\cos\beta_2}\right) = \frac{m_n z}{2}\left(\frac{\sin\beta_2+\cos\beta_2}{\sin\beta_2\cos\beta_2}\right)$$

Also

$$z = \frac{d_2\cos\beta_2}{m_n}$$

Therefore

$$a = \frac{m_n}{2}\times\frac{d_2\cos\beta_2}{m_n}\times\left(\frac{\sin\beta_2+\cos\beta_2}{\sin\beta_2\cos\beta_2}\right) = \frac{d_2}{2}(1+\cot\beta_2)$$

or

$$\cot\beta_2 = \frac{2a}{d_2} - 1 = \frac{2\times 212.132}{150} - 1 \qquad \text{whence } \beta_2 = 28.675''$$

Therefore

$$z_2 = \frac{d_2\cos\beta_2}{m_n} = \frac{150\times\cos 28.675'}{2.5} = 52.64 = z$$

Taking an integral value of z as 52, and substituting

$$\frac{2a}{m_n z} = \frac{\sin\beta_2 + \cos\beta_2}{\sin\beta_2\cos\beta_2}$$

or

$$\frac{a}{m_n z} = \frac{212.132}{2.5\times52} = \frac{\sin\beta_2+\cos\beta_2}{2\sin\beta_2\cos\beta_2}$$

Squaring

$$2.662721 = \frac{1+\sin 2\beta_2}{\sin^2 2\beta_2}$$

This is a quadratic equation, yielding $\beta_2 = 27^\circ\ 59'\ 2.8''$.

Final values : $d_1 = 2.5\times 52/\sin 27^\circ\ 59'\ 2.8'' = 277.052$ mm

$d_2 = 2.5\times 52/\cos 27^\circ\ 59'\ 2.8'' = 147.212$ mm (which is around 150 mm)

$a = (d_1+d_2)/2 = (277.052+147.212)/2 = 212.132$ mm

This tallies with the original value of the centre distance.

It has been stated before that the crossed helical gears have point contact while in mesh. Also, the load carrying capacity is inconsequential. Because of point contact, even light tooth forces generate very high compressive stresses. Moreover, while the sliding of teeth of gear pairs previously discussed is up and down along the tooth profile, in case of crossed helical gears the teeth slide also across one another. This cross-sliding phenomenon gives rise to more friction and the low efficiency of these gears is attributable to frictional loss as one of the contributing factors.

To calculate the load carrying capacity of these gears, wear and heat developed are the deciding design criteria, beam strength calculation being superfluous. Insufficient information is available in this area and that too is not very reliable. As such, only some broad guidelines can be given in this regard.

As regards materials, bronze, non-metallic materials or hardened steel should be used for one of the members comprising the pair when the sliding velocity is in excess of 5 m/s. If we take the ratio $b/m_n = 10$, the following approximate formula, based on contact stress and wear, gives reasonably acceptable results

$$F_{tl} \leq 10 m_n^2 \pi K \tag{3.58}$$

where

F_{tl} = Tangential force of the driver (N)
K = A factor to be taken from Table 3.7 (N/mm²)

Table 3.7 Factors K and C_T for crossed helical gears

Material pair	K (N/mm²)	V_S (m/s)	C_T
CI/CI CI/Steel CI/Bronze	$6/(2+V_s)$	0 to 5	7
Steel/Bronze	$10/(2+V_s)$	5 to 10	4
Steel/Steel (hardened)	$20/(2+V_s)$	≈ 8 to 10	2

The relative sliding velocity v_s is given by

$$v_s = v_1 \sin \Sigma / \cos \beta_2 = v_2 \sin \Sigma / \cos \beta_1 = v_t \sin \beta_1 + v_2 \sin \beta_2 \tag{3.59}$$

The circumferential velocity v_t is given by

$$v_1 = d_1 n_1 / 19100 \text{ m/s, where } d_1 \text{ is in mm and } n_1 \text{ is in rpm}$$

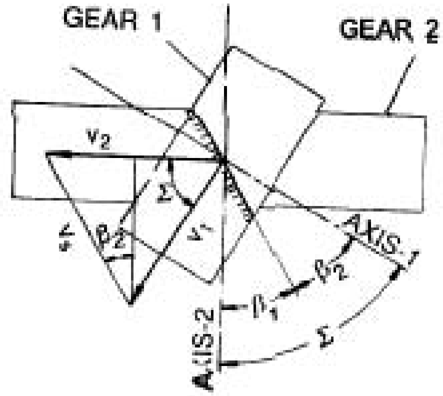
When the pinion is driving the gear, the power loss due to sliding in terms of pinion power P_1 is given by

$$P_s = P_1 \frac{\mu(\tan \beta_1 + \tan \beta_2)}{1 + \mu \tan \beta_1} \tag{3.60}$$

The coefficient of sliding friction μ is a function of the material and the sliding velocity. With good lubrication, its value can be taken as 0.1. For the orientation of the sliding velocity v_s, see the velocity diagram in Fig. 3.12 (a).

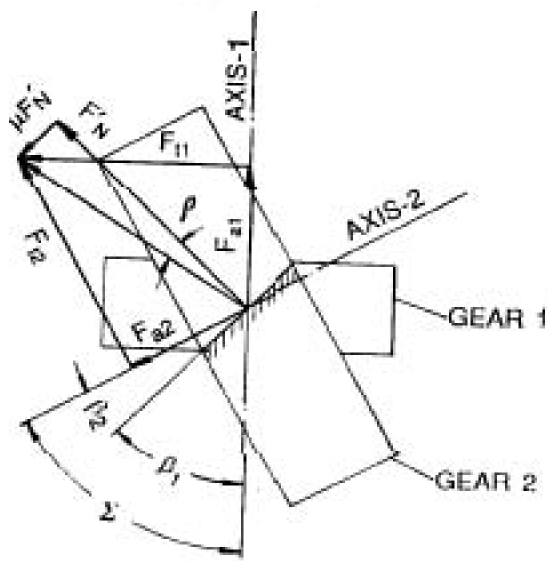
In a crossed helical gear system, the lengthwise cross-sliding of teeth which has been referred to before, has more pronounced effect than the other kind of sliding. For the determination of frictional power loss, therefore, calculations are based on the cross-sliding phenomenon, neglecting the other kind of sliding whose magnitude is comparatively small. The efficiency of toothing is given by

$$\eta = \frac{P_1 - P_s}{P_1} = 1 - \frac{P_s}{P_1} = 1 - \frac{\mu(\tan \beta_1 + \tan \beta_2)}{1 + \mu \tan \beta_1} = \frac{1 - \mu \tan \beta_2}{1 + \mu \tan \beta_1} \tag{3.61}$$



(a)

VELOCITY DIAGRAM



(b)

FORCE **DIAGRAM**

Fig. 3.12 Velocity and force diagrams of crossed-helicaldrive

Here, only the loss of power due to sliding has been considered to arrive at the above expression for efficiency. Besides this, there are other losses, e.g. loss due to toothing action which depends on i, z and m_a, bearing losses and losses due to allied factors. Efficiency of bearing can be generally taken **as 0.9.** The total efficiency is found by multiplying all the individual efficiencies.

From **Eq. 3.61** and differentiating, we get for the maximum (theoretical) efficiency,

$$\tan(\beta_1 - \beta_2) = \mu$$

after setting $d\eta/d\beta_1 = 0$

Putting $\mu = \tan\rho$, where ρ = Angle of friction, we have

$$\tan(\beta_1 - \beta_2) = \tan \rho$$

or

$$\beta_1 - \beta_2 = \tan \rho \tag{3.62}$$

or $\beta_1 - \left(\sum - \beta\right) = \rho$ whence $\beta_1 \left(\sum + \rho\right)/2$

Note that $\beta_1 > \beta_2$. If β_2 exceeds a certain limiting value, the drive from gear No. 1 cannot take place. This value can be derived by putting $\eta = 0$ in Eq. 3.61. We have

$$\eta = \frac{1 - \mu \tan\beta_2}{1 + \mu \tan\beta_1} = 0$$

whence

$$\tan\beta_2 = \frac{1}{\mu} = \frac{1}{\tan\rho} = \cot\rho = \tan(90° - \rho)$$

This shows that transmission of motion from shaft 1 to shaft 2 is theoretically possible only when

$$\beta_2 < (90° - \rho)$$

The practical limiting value of β_2 is still less. When the shaft angle is 90°

$$\eta = \frac{\tan(\beta_1 - \rho)}{\tan\beta_1} \tag{3.63}$$

For high and practically unaltered efficiency during the drive, it is desirable that the helix angle of the driving gear β_1 should lie between 30° and 60°.

In a crossed helical gear drive, the output is limited by the contact pressure and the heat developed. For a reasonable life, the material selected should be wear resistant and an adequate supply of lubricant should be ensured. Allowable contact stress should be around half of that assumed for normal spur or helical gear drive having the same material and conditions. Because of point contact, it serves no purpose to increase the gear widths. The minimum values for a shaft angle of 90° are $b_1 = p_{t2}$ and $b_2 = p_{t1}$, where p_t stands for the transverse circular pitch and b for the width as before. For usual calculations, the value taken is

$$b = 5 \text{ to } 10\, m_n$$

Recalling Eq. 3.58,

$$F_{t1} \leq 10 m_n^2 \pi K = 1000 P_1 / v_1 \tag{3.64}$$

transposition, we get

$$m_n(\text{mm}) \geq \sqrt{\frac{100 P_1(\text{kW})}{\pi v_1(\text{m/s}) K(\text{N/mm}^2)}} \tag{3.65}$$

To check against overheating, the following formula can be used

Factor of safety for temperature.

$$S_T = \frac{d_1 b}{1360 P_1 C_T} \geq 1 \tag{3.66}$$

The values of C_T are to be taken from Table 3.7.

Rewriting Eq. 3.50, we get for shaft angle 90°

$$a = \frac{m_n}{2}\left(\frac{z_1}{\cos\beta_1} + \frac{z_2}{\cos\beta_2}\right) = \frac{m_n}{2}\left(\frac{z_1}{\cos\beta_1} + \frac{z_2}{\sin\beta_1}\right)$$

or

$$\frac{z_1 m_n}{2a}\frac{1}{\cos\beta_1} + \frac{z_2 m_n}{2a}\frac{1}{\sin\beta_1} = 1$$

If the parameters, e.g. a, m_n, z_1 and z_2, are known or are assumed and their values inserted in the above equation, it becomes an equation of fourth degree to solve for the helix angle β_1. This equation can be graphically solved by trial and error, and by recalculation to arrive at the precise values. Such methods are given in detail in relevant books.

Referring to the force diagram in Fig. 3.12(b) in which the tooth flank of the driven gear is shown in operating condition with its driving gear, we can derive the following expressions for the different forces with gear 1 as the driving member

$$F_N' = F_N \cos\alpha$$

where F_N is the normal force on the tooth surface as before, and other symbols have the usual meanings in the following equations,

Tangential forces

$$F_{t1} = \frac{F_N'}{\cos\rho}\cos(\beta_1 - \rho) = \frac{F_N \cos\alpha}{\cos\rho}\cos(\beta_1 - \rho) \tag{3.67}$$

$$F_{t2} = \frac{F_N'}{\cos\rho}\cos(\beta_2 + \rho) = \frac{F_N \cos\alpha}{\cos\rho}\cos(\beta_2 + \rho) \tag{3.68}$$

Axial forces

$$F_{a1} = \frac{F_N'}{\cos\rho}\cos(\beta_1 - \rho) = \frac{F_N \cos\alpha}{\cos\rho}\cos(\beta_1 - \rho) \tag{3.69}$$

$$F_{a2} = \frac{F_N'}{\cos\rho}\cos(\beta_2 + \rho) = \frac{F_N \cos\alpha}{\cos\rho}\cos(\beta_2 + \rho) \tag{3.70}$$

Example 3.3 : *Given* : $\Sigma = 90°$ $n_1 = 1440$ rpm, $n_2 = 300$ rpm, $P_1 = 0.9$ kW, $\alpha = 20°$. To find the gear dimensions and other relevant data of a crossed helical gear drive. Due to space constraints, the diameter of the pinion should not exceed 120 mm.

Solution: In case of a new design, it is customary to assume some reasonable value for one of the gears and then proceed further to determine other parameters. The assumption should be based on such considerations as design requirements, space availability, shaft diameter, allowable circumferential velocity and other factors. In selecting the helix angles, the designer should make sure that the driving gear is provided with the bigger angle. The following guideline can be given

$$\beta_1 \geq \frac{1}{2}\sum \text{ for reduction gear drive}$$

and

$$\beta_1 = \frac{1}{2}\sum \text{ for stepup gear drive}$$

The following data are assumed for the example in question

$$d_1 = 110\text{ mm}, \beta_1 = 60^\circ, \mu = 0.1, v_1 = d_1 n_1/19100 = 110 \times 1440119100 = 8.3 \text{ m/s}.$$

The material selected is hardened steel. $v_s = v_1/\sin\beta_1 = 8.31 \sin 60' = 9.6$ m/s, $K = 20/(2 + v_s) = 201 (2 + 9.6) = 1.72$

$$m_n = \sqrt{\frac{100 P_1}{\pi v_1 K}} = \sqrt{\frac{100 \times 0.9}{\pi \times 8.3 \times 1.72}} = 1.42 \qquad m_n = 2\text{mm (selected)}$$

$$m_{t1} = m_n/\cos\beta_1 = 2/\cos 60' = 4\text{mm}, m_{t2} = 2/\cos 30' = 2.3094\text{mm}$$

$$z_1 = d_1/m_{t1} = 110/4 = 27.5 \approx 28. \text{ Actual value of } d_1 = 28 \times 4 = 112 \text{ mm}$$

$$z_2 = i z_1 = \frac{n_1}{n_2} z_1 = \frac{1440}{300} \times 28 = 134.4 \approx 135$$

Actual value of $i = z_2/z_1 = 135128 = 4.8214$. Actual value of $n_2 = \dfrac{1440}{4.8214} = 298.67$ rpm

Since the numerical values of z_1 and z_2 have no common divisor, hunting tooth action is assured.

$$d_2 = z_2\, m_{t2} = 135 \times 2.3094 = 311.769 \text{ mm}$$

Centre distance

$$a = (d_1 + d_2)/2 = (112 + 311.769)/2 = 211.8845 \text{ mm}$$

If it is desired to have the centre distance as a whole number, it can be taken as 212, and recalculations can be made accordingly to determine the relevant parameters, or the gears may be corrected.

Efficiency $\quad \eta = \dfrac{\tan(\beta_1 - \rho)}{\tan\beta_1}, \mu = \tan\rho = 0.1, \quad$ or $\rho = 5^\circ\, 42'\, 38''$

Therefore

$$\eta = \frac{\tan(60^\circ - 5^\circ\ 42'\ 38'')}{\tan 60^\circ} = 80.3\%$$

$$d_{a1} = d_1 + 2m_n = 112 + 2 \times 2 = 116 \text{ mm}$$

$$d_{a2} = 311.769 + 2 \times 2 = 315.769 \text{ mm}$$

$$b = 10m_n = 10 \times 2 = 20\text{mm}$$

Power loss

$$P_s = P_1(1 - \eta) = 0.9\ (1 - 0.803) = 0.1773 \text{ kW}$$

$$S_T = \frac{112 \times 20}{1360 \times 0.1773 \times 2} = 4.6$$

Since the value of S_T is greater than 1, the system is safe against temperature rise. Check for power transmission

$$v_1 = 112 \times 1440 / 19100 = 8.44 \text{ m/s}$$

$$v_s = 8.44/\sin 60° = 9.75 \text{ m/s}$$

$$K = 20/(2 + 9.75) = 1.702 \text{ N/mm}^2$$

$$\text{Power capacity} = \frac{10 m_n^2 \pi K v_1}{1000} = \frac{10 \times 2^2 \times 3.14 \times 1.702 \times 8.44}{1000} = 1.8 \text{ kW}$$

Since this is greater than the required value of $P_1 = 0.9$ kW, the calculated gear data of the crossed helical gear system are suitable for the purpose.

3.16 Herringbone Gears

It has been explained in Sec. 3.12 that a pair of helical gears meshing with each other develops radial as well as thrust loads. These loads are taken by the bearings on which the shafts carrying the gears are mounted.

In certain operations, the high thrust forces may be objectionable for various reasons and undesirable for the bearings and other components of the equipments. These forces therefore should be avoided as they may lead to vibrations and other complications. One solution lies in using the herringbone or double-helical gears. Such a gear is in effect equivalent to two helical gears of opposite hands, mounted side by side on the same shaft. When two such gears are in mesh, the pair develops equal and opposite thrust reactions with the result that they cancel out the thrust loads (see Fig. 1.5).

Normally, to provide a run-out clearance for the cutting tool, e.g. hob, milling cutter, grinding wheel, and other cutters, a suitable gap is provided between the helices on the gear blank. This gap is in the form of a groove, the depth of which should be somewhat below the root diameter of the gear. The width of this groove required will depend upon the cutting process used. There are, however, gear generators which produce double-helical gears with continuous teeth, i.e. no gap needs to be provided between the opposite-handed helices.

Double-helical gears are generally used in parallel shaft transmissions where smooth, continuous motion in high speed drives is intended. The pitch line velocity usually ranges from 5 m/sec to 15 m/sec in normal applications and it may rise to 60 m/sec or more in special equipments, such as marine reduction gears, turbo-alternators, and similar machines.

In case of low angular speeds and heavy duty drives, like, rolling mills, the teeth are usually cast. For higher speeds coupled with heavy loads where accuracy is of prime importance, the teeth are made by milling or generation methods.

The value of helix angle β used in case of double-helical gears normally lies between 20° to 45°, with higher angles for precision gears and low tooth pressures. As in the case of so many other types of gears, failure in herringbone gear transmission is seldom attributed to the tooth breakage. It is more likely to be due to fatigue. Excessive wear may take place or fatigue may cause sub-surface failures such as pitting or spalling. Surface durability, therefore, should be the design criterion. The contact stresses should be within the permissible limits for wear. Since pinion is the weaker member and subject to a greater amount of wear, it is the general practice to base the power rating calculations of the gear pair on the pinion data.

Three types of designs are usually followed in case of herringbone gears.

(i) Teeth are cut on single solid blank. Normally, pinions are of this type.

(ii) Teeth are cut on two separate blanks and then the gears are joined together by means of bolts or similar fasteners. The magnitude of the helix angle is the same, but the helices are of opposite hands. Dowel pins may be used for alignment.

(iii) Teeth are cut on rings and the ring-gears are then shrink-fitted on the circular stepped seats on the solid hub, machined properly to accept the rings. Grub screws may be inserted in-between the hub and the rings for additional securing.

Proper alignment is of vital importance when the gear is made of separate components. It is to be ensured that the two halves are exact mirror-images of each other. The gears may be post-machined so that alignment with the pinion is perfect and the system runs smoothly when in mesh. There is, however, one type of design of herringbone gears where the teeth on the two halves of the same gear blank are cut staggered to each other. This design is known as a Wuest gear, named after its inventor.

The herringbone gears are generally sturdy in construction, are resistant to shock loads and can take frequent reversal of direction of loads without any noticeable detrimental effects.

Because these gears usually have high helix angles, pinions having a very few number of teeth can be cut without undercutting. The minimum number of teeth of a pinion in such cases may be as low as **4** or **5**. This is a distinct advantage of the herringbone gear, besides others.

A normal practice followed in case of herringbone gears in mesh is to have one of the shafts as a floating shaft. This is done to facilitate equal distribution of load between the two sections of the gear.

The large helix angle of these gears allows more over-lapping of tooth contact which in consequence leads to quieter and smoother operation. Generally, the minimum face width (b) is kept at around $2.3\,p_t/\tan\beta$, where p_t = the circular pitch in the transverse plane. Both the single helical and the herringbone gears are produced using the same design techniques.

The double helical gears, however, have their own limitations as regards application. If it is required to fit a helical gear by face-wise sliding along the teeth so that the gears are slid into mesh by axial displacement in a cluster in which the other members have already been assembled, then obviously this cannot be done in case of a double helical gear. In case of axial thrusts, if both the shafts have to sustain large axial loads caused by some extraneous source, the single helical gears are preferable as the tooth pressure can evenly distribute itself along the width of these gears which is not the case in case of double-handed gears. The herringbone gears are also not suitable in case of application which calls for anchoring of both shafts against axial movement. However, as pointed out earlier, the double-helical gears induce no additional thrust as compared to the single helical gears.

Example 3.3a : In a turbine drive, a pair of double-helical gears is used with following data:

$$z_1 = 31,\; z_2 = 123,\; u = \frac{z_2}{z_1}\; 3.3243,\; m_n = 3\text{ mm},\; \beta = 31^\circ$$

$$\alpha_n = 20^\circ,\; x_1 = +0.14,\; x_2 = -0.14,\; b_1 = b_2 = 2 \times 92 = 184\text{ mm},$$

nominal power, P_1 = 300kW, service factor = 1.1, n_1 = 3225 rpm. It is required to select the proper materials for the gear set.

Solution:

$$\text{Torque}, T_1 = 9550 \times \frac{P_1}{n_1} \times \text{service factor} = 9550 \times \frac{300}{3225} \times 11 = 977\,\text{Nm}.$$

$$d_1 = m_t z_1 = \frac{m_n}{\cos\beta} z_1 = \frac{\smile}{\cos 31^\circ} \times 37 = 129.49 \text{ mm.}$$

Tangential force or transmitted load at the pitch circle

$$F_t = \frac{2000 T_1}{d_1} = \frac{2000 \times 977}{129.49} = 15{,}090 \text{ N}$$

Using Figs 2.48 and 2.49, we get the form factors for pinion and gear having $z_1 = 37, z_2 = 123$, $\beta = 31^\circ, x_1 = +0.14$ and $x_2 = -0.14$, as follows

$$q_{k1} = 2.25$$

$$q_{k2} = 2.25 \text{ (by extrapolation)}$$

The contact ratio factor, 'q_ε' is taken to be 1 in both cases. The bending stresses, as per Eq. 3.40, are given by

$$\sigma_{b1} = \frac{F_t}{bm_n} q_{k1}\, q_\varepsilon = \frac{15090}{184 \times 3} \times 2.25 \times 1 = 62 \text{N} / \text{mm}^2$$

$$\sigma_{b2} = \frac{15090}{184 \times 3} \times 2.25 \times 1 = 62 \text{N} / \text{mm}^2$$

To find the contact stress, we use Eq. 3.43

$$p_{cp} = y_m\, y_P\, y_L \sqrt{\frac{2000\, T_1}{\left(\frac{b}{d_1}\right) d_1^3} \times \frac{u+1}{u}}$$

From Table 2.17 $y_m = 269$ for steel gears, y_p is calculated as per Eq. 3.45 and found to be 1.56. Here, $\alpha_{tw} = \alpha_t$ for S_0- gearing. The transverse pressure angle, α_t, and the base helix angle, β_b, are calculated as per Eqs 3.9 and 3.46, respectively. The value of y_L is taken to be 1. Inserting the above values, we have

$$p_{cP} = 269 \times 1.56 \times 1 \sqrt{\frac{2000 \times 977}{\left(\frac{184}{129.49}\right) 129.49^3} \times \frac{3.3243+1}{3.3243}} = 381 \text{N} / \text{mm}^2$$

Consulting the strength of gear materials given in Appendix E, we find that the materials 45 C 8 or 40Cr 4, after hardening by flame or induction hardening processes, will serve the purpose adequately. The dimensional parameters can be fixed as per Table 3.2 on S_0- gearing of helical gears.

3.17 Replacement of Diametral Pitch Gears by Module Gears

Many machines are equipped with components which are based on the FPS system of measurements. In such equipments, the gears are based on the diametral pitch (DP) system of calculations and are cut or generated by DP cutters. Since the inch system is on the way out, the designer is often asked to replace the DP spur or helical gears by module gears without tampering with the existing parameters like centre distance, gear ratio, and available space in the machine as most of the machine shops these days stock module cutters only.

The above objective can be achieved in two ways as illustrated in the examples which follow. Since the changes in strength properties are negligible and since sufficient factor of safety is usually taken in the initial design calculations itself, checking from the point of view of strength is normally not necessary.

Example 3.4 *Given:* diametral pitch = 10, $z_1 = 21$, $z_2 = 60$.
To replace the DP gears by module gears, keeping the gear ratio and the centre distance the same.

Solution:

$$\text{Centre distance} = \frac{1}{2} \times \frac{1}{DP} \times (z_1 + z_2) = \frac{1}{2 \times 10}(21+60) = 4.05 \text{ inch} = 102.87 \text{ mm}$$

Reduction ratio $i = 60/21 = 2.8571$. As per Appendix D, the nearest module to DP 10 is 2.5. To keep the centre distance unaltered, the gears to be corrected. We proceed as follows

Centre distance $$a = m\frac{z_1 + z_2}{2}\,\frac{\cos\alpha}{\cos\alpha_w} \qquad \text{(as per Eq. 2.35)}$$

or

$$102.87 = 2.5 \times \frac{21 + 60}{2} \times \frac{\cos 20°}{\cos \alpha_w}$$

whence

$$\alpha_w = 22° \, 20' \, 51.25''$$

$$\text{inv}\,\alpha_w = \frac{2 \times \tan 20° (x_1 + x_2)}{z_1 + z_2} + \text{inv}\,20° \qquad \text{(as per Eq. 2.33)}$$

Inserting the values and interpolating for the value of inv α_w from Appendix H, we have

$$0.021061 = \frac{2 \times 0.36397\,(x_1 + x_2)}{81} + 0.014904$$

whence

$$x_1 + x_2 = 0.6851$$

Equation 2.41 yields

$$x_1 = \frac{0.6851}{(2.8571 + 1)} - \frac{0.5(2.8571 - 1)}{(2.8571 + 1)} = 0.4184$$

$$x_2 = 0.6851 - 0.4184 = 0.2667$$

$$d_1 = mz_1 = 2.5 \times 21 = 52.5, \quad d_2 = 2.5 \times 150$$

$$d_{a1} = 2(102.87 + 2.5 - 0.2667 \times 2.5) - 150 = 59.4065 \text{ mm} \qquad \text{(as per Eq. 2.38)}$$

$$d_{a2} = 2(102.87 + 2.5 - 0.4184 \times 2.5) - 52.5 = 156.148 \text{ mm}$$

The final values are

$$z_1 = 21,\ z_2 = 60,\ m = 2.5,\ i = 2.8571,\ d_1 = 52.5,\ d_2 = 150$$

$$d_{a1} = 59.4065,\ d_{a2} = 156.148,\ x_1 m = 1.046,\ x_2 m = 0.66675$$

Check: The amount of centre distance modification, topping and the top clearance are checked as follows

$$2ym = (mz_1 + 2m + 2x_1m) - d_{a1} = 0.1855, \quad \text{or} \quad ym = 0.09275$$

Similarly

$$2ym = (mz_2 + 2m + 2x_2m) - d_{a2} = 0.1855, \quad \text{or} \quad ym = 0.09275$$

Again

$$a_p - a = ym \qquad \text{(as per Eq. 2.36)}$$

or

$$\left[\left(\frac{d_1 + d_2}{2}\right) + x_1 m + x_2 m\right] - a = ym$$

Inserting the value, we get

$$ym = 0.09275$$

Hence all the values of ym tally

Clearance at the top $$c = a - \left[\frac{d_{a1} + d_{a2}}{2} + ym - h\right] \qquad \text{(as per Eq. 2.37)}$$

Putting the relevant values, we get

$$c = 0.625 \text{ mm} = 0.25 \times \text{module } 2.5$$

Hence, the clearance conforms to the value stipulated in the basic rack given in Sec. 2.1.

Example 3.5 *Given:* data as in Example 3.4.
To solve the problem by using helical gears.

Solution: Figure 3.13 shows a pictorial representation of helical gears with different helix angles. A study of this figure reveals the fact that by varying the helix angle, we can adjust the centre distance between a pair of meshing gears without altering the gear ratio or the number of teeth of individual gears. We shall take advantage of this property in the following manner to solve this problem.

$$d_1 = \frac{mz_1}{\cos\beta} = \frac{2.5 \times 21}{\cos\beta}, \quad d_2 = \frac{mz_2}{\cos\beta} = \frac{2.5 \times 60}{\cos\beta}$$

$$a = \frac{1}{2}(d_1 + d_2), \quad \text{or } 102.87 = \frac{1}{2} \times 81 \times \frac{2.5}{\cos\beta}$$

whence $$\beta = 10' 10' 54.29'' \text{ (LH and RH)}$$

Therefore, the final values are

$$d_1 = 53.34 \text{ mm}, \; d_2 = 152.4 \text{ mm},$$

$$d_{a1} = d_1 + 2m = 58.34,$$

$$d_{a2} = 157.4$$

Due to certain restricting factors like insufficient top land due to a high correction factor and difficulty in machine-setting for accurate helix angle, sometimes a combination of the two methods described above is employed to attain the desired values, that is, gears are made both corrected and helical. Such a measure poses no problem if the gears are cut by a generating method. However, while cutting gears in a milling machine, it may not be possible to adhere to the exact helix angle found by calculation due to indexing difficulties of the particular machine. Again, if corrected profiles of gear teeth are to be cut in a milling machine, this necessitates making of a form-tool to conform to the particular tooth profile. This may neither be practically possible nor economically viable. In such cases, gears are cut by standard milling cutters by compromising on the gear ratio, provided that the maintenance of the gear ratio is not that rigid and as such, it can be changed a little without unduly hampering the gear drive. Such a flexible case of gear ratio is illustrated in the Example 3.6.

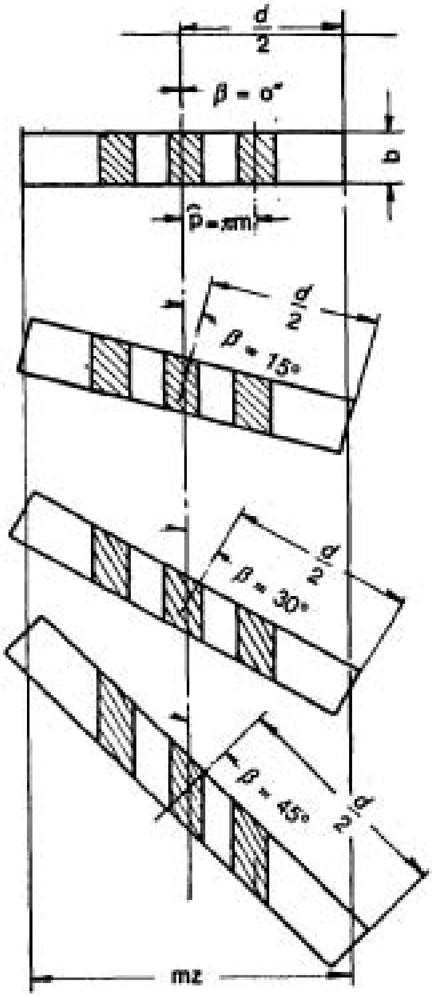


Fig. 3.13 Orientations of helical gear

Example 3.6: *Given:* $m = 2.5$, $a = 102.87$ mm (as in previous example), helix angle can be adjusted to the nearest minute only.
To find z_1 and z_2.

Solution: Taking $\beta = 10^\circ$ at first, neglecting the minutes and seconds of the helix angle of Example **3.5**, we have

$$a = 102.87 = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}(z_1 + z_2)\frac{2.5}{\cos 10^\circ}$$

or

$$z_1 + z_2 = 8104573884$$

Since $z_1 + z_2$ must be an integral number, and since 81 has already been used in the previous examples, we try other numbers.

By transposing, we get

$$\cos \beta = \frac{2.5}{2 \times 102.87}(z_1 + z_2) = 0.012151258\,(z_1 + z_2)$$

By trial and error, we find that one combination is $z_1 + z_2 = 83$. In that case, $\cos\beta$ is practically equal to 1, that is, $\beta = 0^\circ$.

Taking $z_1 = 21$ and $z_2 = 83 - 21 = 62$

$$a = \frac{1}{2} \times 2.5 \times 83 = 103.75 \text{ mm}$$

Obviously, this combination cannot be used as the centre distance should be kept at 102.87 mm. Again by trial, we find that a value of $\beta = 13^\circ\ 34'$ and $z_1 + z_2 = 80$ will serve the purpose as the following calculations show

$$z_1 = 20, z_2 = 60, d_1 = 2.5 \times 20/\cos 13^\circ\ 34' = 51.435 \text{ mm}$$

Similarly, $d_2 = 154.305$ mm

$$a = \frac{1}{2}(51.435 + 154.305) = 102.87 \text{ mm}$$

Hence, this combination can be accepted as it more or less satisfies the conditions. The reader is advised to check for backlash in each case and assign the proper tooth thickness or tooth distance tolerances.

3.18 Efficiency of Helical Gear Drive

When compared to spur gears, it can be seen that the meshing conditions of helical gears have a considerable influence on the efficiency of the system. This is mainly due to the different natures of load distribution patterns in case of the two types of the gear drives during the course of action.

In a mating pair of spur gears, the tooth contact relations and load distribution are relatively simpler than in case of helical gears in mesh. As pointed out in previous chapters, contact begins along the whole of the top edge of the driven tooth in case of spur gears. The frictional loss is greatest at this point. In contrast, the load distribution in case of helical gears takes place along a diagonal over the tooth surface, as explained in Sec. 3.5. One result of such action is that the height of the point of grip between the mating teeth varies continuously along the diagonal, if one considers each individual transverse section of the tooth. This results in a lesser amount of loss due to tooth friction than in case of spur gears with identical data. The frictional loss is roughly about **20%** less.

Equation 2.11 is repeated here

$$\eta = 1 - f\mu\left(\frac{1}{z_1} \pm \frac{1}{z_2}\right)$$

The above equation gives us an expression for the efficiency for spur gear drives. It can be shown that the efficiency for helical gears is given by

$$\eta = 1 - 0.8\cos\beta\, f\mu_{u} \quad - \pm \frac{1}{z_2}\Big) \tag{3.71}$$

In the above equation, the different factors have the same meanings and values as given previously in the relevant sections, (Secs 2.6 and 2.29).

4
Worm and Worm-Wheels

4.1 Fundamentals of Worm and Worm-Wheel Drives

Worm and worm-wheel drives are normally used for non-parallel, non-intersecting, right-angle gear-drive systems where high reduction ratios are involved, though they are also employed as low to medium speed reducers in many applications. A large reduction ratio also automatically means a large multiplication of torque. A variety of worm-drives have been developed which are quite novel and innovative as to their shape, scope of application, kinematic properties and other aspects. In this chapter, we will restrict our discussion only to those types which are most commonly used in usual, conventional applications.

The worm in a worm-drive system is generally the driving member, though this is not always the case. The most commonly used type is the single-enveloping, cylindrical worm. The tooth resembles the thread of a screw which wraps around the root cylinder in a helical fashion. The profile of the thread is either straight or slightly curved, depending upon the type of design used, as discussed later in this chapter. The worm-wheel is hobbed on a blank which is curved widthwise. This is called a "throated" blank, and this is done so that the wheel can envelope the worm. The hob used for generating the worm-gear is an exact duplicate of the worm. In other words, the shape of the worm-tooth defines the shape of the wheel tooth. This means that the wheel tooth must be cut by a tool that produces a profile conjugate with the thread on the worm. The wheel teeth are curved and are thicker at each end than at the middle.

The meshing action in a worm-drive is a combination of sliding and rolling, with sliding prevailing at higher reduction ratios. In this respect, this drive is similar to the crossed-helical gear-drive. But the worm-drive has greater load-carrying capacity than the crossed-helical system because the worm-drive has a line contact whereas the crossed-helical has a point contact only and as such it is not considered for power transmission as explained in Sec. 3.15.

The teeth of the worm in a worm-drive system may or may not be of involute form. This will depend upon the type of tool used in cutting the threads. The tooth profile of the wheel is an involute. A worm may have more than one tooth or thread helix. In gear technology the number of teeth is referred to as the number of "start". Thus, there can be single-start or multi-start worms. The reduction ratio of a worm-wheel set does not depend only on the diameters of the worm and the wheel. To find the reduction ratio, the simplest method is to divide the number of teeth of the wheel (z_2) by the number of starts of the worm (z_1).

The worm and the wheel have the same hand of helix. However, unlike a set of parallel helical gears, the magnitudes of the helix angles of worm and the wheel are quite different. It is customary to specify the "lead angle" of the worm. The lead angle is complementary to the helix angle. With a 90° shaft angle worm-set, the lead angle of the worm is equal to the helix angle of the wheel. It is the axial pitch of worm which is normally used for specification. The axial pitch of the worm is numerically equal to the circular pitch of the worm-wheel. In case of module too, the calculations are based on the axial module and not on the normal module.

Though orientations having other shaft angles are possible, the components of a worm-drive are generally mounted on non-intersecting shafts with 90° shaft angle. While the worm bearings are designed for high thrust loads, the wheel bearings are meant to withstand high radial loads. As far as mounting is concerned, it is imperative that the shafts on which the members of the set are mounted be very correctly aligned. The centre distance must tally perfectly with the calculated value, though some leeway in centre distance can be achieved by using corrected wheels as discussed in Sec. 4.7.

As regards the shape of the worm-thread, several types are in vogue. These will be taken up in Sec. 4.2. Broadly, four types are in common usage — thread which is straight-sided in the axial section, thread which is straight-sided in normal section, thread having a tooth profile which is convex in normal section and concave in axial section, and thread which has an involute profile in the transverse section. Besides these, a fifth type with concave profile of worm-tooth is also used in industrial applications for its many advantages (see Sec. 4.4).

Since a considerable amount of driving energy is dissipated mainly in the form of heat due to the sliding action between the mating components in a worm-set, entailing frictional power loss, it is therefore logical for a designer to endeavour to minimise the coefficient of friction by various means. One way to achieve this is to select dissimilar metals for the worm and the gear. Combinations of worm and gear materials are discussed in several sections of this chapter at relevant places.

Since the worm-gear is produced by the generating method, it can be corrected in the same manner as explained in the case of spur and helical gears. However, as the parameters of the hob cutter and those of the worm remain constant, only the wheel can be corrected and not the worm.

The wide application of the worm-drive system stems from the fact that the system affords to have a design with higher transmission ratio with comparatively lower weight, smaller overall dimensions and space requirements. The arrangement is thus compact and also ensures a smooth and noiseless operation. Self-locking ability or irreversibility of drive, when required, is another advantage which is of utmost importance in applications like hoisting equipment and allied contrivances, because the load can be held in suspended condition when the motive power is withdrawn. Low overall efficiency, high frictional losses giving rise to heat, and comparatively low transmitted power are the main disadvantages.

4.2 Types of Worms

Depending on the type of worm, the worm -tooth can be manufactured in a lathe with a turning tool or in a hobbing machine employing a hob or a circular cutter. It can be produced by other methods as well.

For simple and economical manufacture, the cutting edge of the tool is provided with a simple straight or curved form. In relation to the manufacturing processes involved, the under-mentioned tooth forms of the worm are generally used in practice (Fig.4.1).

(i) Type **Z A** The tooth flank is defined in the axial profile. This type of worm tooth has a trapezoidal form with straight-sided tooth profiles in the axial section. In transverse section. that is, in section at right angles to the axial section, the profile is an Archimedian spiral. When section is taken normal to the tooth, the tooth profile is convex. It corresponds to a screw with a trapezoidal or acme thread. It can be produced by a single poinf tool with cutting edge having the required profile and is located in the axial plane of the worm. Angle α in half axial profile of the tool is the tool angle. This type of worm can also be produced by a pinion type cutter working in an axial plane of the worm.

(ii) Type ZN: This is similar to type ZA except that instead of the axial section, the tooth profile is of trapezoidal form with straight sides in the normal section. Favourable contact relations can be obtained in this type. The tool is of trapezoidal form in the normal section. The tooth profile is slightly curved in the axial section. The profile can be made by a turning tool and approximately by end-milling cutter or face and side milling cutter.

(iii) Type ZK : In this type the tool is of trapezoidal form, but the tooth profile of the worm is convex in normal section and concave in axial section. This type is frequently used for certain advantages it offers as regards cutting processes. Tooth profiles are obtained by means of side and face milling cutter or grinding wheel. Angle α is the tool angle in the half axial profile of the tool and angle γ is the lead angle of the worm.

(iv) Type ZI: This type of worm is analogous to a helical gear with involute toothing and having **small** number of teeth. The worm-thread is an involute helicoid. The tooth form is an involute in transverse section as in the case of a helical gear, convex in normal section and approximately hyperbolic in the axial section. The tooth profile is produced by a trapezoidal shaped turning tool which is properly set or by a grinding wheel or by special hob-cutters.

Besides the above four common types, worm-drive with concave-convex profilqs is also used in the industrial field for its many advantages. This is called a CAVEX drive and is separately treated in Sec. 4.4.

In normal practice, straight cylindrical worms with trapezoidal profile of worm-thread and wheel with involute toothing are used. This type of pairing has theoretically only a line contact. Special profiles like **CAVEX** offer greater load capacity, lesser frictional loss, continuous maintenance of lubricating film and high hydrodynamic lubricant pressure which reduces output loss and wear, impact damping properties and noiselessness. Other advantages and special properties are given in detail in Sec. **4.4**

Besides CAVEX, another similar design is from Brown and Bostock-Renk. Because of their special advantages, these special types of worm gearings with high capacity and specially high efficiency are now fast replacing the conventional types, particularly in such applications as cranes and similar drives where lighter construction with greater power output is of vital importance.

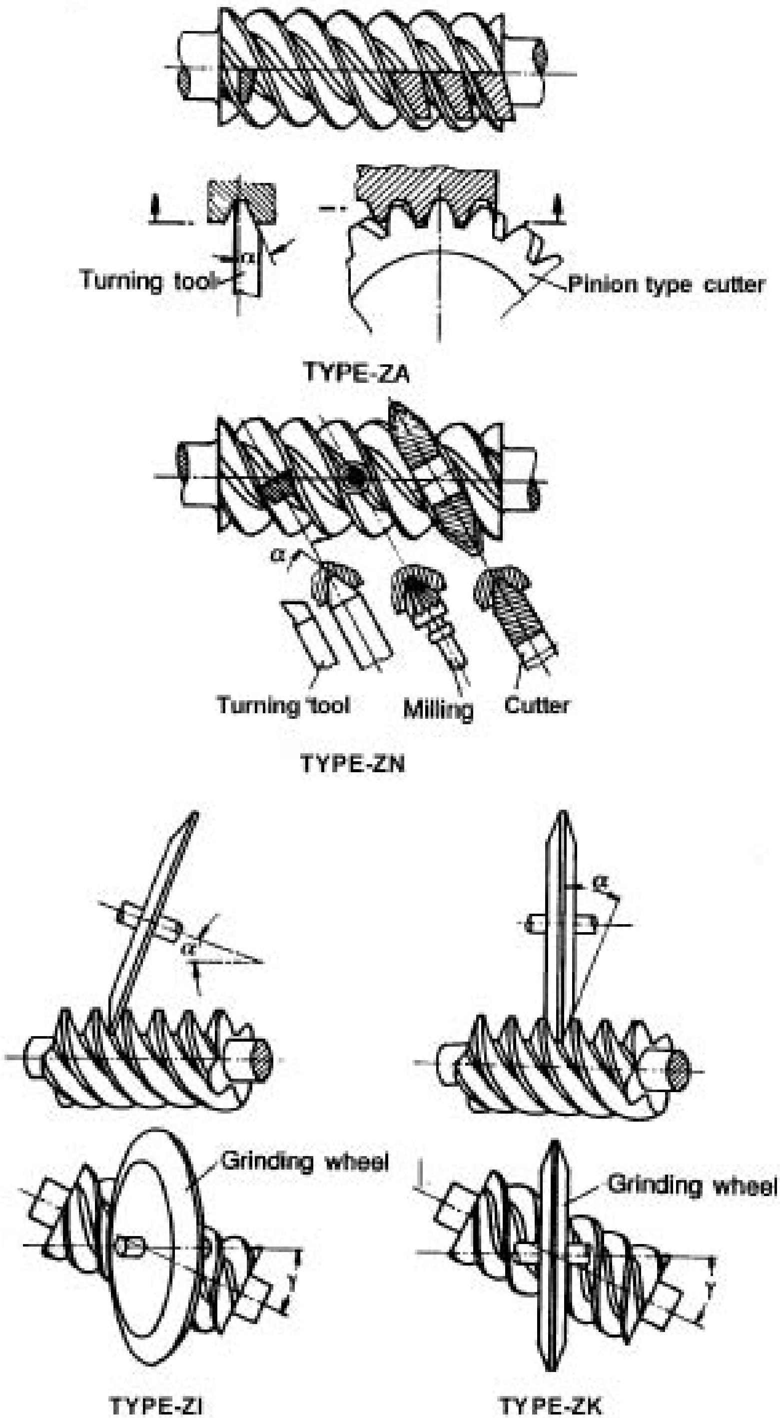


Fig. 4.1 Type of worm flanks

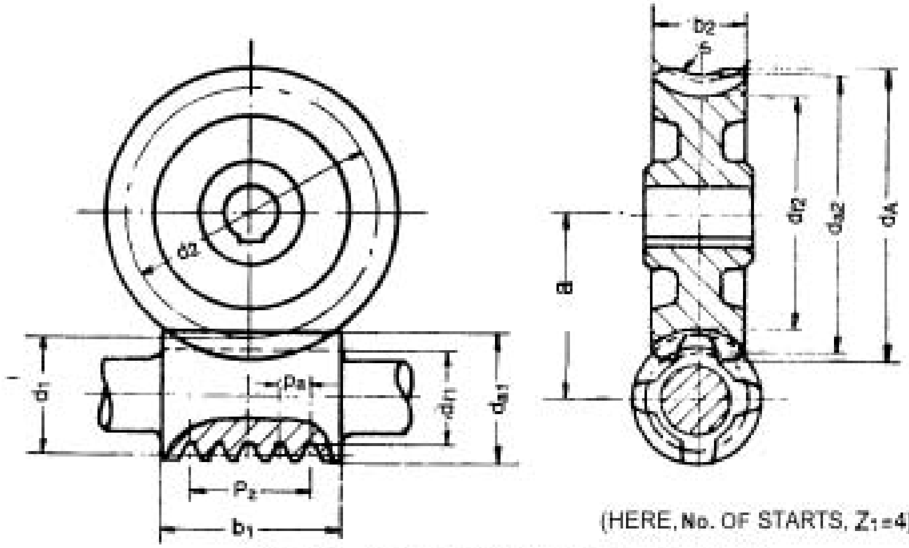


Fig. 4.2 Worm and worm-wheel geometry

A general view of the common worm-drive has been shown in Fig.4.2. The worm and wheel arrangements can also be categorised according to the envelopment point of view. This is explained below:

Single-enveloping worm-gear This type of worm gear has been shown in Fig. **4.3**. The single enveloping worm-set is made in such a way that the wheel wraps around or envelops the worm thereby partially enclosing it. The worm-gear is "throated" to achieve this purpose. In this type

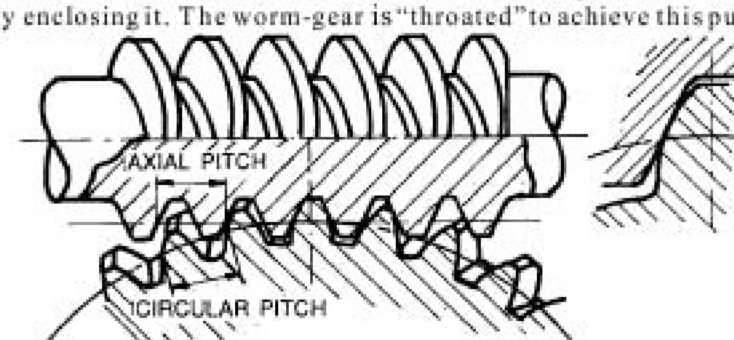


Fig. 4.3 Single enveloping worm-wheel

of configuration, it is theoretically a "line contact". Under load, there is some area contact because of flattening effect. This line contact may extend either across the face width or across the portion of the tooth which is in the region of action. While in action, the line sweeps across the whole width and height of the tooth. Though similar to helical gears as far as the meshing action is concerned, the sliding velocity is much higher in the worm-drive **for** the same pitch line velocity.

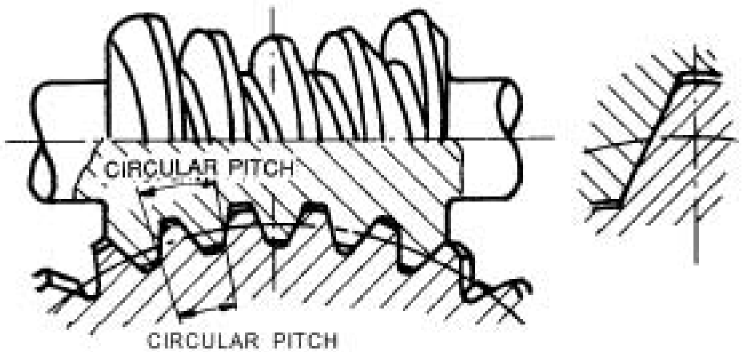


Fig. 4.4 Double enveloping worm-wheel

Double-enveloping worm-set In this type, shown in Fig. 4.4, both the worm and the wheel are throated and, the worm is also curved longitudinally to fit the curvature of the wheel. In double-enveloping type, the system can have "area contact". The alignment must be accurate and the mounting dimensions should be strictly adhered to. The worm and the gear must mutually fit exactly at the middle axially. This type has more tooth surfaces in contact than the single-enveloping type. Consequently, the larger contact area enables a double-enveloping worm-gear system to have increased load carrying capacity and higher efficiency at normal speeds. However, more heat is generated at high speeds and hence, copious lubrication should be assured to prevent scoring and overheating.

The double-enveloping worm-set is also known as the "globoidal worm-gear pair". The worm is sometimes referred to as an "hour-glass" worm because of its peculiar shape. It can be easily seen from Fig.4.4, that the linear pitch is not constant from thread to thread. The profile of the worm thread must also continuously vary and so does the lead angle.

Since more teeth share the load, the unit pressure is low, leading to longer life, provided the lubrication and other aspects are in order. The system, however, is comparatively difficult to manufacture as special cutting tools are required.

Besides the worm and wheel systems described so far, there are a number of other combinations described below which find place in industrial usage.

(i) ***Cylindrical-worm and helical gear*** A standard single-start worm can mesh with a standard helical gear. This simple arrangement is economical and does not require rigid maintenance of alignment or centre distance. Existing helical gear in a system can be made to operate with a suitably designed worm. However, power capacity is low due to point contact.

(ii) ***Enveloping-worm and spur or helical gear*** Here, the worm is throated, hour-glass type, while the gear is either standard involute spur or helical. The system is good for indexing but is not recommended for high power applications. The drive makes a line contact.

(iii) ***Wildhaber worm-drive*** This is same as type (ii), except that the gear has a straight-sided tooth form instead of an involute one. Extremely high accuracy in tooth spacing can be obtained in this type of worm-drive. Axial alignment of worm should be strictly maintained. The drive makes a line contact.

(iv) ***Hindley-worm drive*** This is a double-enveloping worm-drive system and is the only one with area contact among the four types discussed here. This type, with certain modifications, is also known as the "Cone-drive" system which is named after Samuel Cone. The drive has the hour-glass type worm and the gear is throated. In this type, the design is compact and the load transmitting capacity is high. Strict axial alignment is a prerequisite. All teeth are straight sided, and special tooling and equipment are required to cut the teeth. Space requirements are small compared to the output delivered.

4.3 Basic Parameters of Worm and Worm-Wheel

The basic dimensional parameters of the worm and the wheel have been illustrated in Figs 4.5 and 4.6. These parameters as well as other relevant details connected thereof will be discussed in this section.

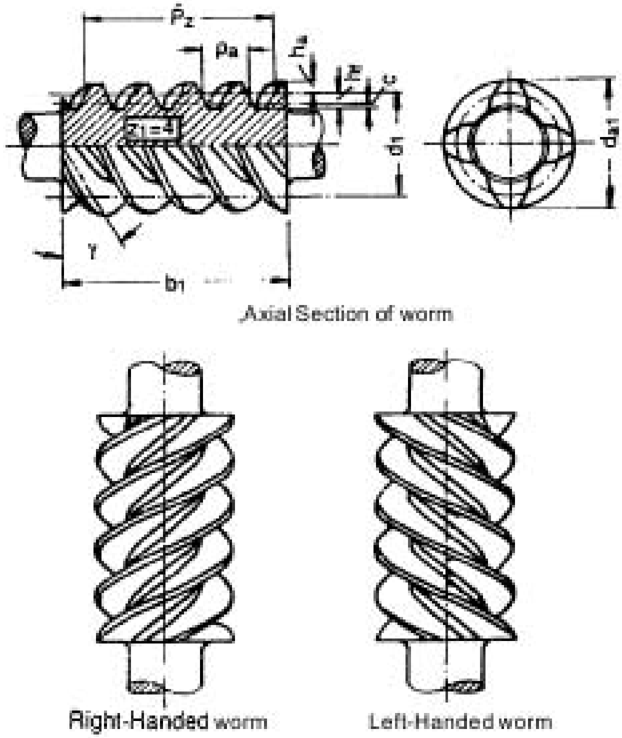


Fig. 4.5 Basic parameters of worm

The reference profile of a worm-drive is taken in the axial section of the worm. The different tooth dimensions, namely, height of tooth, clearance at the tip, tooth thickness, and other measurements are taken in this profile. The definitions and relations of these parameters are given in the following paragraphs.

Number of starts of worm A worm can have a single or more threads or teeth which are wound around the cylindrical body of the worm along the length of the worm. The number of such threads has been termed as the number of starts and is denoted by z_1. The number of starts should not normally exceed 6. If self-locking or irreversibility of drive is desired, then only single start worms are used.

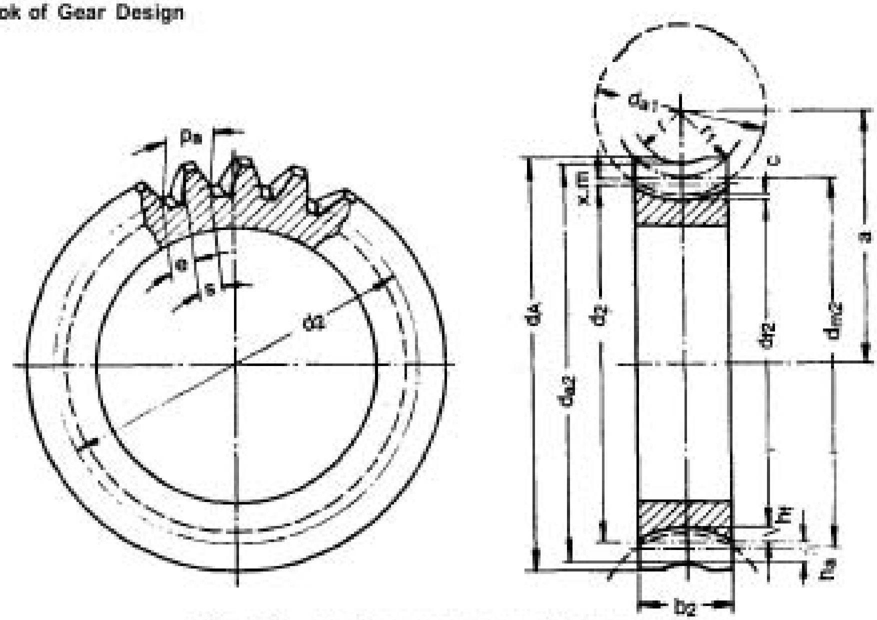


Fig. 4.6 Basic parameters of worm-wheel

Middle circle diameter of worm This is the nominal dimension of the worm. It is denoted by d_1 or d_m. It is the diameter of the reference circle of the worm and is analogous to the pitch circle diameter of a gear. Generally, the value of d_{m1} lies between **25** to ***60 %*** of the centre distance.

Axial module of worm In case of worm, the axial pitch is normally used as a specification factor. The axial module is related to the axial pitch by

$$p_a = \pi m \tag{4.1}$$

where m is the axial module and p_a the axial pitch. Since in case of a worm-drive, it is the axial module which is the reference module on which all calculations and specifications are based, it is customary to denote this module as m, without any subscript. However, some authors use m_a for axial module and m_n for normal module. These two modules are related to each other by

$$m \text{ (or } m_a) = m_n / \cos \gamma_m \tag{4.2}$$

where γ_m is the middle lead angle of the worm. As a guiding value, m can be taken to have a value which lies between $d_{m1}/15$ to $d_{m1}/6$, with an average value of $0.1\,d_{m1}$

Axial pitch of worm The axial pitch is the distance, measured axially, from a point on one tooth to the corresponding point on an adjacent tooth, and is given by

$$p_a = \pi m = p_z / z_1 \tag{4.3}$$

Lead of worm This is denoted by p_z, and is defined as the distance, measured axially, between two consecutive point on the same worm-tooth when the tooth helix makes a complete turn around the axis. If the number of start is one, then $p_z = p_a$. In Fig. *4.5* (top portion) the number of start (z_1) is **4**. Hence, in this case, $p_z = 4\,p_a$.

Lead angle of worm This is the angle subtended between a tangent to the pitch helix (i.e. the helix on the middle cylinder) and a plane normal to the axis of the worm. This is also known as the middle lead angle of the worm and is denoted by γ or γ_m. The helix angle of the worm is complementary to the lead angle, that is, lead angle = 90°−helix angle. Thus, a worm can be thought of as a helical gear whose tooth makes a complete revolution around the pitch cylinder. The lead angle is given by

$$\tan \gamma_m = \frac{p_z}{\pi d_1} = \frac{\pi m z_1}{\pi d_1} = \frac{m z_1}{d_1} \tag{4.4}$$

Form number of worm This is the ratio between the middle circle diameter and the module, and is given by

$$z_F = \frac{d_1}{m} \tag{4.5}$$

This is an important parameter of the worm as it determines the shape of the worm and as a consequence also determines the moment of resistance of the worm against bending. We can also write

$$\tan \gamma = \frac{m z_1}{d_1} = \frac{z_1}{z_F} \tag{4.6}$$

z_F	7	10	17
γ	8.1°	5.7°	3.4°

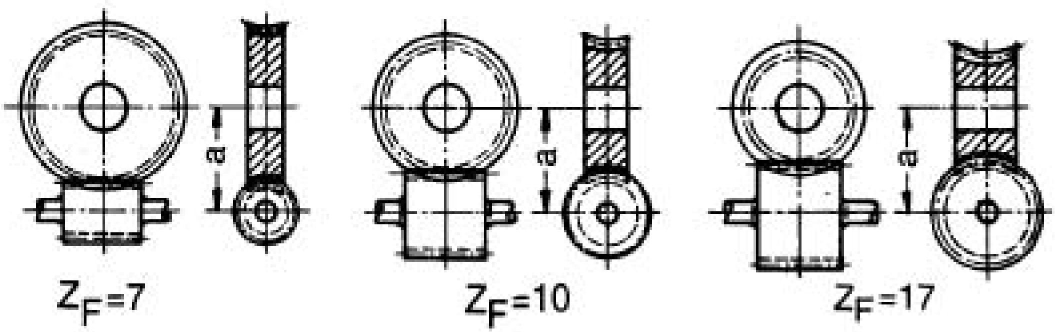


Fig. 4.7 Effect of form number

It can be seen in Fig. **4.7** how the form number affects the same shape of the worm, keeping other parameters of the worm-drives constant, e.g. the centre distance *a*, the transmission ratio *i*, and the number start z_1 = 1. With smaller z_F, the worm diameter is smaller, greater lead angle, greater higher deflection and lower circumferential velocity. With larger value of z, the case is reversed. In practice, z_F lies between 6 and 15. For normal design, an average value of z_F = 10 can be taken.

Addendum, dedendum and whole depth of tooth If the lead angle is small, the addendum and dedendum should be selected in relation to the axial module. Recalling Eq. **4.2**, it can be seen that as the lead angle increases, the axial module also increases for the same normal module. The tooth height also increases. The tooth height for larger lead angle is found with reference to the normal module, as in this way certain unfavourable consequences like peaked tooth on hob as well as on the worm and the wheel can be avoided. Hence, the following practice is generally adopted

$$h_a = m \quad \text{and} \quad h_f = 1.2\,m \quad \text{for} \quad \gamma \le 15^\circ \tag{4.7}$$

$$h_a = m_n \quad \text{and} \quad h_f = 1.2\,m_n \quad \text{for} \quad \gamma > 15^\circ \tag{4.8}$$

$$\text{Tooth height} = h_1 = h_2 = h_a + h_f \tag{4.9}$$

Pressure angle The pressure angles used in worm gearing depend upon the lead angles. The pressure angles must be large enough to avoid undercutting of the gear tooth on the side at which the contact ends. With increased lead angle, the cutting condition of the worm-wheel cutter becomes unfavourable. For manufacturing reasons, greater pressure angles are selected for larger lead angles as shown in Table 4.1. A satisfactory tooth depth is also obtained in the process by choosing the proper pressure angle.

Table 4.1 Recommended pressure angles in relation to lead angle

γ	up to 15°	over 15° up to 25°	over 25° up to 35°	over 35°
γ_n	20°	22.5°	25°	30°

In worm gearing, the pressure angles in the normal and the axial sections have to be differentiated. They bear the following relation

$$\tan \alpha_a = \frac{\tan \alpha_n}{\cos \gamma_m} \tag{4.10}$$

where α_n and α_a are the normal and axial pressure angles respectively.

Tip circle diameter of worm This is given by

$$d_{a1} = d_1 + 2\,h_{a1} \tag{4.11}$$

Root circle diameter of worm This is given by

$$d_{f1} = d_{a1} - 2\,h_1 \tag{4.12}$$

Top clearance Depending upon the type of manufacturing process involved, the value of top clearance (c) is

$$c = 0.167 \text{ to } 0.3\,m \tag{4.13}$$

This value should be as small as possible to enable the worm to have a high moment of resistance and preferred value is $0.2\text{-}0.25\,m$.

Length of Worm The length of worm (b_1) is a function of the magnitude of load as shown

$$b_1 = 2m(1+\sqrt{z_2}) \quad \text{for light to medium duty service} \tag{4.14}$$

$$= 2m\sqrt{2z_2 - 4} \quad \text{for heavy duty service} \tag{4.15}$$

where z_2 = Number of teeth of worm-wheel

Tooth thickness of worm On the middle circle, it is given by

$$s = \frac{\pi m}{2} \cos\gamma = \frac{\pi m_n}{2} \tag{4.16}$$

Pitch circle diameter of worm wheel This is given by

$$d_2 = m z_2 \tag{4.17}$$

For expression regarding the middle circle diameter of the wheel (d_{m2}) for uncorrected and corrected gears, see Sec. 4.7

Throat diameter of worm wheel This is given by

$$d_{a2} = d_2 + 2 h_{a2} \tag{4.18}$$

For corrected gears, the value of h_{a2} should be inserted as per Eqs 4.39 and 4.40 given in Sec. 4.7.

Outside diameter of wheel The value of the outside diameter will depend upon the helix angle

$$d_e = d_{a2} + m \qquad \text{for } \gamma \leq 15^\circ \tag{4.19}$$

$$d_e = d_{a2} + m_n \qquad \text{for } \gamma > 15^\circ \tag{4.20}$$

Root circle diameter of wheel This is given by

$$d_{f2} = d_{a2} - 2h_2 \tag{4.21}$$

Radius of wheel face This is the radius of the circle which forms the surface line of the globoidal worm-wheel, and is given by

$$r = a - \frac{1}{2} d_{a1} \tag{4.22}$$

Helix angle of wheel For proper meshing, the following conditions must be satisfied

Lead angle of worm = Helix angle of wheel
Axial pitch of worm = Circular pitch of wheel

Face width of wheel This is the face to face distance of a worm wheel as measured along the wheel axis. The empirial relations as guidelines for selection are as follows

$$b_2 = 0.4 \text{ to } 0.5 \text{ times } (d_{a1} + 4m) \text{ for bronze wheels} \tag{4.23}$$

$$= 0.4 \text{ to } 0.5 \text{ times } (d_{a1} + 4m) + 1.8\,m \text{ for light metal wheels} \tag{4.24}$$

As per DIN recommendation $b_2 \approx 0.8\, d_1$.

Centre distance The value of the centre distance (a) will depend on the magnitude of correction, if any, and the reader is referred to Sec. 4.7 for its expression.

The dimensional parameters of worm and worm-wheel discussed so far in this section are summarised in Table 4.2

Table 4.2 Dimensional parameters of a worm-drive

Description	***worm***	***Worm-wheel***
Number of teeth	Number of start of worm z_1 Guidelines for selection i: ≥ 30 / 15-29 / 10-14 / 6-9 z_1: 1 / 2 / 3 / 4	$z_2 = iz_1$
Axial module of worm = Transverse module of wheel	$m = \frac{p_a}{\pi} = \frac{m_n}{\cos\gamma} = \frac{p_z}{\pi z_1}$	
Axial pitch of worm = Circular pitch of wheel	$p_a = \pi m = \frac{p_z}{z_1}$	
Lead	$p_z = p_a z_1$	
Lead angle of worm = Helix angle of wheel	$\tan\gamma = \frac{p_z}{\pi d_1} = \frac{m z_1}{d_1} = \frac{z_1}{z_F}$	
Form number of worm	$z_F = \frac{d_1}{m}$	
Diameter of reference circle	Middle circle of diameter of worm = Nominal dimension of worm $= d_1 = d_{m1}$	$d_2 = m z_2$ $(d_{m2} = d_2 \pm 2xm)$
Tip diameter	$d_{a1} = d_1 + 2h_{a1}$	Throat diameter - $d_{a2} = d_2 + 2h_{a2}$
Outside diameter		$d_A = d_{a2} + m\,(\gamma \leq 15^\circ)$ $= d_{a2} + m_n\,(\gamma > 15^\circ)$
Root diameter	$d_{f1} = d_{a1} - 2h_1$	$d_{f2} = d_{a2} - 2h_2$
Reduction ratio	$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{mz_1} = \frac{d_2}{d_1 \tan\gamma}$	
Centre distance	$a = \frac{d_1 + d_2}{2} \pm xm$	
Radius of wheel face		$r = \frac{d_1}{2} - m$
Top clearance	$c = 0.167\,m$ to $0.3\,m$. Preferred value $0.2 - 0.25\,m$	
Addendum For $\gamma \leq 15^\circ$ For $\gamma > 15^\circ$	$h_{a1} = m$ $= m_n$	$h_{a2} = m \pm xm$ $= m_n \pm xm$
Dedendum	$h_{f1} = h_1 - h_{a1}$	$h_{f2} = h_2 - h_{a2}$
Whole depth For $\gamma \leq 15^\circ$ For $\gamma > 15^\circ$		$h_1 = h_2 = 2.2\,m$ $h_1 = h_2 = 2.2\,m_n$

Table 4.2 *(Contd)*

Length of worm	For light or medium load		For bronze wheels	
Face width of wheel	$b_1 \approx 2m\left(1+\sqrt{z_2}\right)$		$b_2 = (0.4 \text{ to } 0.5)\left(d_{a1}+4m\right)$	
	For heavy load $b_1 = 2m\sqrt{2z_2-4}$		For light metal wheel	
			$b_2 = (0.4 \text{ to } 0.5)\left(d_{a1}+4m\right)+1.8m$	
Normal pressure angle (Guiding values)	γ up to 15°	over 15° up to 25°	over 25° up to 35°	over 35°
	α_n 20°	22.5°	25	30°
Axial pressure angle			$\tan\alpha_a = \frac{\tan \alpha_n}{\cos\gamma}$	

Hand of worm A worm may be classified as right-handed or left-handed, depending upon the direction of helix of the worm-thread in which the thread winds around the root cylinder. The worm is said to be "right-handed" if the tooth helix rises from left and goes up towards right as seen from front when the worm is held in an upright position as shown in Fig. 4.5. If the tooth helix veers towards left, then it is a "left-handed" worm. Normally right-handed worms are used in preference to left-handed worms which are used in case of special requirements only.

Direction of rotation in a worm-drive The direction of rotation in a worm and worm-wheel system of drive will depend upon the direction of helix of the worm and the position of worm-wheel. This can be easily seen from Fig. 4.8.

Standardised dimensions of worm gearing To alleviate the tedium of calculation, standard tables (Tables 4.3 and 4.4) have been prepared from which the main parameters of worm and wheel can be obtained. Table 4.3 is based on standard modules as recommended by DIN 780. With a few exceptions, these standard modules by and large tally with those given in IS: 3734. Table 4.4. is also based on the above modules, but here the emphasis is on maintaining standard centre distances. Hence, the worm-wheels are corrected in majority of the cases to attain those standard centre distances.

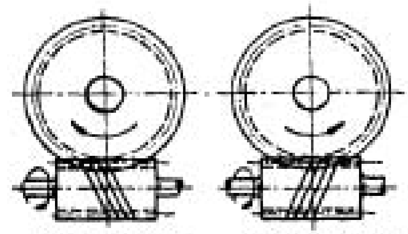


Fig. 4.8 Direction of rotation of worm drive

While designing a worm-drive set, the gear designer should aim at modifying the strength and other calculations involved so that the designed parameters of the components conform to the standard values given in the above tables, without radically hampering the essential design considerations, as far as possible, in normal cases.

Table 4.3 Standard dimensions of worm gearing

Module m	P_x	z_1	z_F	d_m Preferred Series		d_{a1}	d_n	γ_m	Remarks
		1	9.000	—	18	22	13.2	6.3402'	
		1	11.200	22.4	—	26.4	17.6	5.1022'	
		1	14.000	—	28	32	23.2	4.0856'	
		1	17.750	35.5	—	39.5	30.7	3.2245'	SL
2	6.283	2	9.000	—	18	22	13.2	12.5289'	
		2	11.200	22.4	—	26.4	17.6	10.1248'	
		2	14.000	—	28	32	23.2	8.1301'	
		4	9.000	—	18	22	13.2	23.9625'	
		4	11.200	22.4	—	26.4	17.6	19.6538'	
		4	14.000	—	28	32	23.2	15.9454'	
		1	8.480	—	21.2	26.2	15.2	6.7256'	
		1	10.600	26.5	—	31.5	20.5	5.3891'	
		1	13.400	—	33.5	30.5	27.5	4.2670'	
		1	17.000	42.5	—	47.5	36.5	3.3665'	SL
2.5	7.854	2	8.480	—	21.2	26.2	15.2	13.2706"	
		2	10.600	26.5	—	31.5	20.5	10.6847"	
		2	13.400	—	33.5	38.5	27.5	8.4889"	
		4	8.480	—	21.2	26.2	15.2	25.2531"	
		4	10.600	26.5	—	31.5	20.5	20.6742"	
		4	13.400	—	33.5	38.5	27.5	16.6206"	
		1	8.413	—	26.5	32.8	18.9	6.7789'	
		1	10.635	33.5	—	39.8	25.9	5.3717'	
		1	13.492	—	42.5	48.8	34.9	4.2389"	
		1	16.825	53	—	59.3	45.4	3.4011'	SL
3.15	9.896	2	8.413	—	26.5	32.8	18.9	13.3739"	
		2	10.635	33.5	—	39.8	25.9	10.6506"	
		2	13.492	—	42.5	48.8	34.9	8.4317"	
		4	8.413	—	26.5	32.8	18.9	25.4314'	
		4	10.635	33.5	—	39.8	25.9	20.6119'	
		4	13.492	—	42.5	40.8	34.9	16.5113'	
		1	7.075	—	31.5	39.5	21.9	7.2371"	
		1	10.000	40	—	48	30.4	5.7106'	
		1	12.500	—	50	58	40.4	4.5739"	
		1	16.750	67	—	75	57.4	3.4167'	SL
4	12.566	2	7.875	—	31.5	39.5	21.9	14.2500'	
		2	10.000	40	—	48	30.4	11.3100'	
		2	12.500	—	50	58	40.4	9.0902'	
		4	7.875	—	31.5	39.5	21.9	26.9275'	
		4	10.000	40	—	48	30.4	21.8014'	
		4	12.500	—	50	58	40.4	17.7447'	
		1	8.000	—	40	50	28	7.1250'	
		1	10.000	50	—	60	38	5.7106'	
		1	12.600	—	63	73	51	4.5377'	
		1	17.000	85	—	95	73	3.3665'	SL
5	15.708	2	8.000	—	40	50	28	14.0361'	
		2	10.000	50	—	60	38	11.3100'	

(Contd)

Table 4.3 *(Contd)*

Module m	p_a	z_1	z_r	d_m	Preferred Series	d_a	d_f	γ_m	Remarks
		2	12.600	—	63	73	51	9.0196°	
		4	8.000	—	40	50	28	26.5511°	
		4	10.000	50	—	60	38	21.8014°	
		4	12.600	—	63	73	51	17.6125°	
		1	7.937	—	50	62.6	34.9	7.1814°	
		1	10.000	63	—	75.6	47.9	5.7106°	
		1	12.698	—	80	92.6	64.9	4.5028°	
		1	17.778	112	—	124.6	96.9	3.2195°	SL
6.3	19.792	2	7.937	—	50	62.6	34.9	14.1441°	
		2	10.000	63	—	75.6	47.9	11.3100°	
		2	12.698	—	80	92.6	64.9	8.9506°	
		4	7.937	—	50	62.6	34.9	26.7481°	
		4	10.000	63	—	75.6	47.9	21.8014°	
		4	12.968	—	80	92.6	64.9	17.4644°	
		1	7.875	—	63	79	43.8	7.2369°	
		1	10.000	80	—	96	60.8	5.7106°	
		1	12.500	—	100	116	80.8	4.5739°	
		1	17.500	140	—	156	120.8	3.2705°	SL
8	25.133	2	7.875	—	63	79	43.8	14.2500°	
		2	10.000	80	—	96	60.8	11.3100°	
		2	12.500	—	100	116	80.8	9.0902°	
		4	7.875	—	63	79	43.8	26.9278°	
		4	10.000	80	—	96	60.8	21.8014°	
		4	12.500	—	100	116	80.8	17.7447°	
		1	7.500	—	75	95	51	7.5947°	
		1	9.500	95	—	115	71	6.0089°	
		1	11.800	—	118	138	94	4.8439°	
		1	17.000	170	—	190	146	3.3665°	SL
10	31.416	2	7.500	—	75	95	51	14.9314°	
		2	9.500	95	—	115	71	11.8886°	
		2	11.800	—	118	138	94	9.6198°	
		4	7.500	—	75	95	51	28.0670°	
		4	9.500	95	—	115	71	22.8336°	
		4	11.800	—	118	138	94	18.7256°	
		1	7.200	—	90	115	60	7.9072°	
		1	8.960	112	—	137	82	6.3684°	
		1	11.200	—	140	165	110	5.1022°	SL
		1	16.960	212	—	237	182	3.3744°	
12.5	39.270	2	7.200	—	90	115	60	15.5242°	
		2	8.960	212	—	137	82	12.5831°	
		2	11.200	—	140	165	110	10.1248°	
		4	7.200	—	90	115	60	29.5042°	
		4	8.960	112	—	137	82	24.0572°	
		4	11.200	—	140	165	110	19.6536°	

(Contd)

Table 4.3 *(Contd)*

Module m	P_a	z_1	z_F	d_m	d_m Preferred Series	d_{a1}	d_{f1}	γ_m	Remarks
		1	7.000	—	112	144	73.6	8.1300'	
		1	8.750	140	—	172	101.6	6.5198'	
		1	11.250	—	180	212	141.6	5.0795'	
		1	17.500	280	—	312	241.6	3.2705'	SL
16	50.265	2	7.000	—	112	144	73.6	15.9452'	
		2	8.750	140	—	172	101.6	12.8753'	
		2	11.250	—	180	212	141.6	10.0681'	
		4	7.000	—	112	144	73.6	29.7450'	
		4	8.750	140	—	172	101.6	24.5670'	
		4	11.250	—	180	212	141.6	19.5731'	
		1	6.600	—	132	172	84	8.6156'	
		1	8.500	170	—	210	122	6.7097'	
		1	10.600	—	212	252	164	5.3891'	
		1	17.000	340	—	380	292	3.3665'	SL
20	62.832	2	6.600	—	132	172	84	16.8583'	
		2	8.500	170	—	210	122	13.2405'	
		2	10.600	—	212	252	164	10.6847'	
		4	6.600	—	132	172	84	31.2184'	
		4	8.500	170	—	210	122	25.2011'	
		4	10.600	—	212	252	164	20.6742'	

Notes: 1. Left-handed worms are used only in special cases.
2. Self-locking or irreversibility of the worm drive has been indicated by the letters "SL" in the remarks column of the Table. Self-locking is guaranteed only when the worm drive is stationary and is vibration-free.

Table 4.4. Standard centre distance and other parameters of worm gearing

Centre distance a	Transmission ratio i	Module m	d_m	z_1	γ_m	z_2	x	Remarks
	7.25	2.5	26.5	4	20.6742'	29	+0.2000	
	(9.5)	2	22.4	4	19.6536'	38	+0.4000	
	—	—	—	—	—	—	—	
	14.5	2.5	26.5	2	10.6847'	29	+0.2000	
	(19)	2	22.4	2	10.1248'	38	+0.4000	
	—	—	—	—	—	—	—	
50	29	2.5	26.5	1	5.3891'	29	+0.2000	
	(38)	2	22.4	1	5.1022'	38	+0.4000	
	—	—	—	—	—	—	—	
	62	1.25	22.4	1	3.1939'	62	+0.4000	SL
	(83)	1	17	1	3.3664'	83	0.0000	SL
	—	—	—	—	—	—	—	
	7.25	3.15	33.5	4	20.6119'	29	+0.1825	
	(9.75)	2.5	26.5	4	20.6742'	39	+0.4000	
	12.75	2	22.4	4	19.6536'	51	+0.4000	
	14.5	3.15	33.5	2	10.6506'	29	+0.1825	
	(19.5)	2.5	26.5	2	10.6847'	39	+0.4000	
63	25.5	2	22.4	2	10.1248'	51	+0.4000	
	29	3.15	33.5	1	5.3717'	29	+0.1825	

(Contd)

Table 4.4 *(Contd)*

Centre distance *a*	*Transmission ratio* *i*	*Module* *m*	d_m	z_1	γ_m	?	*x*	*Remarks*
	(39)	2.5	26.5	1	5.3891'	39	+0.4000	
	51	2	22.4	1	5.1022'	51	+0.4000	
	61	1.6	28	1	3.2705'	61	+0.1250	SL
	(82)	1.25	22.4	1	3.1939-	82	+0.4400	SL
	109	1	17	1	3.3664'	109	0.0000	SL
	7.5	4	40	4	21.8014'	30	0.0000	
	(10)	3.15	33.5	4	20.6119'	40	+0.0794	
	13.25	2.5	26.5	4	20.6742'	53	+0.2000	
	15	4	40	2	11.3100'	30	0.0000	
	(20)	3.15	33.5	2	10.6506'	40	+0.0794	
	26.5	2.5	26.5	2	10.6847'	53	+0.2000	
80	30	4	40	1	5.7106'	30	0.0000	
	(40)	3.15	33.5	1	5.3717'	40	+0.0794	
	53	2.5	26.5	1	5.3891"	53	+0.2000	
	62	2	35.5	1	3.2245'	62	+0.1250	SL
	(82)	1.6	28	1	3.2705'	82	+0.2500	SL
	110	1.25	22.4	1	3.1939'	110	+0.0400	SL
	7.5	5	50	4	21.8014'	30	0.0000	
	(10)	4	40	4	21.8014'	40	0.0000	
	13	3.15	33.5	4	20.6119'	52	+0.4286	
	15	5	50	2	11.3100'	30	0.0000	
	(20)	4	40	2	11.3100'	40	0.0000	
	26	3.15	33.5	2	10.6506'	52	t0.4286	
100	30	5	50	1	5.7106'	30	0.0000	
	(40)	4	40	1	5.7106'	40	0.0000	
	52	3.15	33.5	1	5.3717'	52	+0.4286	
	63	2.5	42.5	1	3.3665"	63	0.0000	SL
	(82)	2	35.5	1	3.2245'	82	+0.1250	SL
	107	1.6	28	1	3.2705'	107	+0.2500	SL
	7.25	6.3	63	4	21.8014'	29	+0.3413	
	(10)	5	50	4	21.8014'	40	0.0000	
	13	4	40	4	21.8014'	52	+0.2500	
	14.5	6.3	63	2	11.3100'	29	+0.3413	
	(20)	5	50	2	11.3100'	40	0.0000	
	26	4	40	2	11.3100'	52	+0.2500	
125	29	6.3	63	1	5.7106'	29	+0.3413	
	(40)	5	50	1	5.7106'	40	0.0000	
	52	4	40	1	5.7106'	52	+0.2500	
	62	3.15	53	1	3.4011°	62	+0.1111	SL
	(83)	2.5	42.5	1	3.3665'	83	0.0000	SL
	107	2	36.5	1	3.2245'	107	+0.1250	SL
	7.5	8	80	4	21.8014"	30	0.0000	
	(10)	6.3	63	4	21.8014"	40	+0.3968	
	13.5	5	50	4	21.8014'	54	0.0000	
	15	8	80	2	11.3100'	30	0.0000	
	(20)	6.3	63	2	11.3100"	40	+0.3968	
	27	5	50	2	11.3100'	54	0.0000	
160	30	8	80	1	5.7106'	30	0.0000	

(Contd)

Table 4.4 *(Contd)*

Centre distance a	Transmission ratio i	Module m	d_m	z_1	γ_m	z_2	x	Remarks
	(40)	6.3	63	1	5.7106'	40	+0.3968	
	54	5	50	1	5.7106'	54	0.0000	
	63	4	67	1	3.4167'	63	+0.1250	SL
	(84)	3.15	53	1	3.4011'	84	+0.3810	SL
	111	2.5	42.5	1	3.3665'	111	0.0000	SL
	7.5	10	95	4	22.8336'	30	+0.2500	
	(10)	8	80	4	21.8014'	40	0.0000	
	13.25	6.3	63	4	21.8014'	53	+0.2460	
	15	10	95	2	11.8886'	30	+0.2500	
	(20)	8	80	2	11.3100'	40	0.0000	
	26.5	6.3	63	2	11.3100'	53	+0.2400	
200	30	10	95	1	6.0089'	30	+0.2500	
	(40)	8	80	1	5.7106'	40	0.0000	
	53	6.3	63	1	5.7106'	53	+0.2460	
	63	5	85	1	3.3665'	63	0.0000	SL
	(83)	4	67	1	3.4167'	83	+0.1250	SL
	110	3.15	53	1	3.4011'	110	+0.0794	SL
	7.75	12.5	112	4	24.0572'	31	+0.0100	
	(10)	10	95	4	22.8336'	40	+0.2500	
	13	8	80	4	21.8014'	52	+0.2500	
	15.5	12.5	112	2	12.5831'	31	+0.0100	
	(20)	10	95	2	11.8886'	40	+0.2500	
	26	8	80	2	11.3100'	52	4.2500	
250	31	12.5	112	1	6.3684'	31	+0.0100	
	(40)	10	95	1	6.0089'	40	+0.2500	
	52	8	80	1	5.7106'	52	+0.2500	
	61	6.3	112	1	3.2195'	61	+0.2937	SL
	(83)	5	85	1	3.3665'	83	0.0000	SL
	108	4	67	1	3.4167'	108	+0.1250	SL
	7.5	16	140	4	24.5670'	30	+0.3125	
	(10.25)	12.5	112	4	24.0572'	41	+0.2200	
	13.25	10	95	4	22.8336'	53	+0.2500	
	15	16	140	2	12.8753'	30	+0.3125	
	(20.5)	12.5	112	2	12.5831'	41	+0.2200	
	26.5	10	95	2	11.8886'	53	+0.2500	
315	30	16	140	1	6.5198'	30	+0.3125	
	(41)	12.5	112	1	6.3684'	41	+0.2200	
	53	10	95	1	6.0089'	53	+0.2500	
	60	8	140	1	3.2705'	60	0.0000	SL
	(82)	6.3	112	1	3.2195'	82	+0.1111	SL
	109	5	85	1	3.3665'	109	0.0000	SL
	7.75	20	170	4	25.2011'	31	+0.2500	
	(10.25)	16	140	4	24.5670'	41	+0.1250	

(Contd)

Table 4.4 *(Contd)*

Centre distance a	Transmission ratio i	Module m	d_m	z_1	γ_m	z_2	x	Remarks
400	13.75	12.5	112	4	24.0572'	55	+0.0200	
	15.5	20	170	2	13.2405'	31	+0.2500	
	(20.5)	16	140	2	12.8753'	41	+0.1250	
	27.5	12.5	112	2	12.5831'	55	+0.0200	
	31	20	170	1	6.7097'	31	+0.2500	
	(41)	16	140	1	6.5198'	41	+0.1250	
	55	12.5	112	1	6.3684'	55	+0.0200	
	63	10	170	1	3.3665'	63	0.0000	SL
	(82)	8	140	1	3.2705'	82	+0.2500	SL
	109	6.3	112	1	3.2196'	109	+0.1032	SL
500	—	—	—	—	—	—	—	
	(10.25)	20	170	4	25.2011'	41	+0.2500	
	13.25	16	140	4	24.5670'	53	+0.3750	
	—	—	—	—	—	—	—	
	(20.5)	20	170	2	13.2405'	41	+0.2500	
	26.5	16	140	2	12.8753'	53	+0.3750	
	—	—	—	—	—	—	—	
	(41)	20	170	1	6.7097'	41	+0.2500	
	53	16	140	1	6.5198'	53	+0.3750	
	63	12.5	212	1	3.3744'	63	+0.2000	SL
	(83)	10	170	1	3.3665'	83	0.0000	SL
	107	8	140	1	3.2706'	107	+0.2500	SL

Notes: 1. In Table 4.4, SL stands for self-locking as in Table 4.3.
2. The transmission ratios $i = 10$, $i = 20$, $i = 40$ and $i = 80$ are fundamental transmission ratios. These ratios are shown in parenthesis in Table 4.4.

4.4 Worm-Drive with Concave-Convex Profile

The different types of worm which are commonly used, namely, types ZA, ZN, ZK and ZI, have been described in Sec. 4.2. These usual types of worm-profiles are not satisfactory as regards the requirements for a good and close contact between the tooth surfaces of worm and wheel, and for the creation of adequate pressure of the lubricating medium. Favourable operational conditions for the above requirements can be attained by worms with teeth having concave profile instead of the usual straight or convex curvature. This type of profile has been developed by Prof. Niemann and is known as CAVEX type of tooth profile.

The worm-wheel teeth are convex in profile which fit snugly into the corresponding tooth space of the worm during meshing action. Contrary to the usual types of worms where the pitch or working line is situated at the middle of the teeth, in the CAVEX type it lies on the tip of the worm. In other words another speciality of this type of design is that the tip cylinder of the worm has been made to be identical with the pitch or rolling or working cylinder of the worm.

The cutter employed to produce the worm teeth has a convex, circular-arc profile, which gives a concave shape to the worm-tooth profile. The cutter may be a milling cutter or a grinding wheel which has a circular-arc profile in normal section at right angles to the course of the tooth-gap.

Due to the ideal contact conditions between the concave worm-tooth surface and the convex wheel-tooth surface, the CAVEX drive has several advantages. In this respect, this drive has such features which resemble the properties of the internal gearing. The many advantages which this type of design offers are summarised below.

1. Due to favourable contact relations between the concave-convex tooth profiles of the component members of the worm and wheel set, the contact pressure or Hertz stress distribution takes place in a wider area for the same applied tooth force as compared to the conventional worm-drive. Consequently, the surface pressure per unit area is much less.
2. The close contact of flanks helps in the maintaining of a lubricating oil film.
3. The hydroaynamic pressure of the lubricant is greater.
4. Wear and heating as well as the power losses are low.
5. Without weakening the worm-tooth, a high bending resistance and load carrying capacity of the worm-wheel teeth are attained, assuring a high security against breakage of teeth.
6. The efficiency is high, attaining around 96%.
7. The system is impact-damping and practically noiseless.
8. Higher power can be transmittedper unit volume of space required and weight ofmachinery is comparatively lower.
9. Higher transmission ratios can be achieved at relatively low space requirements.
10. Eddy loss in oil is very small at high circumferential velocities.
11. For the same allowable temperature or wear limits, the system can produce more output to the tune of 125 to 150% as against the usual worm toothing conforming to the same conditions.
12. Operational life of this type of gearing is more.
13. The overload capacity is much higher due to the greater root thickness of teeth of the worm-wheel.
14. The CAVEX toothing used in power transmitting gear boxes also ensures extremely high, momentary peak torques without any detrimental effects on the system.

The comparative configurations of the tooth-shapes, pressure distribution patterns and the efficiency charts of the conventional and CAVEX drives are shown in Fig. 4.9.

It has been pointed out before that in CAVEX type of worm-gearing, the working circle of the worm coincides with the tip circle of the worm. This is the normal practice. However, depending upon the design considerations, the diameter of the working circle may have a range which is normally given by

$$\text{Working circle diameter } d_{w1} = (d_1 + m) \text{ to } (d_1 + 2\text{m})$$

where d_1 = The middle circle diameter of the worm

When the upper limit is taken, then

$$d_{w1} = d_1 + 2m = d_{a1} = \text{Tip circle diameter of the worm}$$

Other relevant diameters are

$$d_{f1} = d_1 - 2.32m$$

$$d_2 = mz_2$$

Centre distance $$a = \frac{d_{01} + d_2}{2}$$

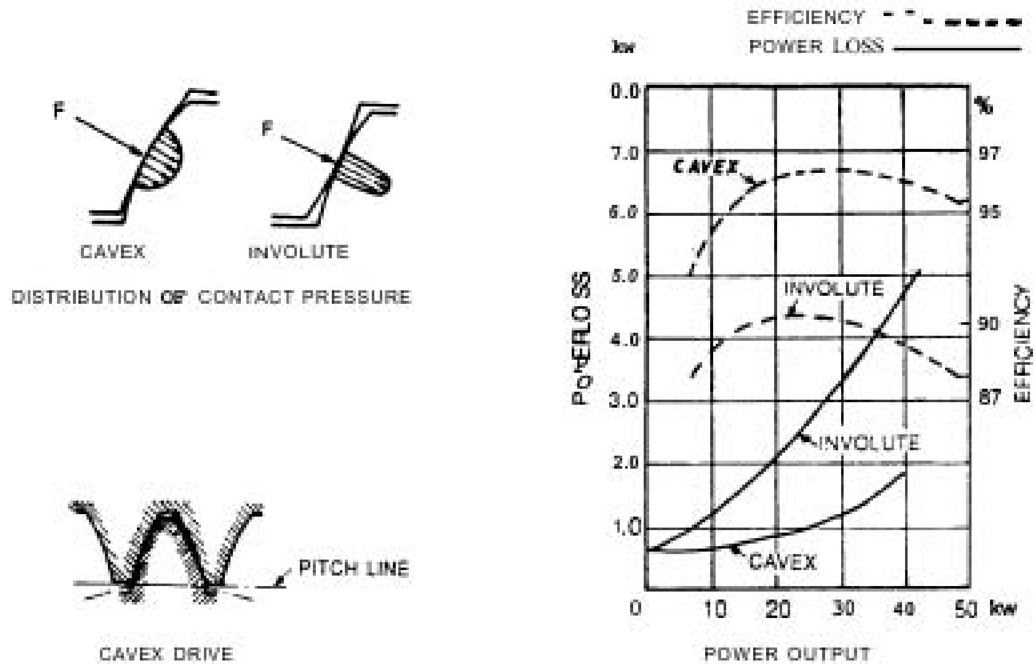


Fig. 4.9 Characteristics of CAVEX drive

4.5 Force Analysis of Worm-Drive

The determination of forces acting on worm and worm-wheel can be done in a similar way as in the case of crossed-helical gears. Neglecting friction for the time being, the only tooth force F_N is the one acting normal to the tooth profile (Fig. **4.10**).

Force F_N can be resolved into three mutually perpendicular component – the tangential component F_t, the radial component F_r, and the axial component F_a.

For the sake of simplification of calculation we assume that the transmission of force takes place only at the pitch point P. In normal section N-N, force F_N, which acts perpendicular to the tooth profile, is inclined at an angle of α_n (the normal pressure angle) as shown in the figure.

If the effect of friction is not taken into account, then the resolved components forces of F_N would be

$$\text{Axial force} = F_N \cos \alpha_n \cos \gamma$$

$$\text{Tangential force} = F_N \cos \alpha_n \sin \gamma$$

$$\text{Radial force} = F_N \sin \alpha_n$$

However, a frictional force is created, which is given by

$$F_{Nf} = \mu F_N$$

where μ = the coefficient of friction = tan ρ and angle ρ is the angle of friction.

This frictional force acts along the worm-tooth surface as shown in Fig. **4.10.** When resolved, it produces two components:

$$\mu F_N \sin\gamma \quad \text{and} \quad \mu F_N \cos\gamma$$

It can be seen from the figure that the axial component of the frictional force acts in the opposite direction to the axial force F_a, whereas the tangential component acts in the same direction as the force F_t. Hence, for a worm driving the gear, the expressions for the different resultant forces effective on the worm are given by

$$\text{Axial force } F_{a1} = F_N \cos\alpha_n \cos\gamma - \mu F_N \sin\gamma \quad (4.25)$$

$$\text{Tangential force } F_{t1} = F_N \cos\alpha_n \sin\gamma + \mu F_N \cos\gamma \quad (4.26)$$

$$\text{Radial force } F_r = F_N \sin\alpha_n = F_{a1} \tan\alpha_a \quad (4.27)$$

The axial pressure angle (α_a) is given by

$$\tan\alpha_a = \frac{\tan\alpha_n}{\cos\gamma}$$

In Eq. 4.27, the term $\mu F_N \sin\gamma$ of Eq. 4.25 is neglected to arrive at the approximate expression. In a worm-drive system, the transmitted load is given by

$$F_{t2} = \frac{T_2}{r_2}$$

Subscripts 1 and 2 refer to the worm and gear respectively as before, T_2 is the output torque available at the gear shaft, and r_2 is the mean radius of the gear. Since the axes of the worm and the gear are at right angles to each other, we have

The total axial force acting on the worm = The transmitted load by the gear

Their absolute values are the same, but the direction or sense is opposite. Therefore, in terms of absolute values, we have from Eq. 4.25

$$F_{t2} = F_{a1} = F_N \cos\alpha_n \cos\gamma - \mu F_N \sin\gamma$$

Similarly

$$F_{t1} = F_{a2} = F_N \cos\alpha_n \sin\gamma + \mu F_N \cos\gamma$$

Therefore, the torque input to the worm is given by

$$T_1 = F_{t1} r_1 \quad (4.28)$$

Taking $\cos\alpha_n \approx 1$ and $\mu = \tan\rho$, we have, for a worm driving the system

$$F_{t1} = F_{a1} \frac{\sin\gamma + \mu\cos\gamma}{\cos\gamma - \mu\sin\gamma}$$

$$= F_{a1} \frac{\sin\gamma + \tan\rho\cos\gamma}{\cos\gamma - \tan\rho\sin\gamma} \times \frac{\cos\gamma}{\cos\gamma}$$

$$= F_{a1} \frac{\tan\gamma + \tan\rho}{1 - \tan\rho\tan\gamma} = F_{a1}\tan(\gamma+\rho) \quad (4.29)$$

$$= F_{t2}\tan(\gamma+\rho) \quad (4.30)$$

With the gear driving the worm, the above equation becomes

$$= F_{t1} = F_{a1} \tan(\gamma - \rho) \quad (4.3)$$

In Fig. 4.10 v_1 is the circumferential velocity of the worm. The sliding velocity is represented by v_s. These are important parameters for calculations connected with the efficiency of worm-drive.

4.6 Bearing Forces in a Worm-Drive

The procedures for the determination of magnitudes of forces acting on a worm-drive system have been discussed in Sec. 4.5. For calculating loads acting on bearings on which the worm-shaft and the wheel-shaft are mounted, the initial step is to calculate the tangential force F_t, the axial force F_a, and the radial force F_r. After ascertaining these forces, we can proceed to find the forces acting on the two bearings of the worm-shaft and the two bearings of the gear-shaft as detailed under.

To determine the bearing forces, it is convenient to consider the different forces as acting on two mutually perpendicular planes—the X-Z plane and the Y-Z plane, as shown in Fig. 4.11. In a worm and wheel system, each component has its own plane of reference containing its axis and the common perpendicular.

From Fig. 4.11, we can arrive at the following expressions for the bearing loads. In each case, subscripts BI and BII refer to bearing Nos. 1 and 2 respectively in both the cases of worm and gear. Subscripts x and y stand for reference planes X-Z and Y-Z respectively. In expressions for forces, subscript 1 refers to worm and subscript 2 refers to the wheel. For dimensions pertaining to lengths, W and G are the subscripts for the worm and the gear respectively.

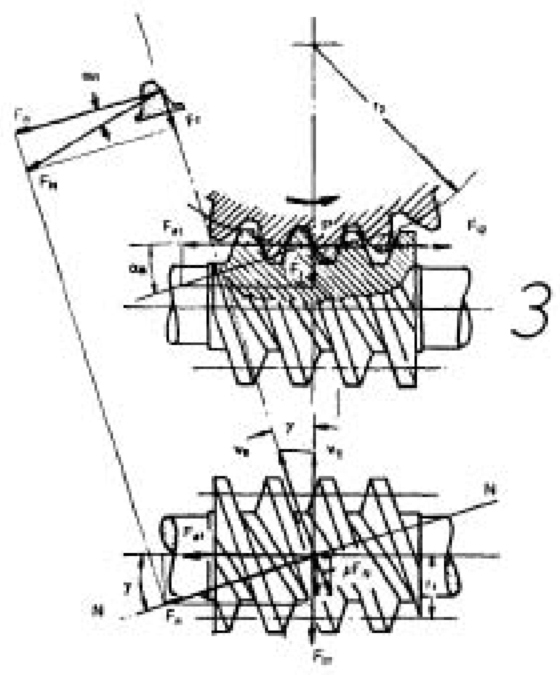


Fig. 4.10 Forces in a worm-drive system

1. Bearing loads on Worm-shaft

$$F_{BIx_1} = \frac{F_{a1}r_1 + F_{r1}L_{W2}}{L_W}$$

$$F_{BIIx_1} = \frac{F_{r1}L_{W1} - F_{a1}r_1}{L_W}$$

$$F_{BIy_1} = \frac{F_{t1}L_{W2}}{L_W}$$

$$F_{BIIy_1} = \frac{F_{t1}L_{W1}}{L_W}$$

"he resultant radial loads on the bearings are given by

$$F_{BI_1} = \sqrt{\left(F_{BIx_1}\right)^2 + \left(F_{BIy_1}\right)^2} \tag{4.32}$$

$$F_{BII_1} = \sqrt{\left(F_{BIIx_1}\right)^2 + \left(F_{BIIy_1}\right)^2} \tag{4.33}$$

Besides the above radial forces, the worm-shaft bearings are subjected to the axial thrust of magnitude F_{a1}.

2. Bearing loads on gear-shaft

$$F_{BIx_2} = \frac{F_{a2}r_2 + F_{r2}L_{G2}}{L_G}$$

$$F_{BIIx_2} = \frac{F_{a2}r_2 - F_{r2}L_{G1}}{L_G}$$

$$F_{BIy_2} = \frac{F_{t2}L_{G2}}{L_G}$$

$$F_{BIIy_2} = \frac{F_{t2}L_{G1}}{L_G}$$

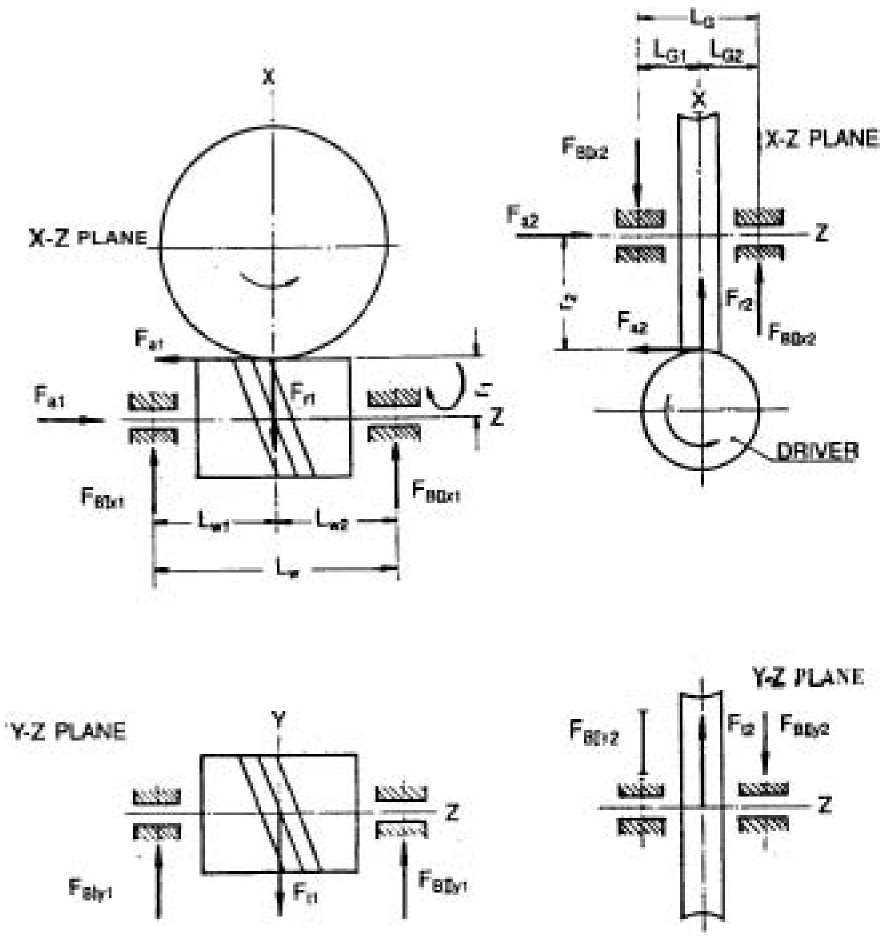
The resultant radial loads on the bearings are given by

$$F_{BI_2} = \sqrt{\left(F_{BIx_2}\right)^2 + \left(F_{BIy_2}\right)^2} \tag{4.34}$$

$$F_{BII_2} = \sqrt{\left(F_{BIIx_2}\right)^2 + \left(F_{BIIy_2}\right)^2} \tag{4.35}$$

Besides the above radial forces, the gear-shaft bearings are also subjected to the axial force of magnitude F_{a2}.

For the proper selection of the anti-friction bearings, the relevant manufacturer's catalogues should be consulted. These bearings should be capable of withstanding both the radial and axial loads. After the determination of these loads, their values should be inserted in the formulae contained in those catalogues to determine the equivalent loads. After that, the proper selection



(a) Worm shaft bearing forces

(b) Worm gear bearing forces

Fig. 4.11 Bearing forces in a worm drive

of the bearings can be made. These aspects are elaborately dealt with in the catalogues and manuals of standard bearing manufacturers.

To facilitate the appropriate selection of bearings, the designer is recommended to have easy access to such catalogues. One example of bearing selection is given in Sec. 3.13 to illustrate the framework of selection.

4.7 Correction in Worm-Drive System

Like spur and helical gears, profile correction becomes necessary to avoid undercutting in case of worm-wheels with small number of teeth, the relevant relations being similar which are given as follows.

Minimum number of teeth to avoid undercutting

$$z_{min} = \frac{2}{\sin^2 \alpha} \tag{4.36}$$

As indicated earlier, the pressure angle is a function of the lead angle chosen, and it should vary according to the magnitude of the lead angle for proper functioning of the worm-drive system. The pressure angle is normally 20° for general application with usual values of lead angles. The values of the minimum number of teeth are

$$z_{min} = 17 \text{ for } \alpha = 20°$$
$$= 30 \text{ for } \alpha = 15°$$

The profile correction factor is given by

$$x = 1 - \frac{z_2}{z_{min}} = 1 - \frac{z_2 \sin^2 \alpha}{2}$$

The amount of profile correction, therefore, is

$$xm = \left(1 - \frac{z_2 \sin^2 \alpha}{2}\right) m \tag{4.37}$$

Referring to Fig. 4.6, the relation of the middle circle diameter of the worm-wheel d_{m2} and the pitch circle diameter of the worm d_1 (sometimes called the middle circle diameter of the worm) in a corrected gearing is given by

$$d_{m2} = 2a - d_1$$

or the centre distance

$$a = \frac{d_{m2} + d_1}{2} = \frac{(d_2 + 2xm) + d_1}{2} = \frac{d_1 + d_2}{2} + xm \tag{4.38}$$

Equation 4.38 is the expression for the corrected centre distance. If there is no correction, then obviously $d_{m2} = d_2$. Therefore, the profile correction xm is the difference between the middle circle radius and the pitch circle radius of the worm-wheel. The correction is positive if the middle circle is greater than the pitch circle and is negative when it is smaller.

The following practice is generally adopted for calculation of the worm-wheel addendum in case of wheel with correction

$$h_{a2} = m \pm xm \quad \text{for } \gamma \leq 15° \tag{4.39}$$

$$= m_n \pm xm \quad \text{for } \gamma > 15° \tag{4.40}$$

4.8 Deflection of Worm-Shaft

In a worm and worm-wheel drive system, the worm is subjected to bending and deflection. Since perfect contact relations and tooth action are possible only in case of precise bearing mountings and accurate toothing, and since such conditions are not practically attainable, some amount of deformation during service has to be allowed, the magnitude of which is based on the experience of the designer and the judicious use of the empirical relations. It goes without saying that this permissible value should not be exceeded.

It is desirable to have comparatively large diameter of the worm-shaft, and the distance between the bearings should be kept as small as possible commensurate with the other design parameters, so that the least amount of change of shape takes place. In this respect, the form number of worm, (Sec. 4.3) plays a major role and, therefore this factor should be carefully chosen. The following relations are relevant for calculation of deflection.

From mechanics, we can establish the following formula.

$$f = \frac{F_1 L_W^3}{48EI} \tag{4.41}$$

where f (mm) = The deflection of the worm shaft

L_W (mm) = The distance between the bearings holding the worm shaft. It is assumed here that the worm is mounted at the middle of this distance (see Fig. 4.11 Sec. 4.6)

For rough calculation L_W = 1.5x centre distance a

$F_1 (N)$ = The vector sum of the tangential force F_{t1} and the radial force F_{r1}. This resultant force (F_1) which deflects the worm shaft, is given by

$$F_1 = \sqrt{(F_{t1})^2 + (F_{r1})^2} \tag{4.42}$$

Symbol E (N/mm²) is the modulus of elasticity of the worm material and I (mm⁴) is the area moment of inertia of the cross-section of the worm shaft. Here, I is taken to be approximately constant throughout the length of the shaft.

Permissible deflection is given by

$$f_p = \frac{d_{m1}}{1000} \tag{4.43}$$

The following factor of safety against deflection (S_d) should be adhered to during design

$$S_d = \frac{f_p}{f} \geq 1 \tag{4.44}$$

4.9 Bending Stress Calculations

In a worm-drive system, the worm-wheel teeth are to be checked against bending. The factor of safety against bending is given by

$$S_b = \frac{\sigma_{lim}}{\sigma_{max}} = \sigma_{lim} \frac{\pi m_x \hat{b}}{F_{t2max}} \tag{4.45}$$

The value of S_b should be equal to or more than 1. Here

σ_{lim} (N/mm²) = Limiting value of allowable load factor, as given in Table 4.5 for different types of wheel materials commonly in use

σ_{max} (N/mm²) = Maximum bending stress which may occur during operation

F_{t2max} (N) = Maximum tangential force which may act on the wheel when referred to the middle circle diameter

$\hat{b}$ (mm) = Arc length of tooth width of wheel at root $= r' \pi \frac{\phi}{180°}$

where $r' = \frac{d_{f1}}{2} + c$ and angle ϕ (in degree) is given by $\sin \frac{\phi}{2} = \frac{b_2}{2r'}$

Table 4.5 Limiting values of load factor for different wheel materials
(Materials of worm is steel in each case)

Material of wheel	Type of worm profile		
	ZA and ZN	ZK and ZI	CAVEX
		σ_{lim} (N/mm²)	
Centrifugally cast Cu–Sn bronze	23.5	29.4	39.2
Aluminium alloys	11.3	14.0	18.6
Al–Si alloy	7.5	9.3	12.5
Zn alloys			
Cast iron	11.8	14.7	19.6

4.10 Contact Stress Calculations

In designing a worm-drive, it is of vital importance to check the system against failure due to the detrimental effect of contact stress. The factor of safety in this case is given by

$$S_c = \frac{p_{lim}}{p} \tag{4.46}$$

where p_{lim} (N/mm²) = Limiting value of contact pressure (Values for common materials are given in Table 4.6)

p(N/mm²) = Actual contact pressure created in the system

The expression for the actual contact pressure is as follows

$$p = \frac{F_{t2}}{d_{m1} d_{m2} K_c} \tag{4.47}$$

The value of S_c should normally lie between 0.6 to 2.2, depending upon the allowable wear.

Factor K_s takes into account the effect of the middle lead angle, its values are given in Table 4.7.

Table 4.6 Limiting values of contact pressure

Based on Zahnraeder, Zirpke, 11th edition, 1980, table no. 23, p. 344, VEB Fachbuchverlag, Leipzig.

Material of wheel	Steel worm	
	Unhardened	Hardened and ground
	p_{lim} (N/mm²)	
Centrifugally cast Cu–Sn bronze	3.5	5.9
Aluminium alloy	1.5	3.1
Al–Si alloy	—	3.3
Zn alloy	1.3	—
Cast iron	1.8	2.9

Table 4.7 Factor K_c In relation to lead angle

Based on Zahnraeder, Zirpke, 11th edition, 1980, table no. 24, p. 344, VEB Fachbuchverlag, Leipzig.

Profile type	$\tan \gamma_m$ =	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.0	0.9
ZA, ZN, ZK and ZI	K_c =	0.41	0.36	0.32	0.29	0.265	0.248	0.233	0.223	0.215	0.213
CAVEX (concave-convex)	K_c =	0.40	0.40	0.46	0.445	0.433	0.425	0.420	0.417	0.415	0.415

4.11 *Effect of* Heat Generation

The relative sliding action between the teeth of worm and wheel causes generation of a considerable amount of heat. Particularly in heavy duty worm-drives, arrangement for heat dissipation should be provided. This is normally achieved by providing fans which are fitted inside the gear box housing or the housing may be so cast as *to* have cooling ribs or both the above measures can be taken.

The effect of heat generation is an important design criterion for the worm-drive. Some researchers in the field have developed a set of empirical relations which determine the temperature safety factor for normal operation of the drive. The relevant equations are given as follows

Temperature safety factor

$$S_t = \left(\frac{a}{10}\right)^2 \frac{K_1 K_2 K_3 K_4}{136\, P_1} \tag{4.48}$$

The value of S_t should be equal to or greater than 1 for smooth service. As before, **a** is the centre distance in mm and P_1 is the transmitted power of the worm in kW. By transposing Eq. 4.48, we get the required minimum value of the centre distance

$$a = 10\sqrt{\frac{136 P_1}{K_1 K_2 K_3 K_4}} \tag{4.49}$$

Factors K_1 to K_4 take care of the influences of type of cooling, transmission ratio, material and design of the drive. Factor K_1 is given by

$$K_1 = \left(1 + \frac{K_F}{1+K_F}\right)\left(\frac{100}{D_F} + K_F\right) \tag{4.50}$$

Equation 4.50 is valid for worm-drives which are situated in a place with sufficient air circulation. The fan factor K_F is given by the following equations with worm rotating at a speed of n_1 rpm.

For drive without fan

$$K_F = 1.4\sqrt[3]{\left(\frac{n_1}{1000}\right)^2} \tag{4.51}$$

For drive with fan

$$K_F = 3.1 \sqrt[3]{\left(\frac{n}{1000}\right)^2} \tag{4.52}$$

The duty factor D_F is expressed as per cent of running time per hour. Thus, $D_F = 20$ if the drive runs for an average time of 12 min/hr, its value will be 100 for a continuous drive system.

For a system where the worm is the driving member, the value of K_2 can be taken from Table 4.8.

The material factor K_3 will depend on the material pairings. This is given in Table 4.9 for cylindrical worms belonging to the types given in Sec. 4.2.

Table 4.8 Values of K_2 vis-a-vis transmission ratio i

Based on Zahnraeder, Zirpke. 11th edition, 1980, table no. 20, p. 340, VEB Fachbuchverlag, Leiptig.

i =	5	7.5	10	15	20	25	30	40	50	60
K_2 =	1.16	1.10	1.00	0.81	0.68	0.59	0.52	0.41	0.32	0.28

Table 4.9 Material factor K_3

Based on Zahnraeder, Zirpke, 11th edition, 1980, table no. 21, p. 343, VEB Fachbuchverlag, Leizig.

Material		K_3
Worm	*Wheel*	
Steel, hardened and ground	Centrifugally cast Cu–Sn bronze	1.0
	Aluminium alloys	0.87
	Cast iron	0.80
Steel hardened and tempered, not ground	Cu–Sn bronze	0.67
	Zinc alloys	0.67
	Aluminium alloys	0.58
	Cast iron	0.55
Cast iron, not ground	Centrifugally cast Cu–Sn bronze	0.87
	Cast iron	0.80

For worm-drive with concave-convex profile described in Sec. 4.4, the value of K_3 is modified, depending on the sliding velocity v_s. The value of K_3, as given in Table 4.9, is to be multiplied by the factor f given in Table 4.10.

Table 4.10 Factor f in relation to sliding velocity v_s

v_s (m/sec) =	0.1	0.5	1.1	2.0	4.0	8.0	12.5	16.0
f =	1.12	1.19	1.25	1.33	1.47	1.61	1.67	1.70

Factor K_4 will depend upon the type of design of the system. It is given in table 4.11.

Table 4.11 Design type factor K_4

Based on Zahnraeder, Zirpke, 11th edition, 1980, table no. 22, p. 343 VEB Fachbuchverlag, Leipzig.

Type of design	K_4
Worm at the bottom with worm supplying the lubricant	1.0
Other configuration of worm with wheel supplying the oil	0.6
Additional oil cooling arrangement provided, e.g., spray	above 1.0

4.12 Design Criteria of Worm-Drive

In designing a worm-drive, many disparate factors are taken into consideration. The designer should be fully conversant with these factors and should take decision according to his discretion. In this section, we shall deal with the factors involved and the guidelines thereof for proper selection of the parameters of the drive.

To simplify elaborate calculation procedures and to achieve standardisation of worm-drives, dimensional standard tables have been made which are based after taking into consideration the common design factors (Tables 4.3 and 4.4 in Sec. 4.3). In these tables relevant data, such as the module, transmission ratio, number of teeth, centre distance, correction factor required and other aspects like diameters, lead angles and self-locking possibilities have been compiled.

Material

The materials for worm and wheel should be carefully selected after considering operational conditions, scope of the drive, ensuing power losses and other factors. For normal power transmission, steel worms and phosphor bronze wheels are extensively used. Worms are generally hardened and ground.

The materials commonly used for worm and wheel, their properties and combinations have been given in Table 4.12.

Table 4.12 Materials and stresses for worm gearing

Symbol	Material	Condition	UTS σ_{ut} N/mm²	Hardness HB N/mm²	Allowable stress σ_p N/mm²	Allowable pairing with	Remarks
			Materials for worm				
A	Case hardending steel	Hardened	520 (crore)	6000 (case)	—	1, 2 and 3	Flanks ground
B	Heat treatable steel	Heat treated	600-900	1800-2080	—	1 and 2	Flanks cut on lathe

Materials for worm-wheel

1	Phosphor bronze	Sand casting	190	600-950	80	A and B	
2	Cast iron	Sand casting	220-260	1970-2410	55	A and B	Sliding velocity up to 3 m/s
3	Special Al-alloy	Chilled casting	200-220	950-1050	40	A	—

Worm-wheels are also made of following materials.

Centrifugally cast phosphor bronze for heavy duty wheels, high contact pressure, high wear resistance, high impact resistance. UTS = 320 N/mm², hardness = HB 1150 N/mm², σ_p = 120 N/mm².

Zinc alloys for low temperature. UTS = 220 N/mm², HB = 800 N/mm², σ_p = 40 N/mm². Synthetic material for low velocity (< 2 m/s). UTS = 150 N/mm², HB = 350 N/mm², σ_p = 35 N/mm².

Determination of Worm-Drive Parameters

Several limiting factors come into consideration while designing a worm-drive set. These are: (*i*) Deflection of worm shaft, (*ii*) bending stress developed in wheel teeth, (*iii*) contact stress developed, and (iv) Heat developed.

These factors have already been discussed in Secs 4.8 to 4.11. For designing the pair, the stress which necessitates the selection of the biggest dimension is the ultimate deciding factor.

Surface stress, sliding velocity and sliding properties of paired materials, viscosity and efficacy of the lubricant are the factors which influence wear. Wear is caused by the rubbing action between the mating surfaces or as a result of contact pressure which may lead to pitting. In the sphere of boundary lubrication, wear caused by rubbing is unavoidable. In hydrodynamic lubrication, there is no metal to metal contact. Such condition is possible if the sliding velocity between the mating surfaces is sufficiently large. However, due to continuous loading and unloading of the teeth, fatigue failures take place which are apparent on the tooth surfaces in the form of pitting, serrations and flaking of the tooth materials.

Load rating of worm-drive using formulae given in IS: 7443 will be discussed in Sec. 4.13. This section outlines how worm and wheel parameters are derived from the first principles. The following example illustrates the procedures involved.

Example 4.1 *Given:* $n_1 = 960$ rpm, $i = 52$, output torque $T_2 = 2000$ Nm, material of worm heat treatable steel, material of wheel phosphor bronze, lubrication by pressurised oil.

To find the relevant dimensions of the components of the worm-drive.

Solution: The following strength-based formula takes care of the limiting factors mentioned before

$$F_{t2}(N) = c\,b\,p_a \tag{4.53}$$

where

$$b\,(\text{mm}) = \text{The useful width of wheel} \approx 2m\sqrt{z_v + 1} \tag{4.54}$$

Recalling equations from Sec. 4.3, we have $z_v = d_1/m = z_1/\tan\gamma_m$, and $p_a = \pi m$.

Factor c depends upon service conditions, circumferential velocity, material, and it takes into consideration the effects of wear, heat generated and other limiting factors as stated before: The value of c is to be take from Fig. 4.12. The following points are to be noted.

1. In Fig. 4.12, the solid curves are valid for normal, continuous service and splash lubrication.
2. The dotted curves are meant for oil cooling and pressurised oil lubrication.
3. For intermittent service, c may be taken at some intermediate point between solid and dotted curves proportionately.
4. In Fig. 4.12 c is valid for $z_2 = 30$. For other number of teeth., multiply this value of c by factor f_z given in Fig. 4.13.
5. Table 4.12 is to be read along with Fig. 4.12 where combinations of worm and wheel of different materials are given. Thus, for example, combination 1 B means a worm of symbol B and a wheel of symbol 1.

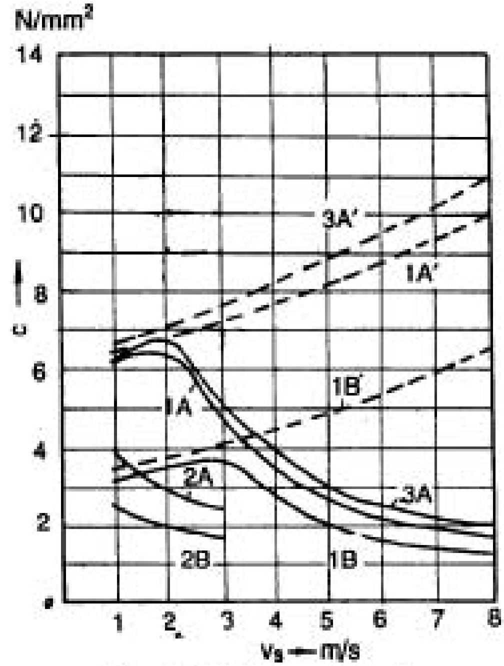


Fig. 4.12 Values of factor c

Assuming about 76% efficiency of the set, we have

Input power of the worm $P_1 = \frac{T_2}{9550} \times \frac{n_1}{i} \times \frac{1}{0.76} = 5\text{kW}$

Module

$$m = 10 \sqrt[3]{\frac{1933 P_1 \eta}{c \Delta z_1 n_1}} \tag{4.55}$$

The value of factor Δ ($= b/p_a$) usually lies around 2 to 2.5. The following data are taken

$z_1 = 1$, sliding velocity $v_s = 4$ m/s, $\Delta = 2.1$ Since $i = 52$, $z_2 = z_1 i = 52$

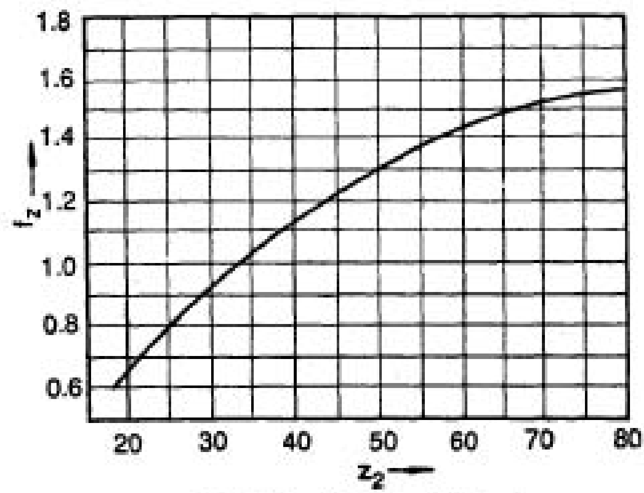


Fig. 4.13 Values of factor f_z

Based on Die Trafaehigkeit der Zahnraeder, Thomas and Charchut, 7th Edition, 1971, fig. no. 80, p. 187. Carl Hanser Verlag, Munich

Material combination 1B is taken, whence we get from Fig. 4.12 corresponding to curve 1B' and from Fig. 4.13 corresponding to $z_2 = 52$, $c \approx 6.3$. Now, using Eq. 4.55

$$m = 10\sqrt[3]{\frac{1933\times5\times0.76}{6.3\times2.1\times1\times960}} \approx 8\,\text{mm (taken)}$$

As

$$A = 2.1 = b/P_a = b/\pi m,\quad b = 2.1\times3.14\times8 = 52.8\text{ mm}$$

Also

$$b = 2m\sqrt{z_F+1} \qquad \text{or } 52.8 = 2\times8\sqrt{z_F+1} \text{ or } z_F = 9.89 \approx 10$$

$$z_F = d_{m1}/m = z_1/\tan\gamma_m = 10 \text{ whence } d_{m1} = 80\text{mm}$$

and

$$\gamma_m = 5.7106' = 5' \, 42' \, 38.14''$$

Since $m = 8 = 0.1 \times d_{m1}$, this tallies with the average value relation given in Sec. 4.3. Using Eq. 4.75 and Table 4.15 given in Sec. 4.14 on efficiency, we have

$$\mu = \frac{0.051\times y}{\sqrt{0.4+v_s}} = \frac{0.051\times 1.5}{\sqrt{0.4+4}} \approx 0.03 \text{ (taken)}$$

$\eta = \tan\gamma_m / \tan(\gamma_m + \rho)$ whence ρ = 171836' also $\tan\rho = \mu$,

Putting the relevant values, we get $\eta = 0.767$, which more or less tallies with our original assumption

$d_2 = 8 \times 52 = 416$ mm Centre distance $a = (80 + 416)/2 = 248$ mm.

Taking the standard centre distance of 250 mm and resorting to positive correction, we have

$$d_{a2} = d_2 + 2m + 2xm = 416 + 16 + 16x = 416 + 16 + 2(250 - 248)$$

$$\text{or} \quad 16x = 4 \quad \text{or} \quad x = 0.25$$

Other dimensions can be fixed by consulting Tables 4.3 and 4.4. To match the set with the standard drive as given in those tables, certain approximations of the calculated values have been made. Rigid adherence to calculated values lead to somewhat different parameters.

Check: From Eq. 4.48 (Sec. 4.11)

$$S_t = \left(\frac{a}{10}\right)^2 \frac{K_1 K_2 K_3 K_4}{136 P_1}$$

$$K_p = 3.1 \sqrt[3]{\left(\frac{960}{1000}\right)^2} = 3 \quad K_1 = \left(1 + \frac{3}{4}\right)\left(\frac{100}{100} + 3\right) = 7$$

(taking 100% duty factor)

$$K_2 = 0.31 K_3 = 0.67 K_4 = 1.0$$

Putting the values, we get $S_t = 1.33 > 1.0$

From Eq. 4.46 (Sec. 4.10) $S_c = p_{lim}/p$. The value of p_{lim} is taken as 1.5

$$p = F_{t2}/d_{m1}\, d_{m2}\, K_c \quad \text{Inserting values } F_{t2} = 6.3 \times 52.8 \times 3.14 \times 8 = 8356 \text{ N}$$

$$K_c = 0.36, \; d_{m1} = 80, \; d_c = 416 + 4 = 420$$

Therefore

$$S_c = 2.17, \text{ which lies within range}$$

From Eq. 4.45 (Sec. 4.9)

$$S_b = \sigma_{lim}/\sigma_{max} = \sigma_{lim} \pi m_n b / F_{t2max}$$

$$b = r' \pi \frac{9}{180°}, \quad r' = \frac{d_{a1}}{2} + c = \frac{80}{2} + 2 = 42$$

$$\sin\frac{\phi}{2} = \frac{b_2}{2r'} = \frac{52.8}{84}, \text{ whence } \phi = 78'$$

As per Eq. 4.23 (Sec. 4.3) $b_2 = 0.4$ to 0.5 times $(d_{a1} + 4m)$

If the higher value is taken, then $b_2 = 64$ mm. If $b_2 = b = 52.8$ is taken, then also the coefficient lies between 0.4 and 0.5.

$$b = 42 \times 3.14 \times 78 / 180 = 57.2 \text{ mm}$$

Inserting the values, we get

$$S_b = 12 \frac{3.14 \times 8 \times \cos 5.7106° \times 57.2}{8356} = 2.05 > 1$$

From Eq. 4.44 (Sec. 4.8) $S_d = f_p/f$

$$f_p = d_{m1}/1000 = 80/1000 = 0.08 \text{ mm}, \; f = F_1 L_w^3 / 48EI.$$

Taking worm shaft diameter = 50 mm, $I = \frac{\pi}{64} \times 50^4 = 306800 \text{ mm}^4,$

$$E = 206000\ \text{N/mm}^2,\ L_w \approx 1.5a = 1.5 \times 250 = 375\ \text{mm},\ F_1 = \sqrt{(F_{t1})^2 + (F_{r1})^2}$$

Referring to **Eq. 4.30** (Sec. **4.5**) we have

$$F_{t1} = F_{t2} \tan(\gamma + \rho) = 8356 \times \tan 7.42896^\circ = 1090$$

$$F_{r1} = F_N \sin \alpha_n \approx F_{a1} \tan \alpha_x = F_{t2} \tan \alpha_n / \cos \gamma = 8356 \tan 20^\circ / \cos 5.7106^\circ = 3056$$

F_{t1} can also be found by using the formula

$$F_{t1} = 2000\, T_1 / d_{m1} = (2000/80) \times 9550 \frac{P_1}{n_1} = 25 \times 9550 \times \frac{5}{960} = 1243\ \text{N}$$

The difference in the two values of F_{t1} stems from the fact that many assumptions and approximations have been made.

As

$$F_1 = \sqrt{(1090)^2 + (3056)^2} = 3244$$

$$f = \frac{3244 \times 375^3}{48 \times 206000 \times 306800} = 0.056$$

Therefore

$$S_d = f_p / f = 0.081/0.056 = 1.43 > 1$$

The above design, therefore, is safe against all the conventional failures as the above check methods show. Remaining dimensions of the set can be calculated by using the various formulae and tables given in Sec. **4.3.**

4.13 Load Rating of Worm-Drive

The code for power rating of worm and worm-wheel system of drive has been laid down in IS: **7443.** Since in this Indian Standard the unit of force is kgf, this unit is used as such, as in the case of IS: **4460** dealing with the method of load rating for spur and helical gears discussed in Sec.2.25. For conversion of kgf into the SI unit of force, see Table in Appendix T. Also, the same symbols and subscripts for the different parameters have been mainly maintained as given in the IS. The explanations of notations which have been previously used in this chapter and elsewhere are not repeated here.

Symbols

y_z = Zone factor as per Table **4.13**

x_{c1}, x_{c2} = Speed factors for worm and worm-wheel; these correspond to the combination of rotational speed and rubbing speed (see Fig. **4.14**)

x_{b1}, x_{b2} = Speed factors for strength, these correspond to the rotational speed only (see Fig. **4.15**)

S_{c1}, S_{c2} = Surface stress factors, these depend on the combination of materials and are given in Table **4.14**

S_{b1}, S_{b2} = Bending stress factors, these correspond to the material used and are given in Table **4.14**

l_r = Length of root of worm-wheel teeth

Formulae

$$\text{Rubbing speed } v_s = \frac{\pi d_{m1} n_1 \sec \gamma}{60,000} \text{ (m/s)} \quad (4.56)$$

(v_s should not exceed 12.5 m/s)

Normal rating This is defined as the rating corresponding to a total running period of **26,000** hours when the system is subjected to a running time of **12** hr/day. For any other life period, the following formulae should be used which give the multiplication factors K_w and K_s to be applied to the permissible power or worm-wheel torque T_2 as found by using the formulae given later

$$K_w = \left(\frac{27,000}{1000+t_{ew}}\right)^{1/3} \text{ for wear} \quad \mathbf{(4.57)}$$

$$K_s = \left(\frac{26,200}{200+t_{eb}}\right)^{1/7} \text{ for strength} \quad \mathbf{(4.58)}$$

Factors t_{ew} and t_{eb} are total equivalent running time in hours. For factors involving shock loads, see IS: **7403.**

Permissible torque for wear The output torques of the worm-wheel in normal running conditions are determined by relations **4.59** and **4.60.**

$$0.19\, x_{c1}\, S_{c1}\, y_z\, m d_{m2}^{1.8} \quad \text{(kgf cm)} \quad (4.59)$$

$$0.19\, x_{c2}\, S_{c2}\, y_z\, m d_{m2}^{1.8} \quad \text{(kgf cm)} \quad \mathbf{(4.60)}$$

Permissible torque for strength The corresponding torques are given by

$$0.18\, x_{b1}\, S_{b1}\, m l_r\, d_{m2} \cos \gamma \quad \text{(kgf cm)} \quad \mathbf{(4.61)}$$

$$0.18\, x_{b2}\, S_{b2}\, m l_r\, d_{m2} \cos \gamma \quad \text{(kgf cm)} \quad (4.62)$$

In both of the above cases, the lower of the two values is to be taken for calculation purposes.

The length of root of worm-wheel tooth is given by

$$l_r = (d_{a1} + 2c) \sin^{-1}\left(\frac{b}{d_{a1}+2c}\right) \quad \mathbf{(4.63)}$$

where

$$b = \text{Useful width of the worm - wheel } = 2m\sqrt{z_p + 1} \quad \text{(cm)} \quad \mathbf{(4.64)}$$

Power rating For normal service conditions, the power is given by

$$P = \frac{T_2 n_2}{97,442} \text{ (kW)} \quad (4.65)$$

Here, T_2 is the smallest of the four worm-wheel torque values as determined by the above four relations (Eqs **4.59-4.62).**

Equivalent running time When the load is not steady, the following formulae should be used for determining the equivalent running time.

$$t_e = t_1 + t_2\left(\frac{n_2'}{n_1'}\right)\left(\frac{T_2'}{T_1'}\right)^3 + t_3\left(\frac{n_3'}{n_1'}\right)\left(\frac{T_3'}{T_1'}\right)^3 + \ldots \quad (4.66)$$

Here

t_e = Equivalent running time in hours per cycle for wear at torque T_1' and n'_1

T'_1 = Maximum torque acting for t_1 hours at a mean speed of n'_1

n'_1 = Mean speed during which T'_1 acts

Other symbols with their respective subscripts stand for smaller torques effective during the period in question. The relevant formula for strength is similarly given by

$$t_b = t_1 + t_2\left(\frac{n_2'}{n_1'}\right)\left(\frac{T_2'}{T_1'}\right)^7 + t_3\left(\frac{n_3'}{n_1'}\right)\left(\frac{T_3'}{T_1'}\right)^7 + \ldots \quad (4.67)$$

The total equivalent running times in hours at torque T'_1 and speed n'_1 are given by

For wear $t_{ee} = t_e$ x total number of cycles during the expected life (4.68)

For strength $t_{eb} = t_b$ x total number of cycles during the expected life (4.69)

In the above equations, since the torques T appear as ratios, they can be replaced by the loads or forces F, giving the same results.

Example 4.2. *Given:* $z_1 = 2$, $z_2 = 30$, $m = 10$, centre distance $a = 200$ mm, $n_1 = 1000$ rpm. To find the power rating required for the system for a total equivalent running time of 40,000 hours.

Solution: To arrive at the standardised values for the two components of the drive, data from the tables 4.3 and 4.4 will be used.

Materials selected:

Case hardened carbon steel for worm

Centrifugally cast phosphor bronze for wheel

Transmission ratio $i = 30/2 = 15$, Output speed = Speed of wheel, $n_2 = \dfrac{1000}{15} = 66.7$

Right-handed drive is selected.

From the above tables, the following data are taken

$$z_p = 9.5,\ \gamma = 11.8886^\circ,\ d_{m1} = 95\text{ mm},\ d_{a1} = d_{m1} + 2\text{nt} = 115\text{ mm}$$

Centre distance
$$a = \frac{d_{m1} + d_{m2}}{2} = \frac{d_{m2} + (d_2 + 2\ xm)}{2}$$

$$= \frac{95 + (30\times10+2\times0.25\times10)}{2} = \frac{95 + 305}{2} = 200\text{ mm}$$

This tallies with the given centre distance. Correction factor $x = +0.25$ from Table 4.4.

$$v_s = \pi d_{m1} n_1 \sec\gamma/1000 \times 60 = 3.14 \times 93 \times 1000 \times \sec 11.8886^\circ/1000 \times 60 = 5.08\text{ m/s}$$

This value is less than 12.5 m/s which is the maximum value allowed in the code as stated before. From Tables 4.13 and 4.14 and Figs 4.14 and 4.15, we get the following data on the materials selected

$$S_{c1} = 4.93,\ S_{c2} = 1.55,\ S_{b1} = 28.20,\ S_{b2} = 7.00,\ y_z = 1.223,$$
$$x_{c1} = 0.125,\ x_{c2} = 0.25,\ x_{b1} = 0.27,\ x_{b2} = 0.45$$

From Eq. 4.63, we get $l_r = 68.5$, taking top clearance $c = 0.2$ m = 2 mm

Table 4.13 Zone factory y_z

z_1 \ z_2	6.	6.5	7	7.5	8	8.5	9	9.5	10	11	12	13	14	16	17	18	20
	y_z																
1	1.045	1.048	1.052	1.065	1.084	1.107	1.128	1.137	1.143	1.160	1.202	1.260	1.318	1.374	1.402	1.437	1.508
2	0.991	1.028	1.055	1.099	1.144	1.183	1.214	1.223	1.231	1.250	1.280	1.320	1.360	1.418	1.447	1.490	1.575
3	0.822	0.890	0.969	1.109	1.209	1.260	1.305	1.333	1.350	1.365	1.393	1.422	1.442	1.502	1.532	1.580	1.674
4	0.826	0.883	0.981	1.098	1.204	1.301	1.380	1.428	1.460	1.490	1.515	1.545	1.570	1.634	1.666	1.710	1.798
5	0.947	0.991	1.050	1.122	1.216	1.315	1.410	1.490	1.550	1.610	1.632	1.652	1.675	1.735	1.765	1.805	1.886
6	1.132	1.145	1.172	1.220	1.287	1.350	1.438	1.521	1.588	1.675	1.694	1.714	1.733	1.789	1.818	1.854	1.928
7			1.316	1.340	1.370	1.405	1.452	1.540	1.614	1.704	1.725	1.740	1.760	1.817	1.846	1.880	1.950
8					1.437	1.462	1.500	1.557	1.623	1.715	1.738	1.753	1.778	1.838	1.868	1.898	1.960
9							1.573	1.604	1.648	1.720	1.743	1.767	1.790	1.850	1.880	1.910	1.970
10							1.410	1.557	1.680	1.728	1.748	1.773	1.798	1.858	1.888	1.920	1.980
11										1.732	1.753	1.777	1.802	1.862	1.892	1.924	1.987
12										1.715	1.760	1.780	1.806	1.868	1.895	1.927	1.992
13												1.784	1.806	1.867	1.898	1.931	1.998
14													1.811	1.871	1.900	1.933	2.000

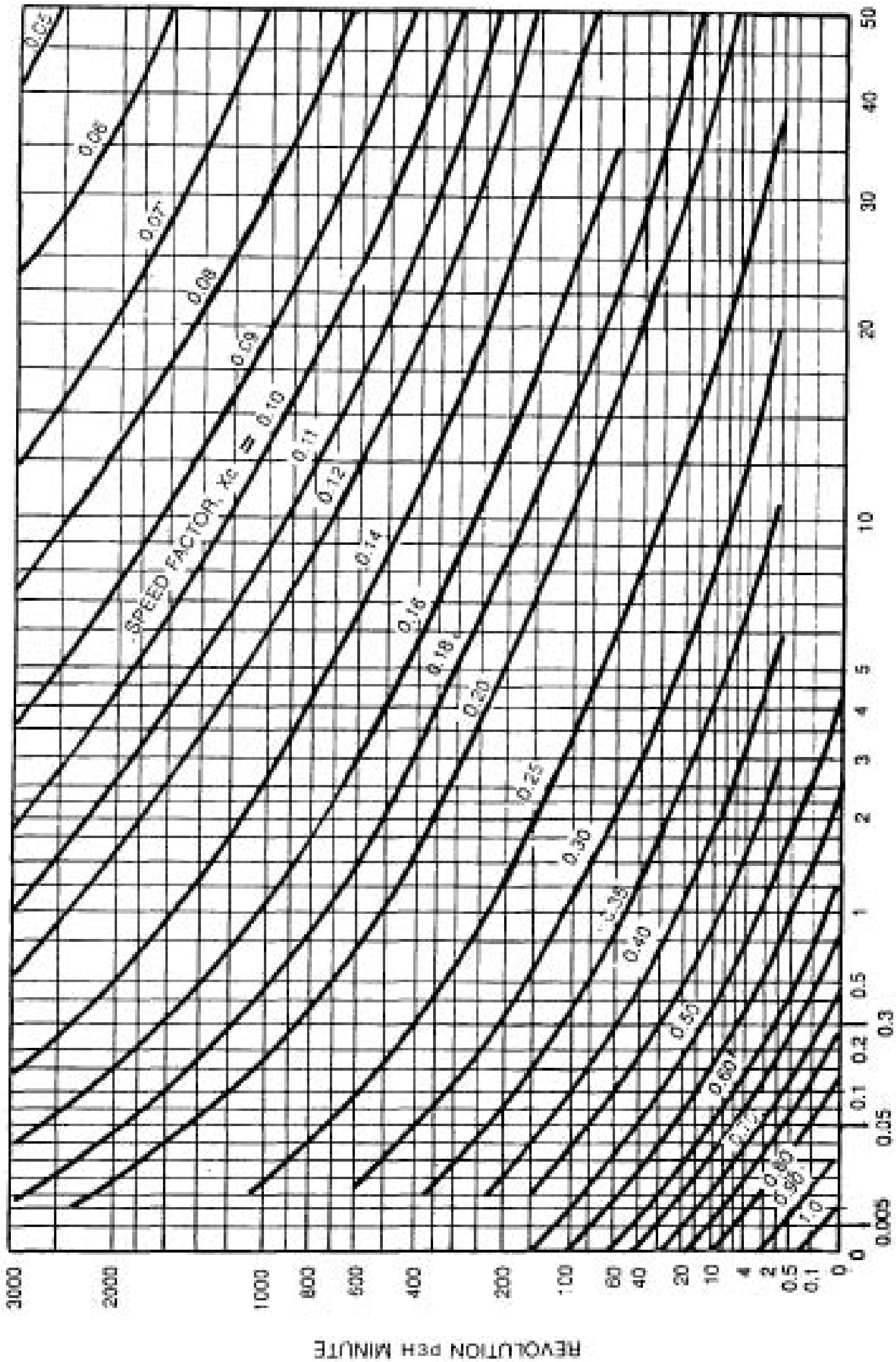


Fig. 4.14 Speed factors for worm gears for wear X_c

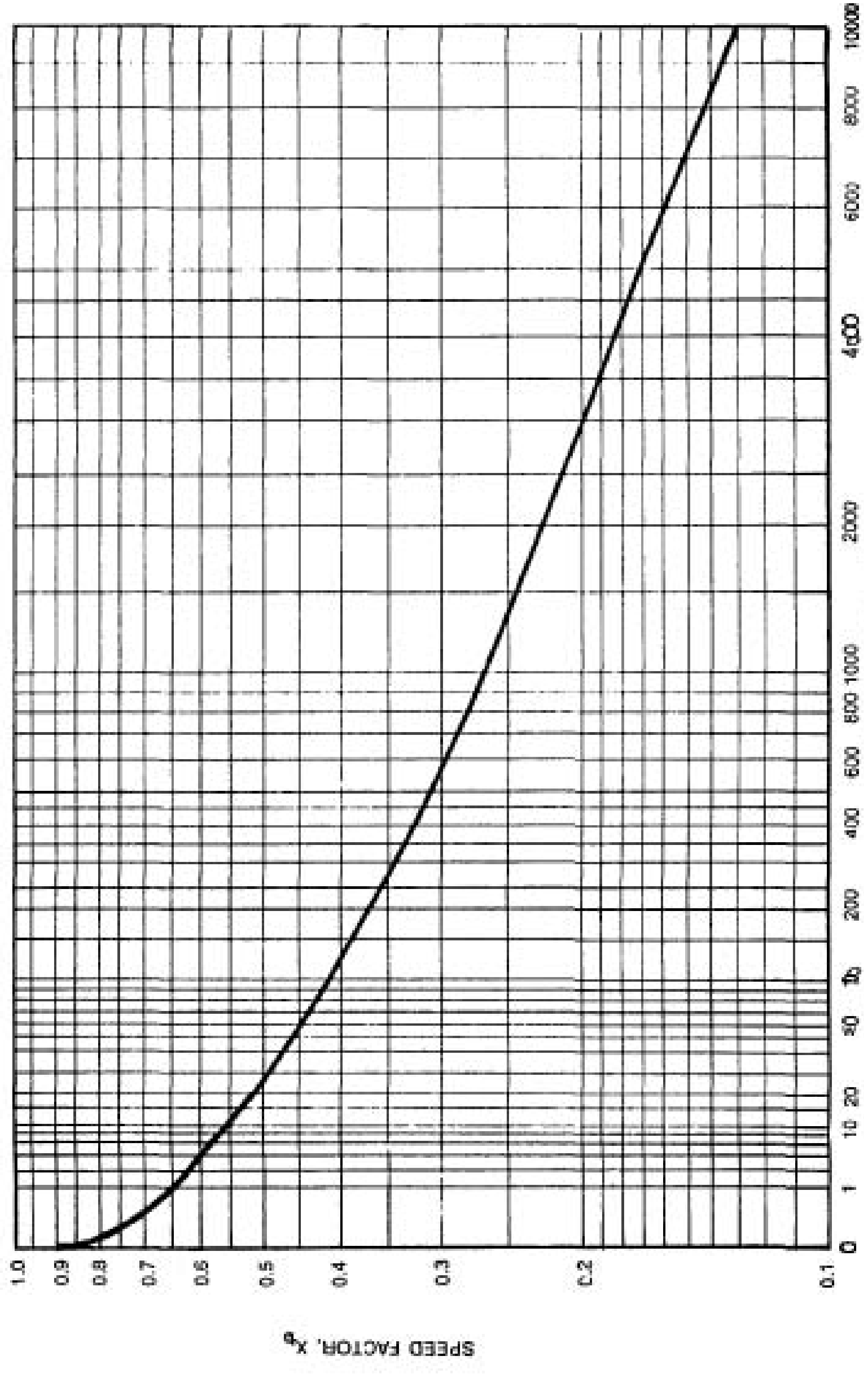


Fig. 4.15 Speed factor for worm gears for strength x_b

Table 4.14 Stress factors for worm and gear S_b and S_c

	Materials	IS: Reference	Bending stress factor S_b	Surface stress factor S_c when running with A	B	C	D	E
A	Phosphor bronze, centrifugally cast	IS: 28-1958	7.00		[illegible]	-		
	Phosphor bronze sand cast chilled	—	[illegible]4	—	0.63	0.63	0.76	1.27
	Phosphor bronze sand cast	—	5.00	—	0.47	0.47	0.54	1.06
B	Grey cast iron	Grade, 20 IS : 210-1970	4.09	0.63	0.42	0.42	0.42	—
C	0.4% carbon steel normalized	C 40 IS: 1570-1961	14.10	1.10	0.76	—	0.42	
	[illegible] case hardening steel	C 10, C 14 IS: 4432-1967	[illegible]	[illegible]	[illegible]	—	—	1.55
E	3% nickel and nickel molybdenum case hardened steel	16 Ni 80 Cr 60 20 Ni 2 Mo 25 IS : 4432-1967	33.11	5.41	3.10	—	—	1.55
	3½ nickel chromium	13 Ni 3 Cr 80 15 Ni 4 Cr 1 IS: 1570-1961	35.22	6.19	3.10	-	—	1.55

Putting these values in the relevant equations, we get the following values of torque

$$0.19\ x_{c1}\ S_{c1}\ y_z\ m d_{m2}^{1.8} = 0.19 \times 0.125 \times 4.93 \times 1223 \times 10 \times 305^{1.8} = 42430 \text{ kgf cm}$$

Similarly, using Eqs. 4.60, 4.61 and **4.62**, and inserting the respective values, we get the following torques values **26,680** kgf cm, 280,609 kgf cm, and **116091** kgf cm.

Taking the lowest value of torque, viz. **26,680** kgf cm, we have

$$\text{Normal power rating} = \frac{26680 \times 66.7}{97442} = 18.3 \text{ kW} \qquad \text{(as per Eq. 4.65)}$$

Using **Eq. 4.57**, we have for a life of 40,000 hours,

$$K_u = \left(\frac{27{,}000}{1000 + 40{,}000}\right)^{1/2} = 0.87$$

Therefore the required rating = **18.3x** 0.87 ≈ 16 kW

4.14 Efficiency ot Worm-Drive

As a power transmitting contrivance, the efficiency of the worm and worm-wheel system is rather low compared to other types of gear drives with similar capacity. A large amount of frictional loss occurs due to the relative sliding between the mating surfaces of worm and wheel under load which is the main cause of the low efficiency.

The efficiency of this system is a function of many factors, viz. lead angle, surface finish, type and amount of lubrication, type of design of the system, and other factors. Efficiency increases with increasing lead angle. Better efficiency can be achieved by using multi-start worms with small diameter. Good results are also obtained by using rigid, non-yielding worms with smooth, ground or polished flanks.

The basis for calculating the efficiency of the worm-drive system is to compare the force without friction with the force with friction. Both the components of the system can drive each other, depending on the design requirements. Denoting the angle of friction by ρ, and the coefficient of friction by μ where $\mu = \tan \rho$, we can arrive at the following expressions for the efficiency of the system

With worm driving, the efficiency is given by

$$\eta = \frac{\tan \gamma}{\tan(\gamma + \rho)} \tag{4.70}$$

With wheel driving, the efficiency is given by

$$\eta' = \frac{\tan(\gamma - \rho)}{\tan \gamma} \tag{4.71}$$

If the lubrication is proper and if the system consists of a hardened and ground worm meshing with an accurately machined wheel, then the efficiency will mainly depend on the lead angle and the coefficient of friction, μ.

Transposing **Eqs** 4.70 and 4.71, we can write

$$\eta = \frac{1 - \mu \tan \gamma}{1 + \dfrac{\mu}{\tan \gamma}} \tag{4.72}$$

$$\eta' = \frac{1 - \dfrac{\mu}{\tan \gamma}}{1 + \mu \tan \gamma} \tag{4.73}$$

Simplifying we get

$$\eta' = 2 - \frac{1}{\eta} \tag{4.74}$$

Experiments have confirmed the fact that the coefficient of friction is dependent on the sliding velocity. Denoting the sliding velocity along the worm middle circle as v_s, the following empirical relation can be established

$$\mu = \frac{0.051 \times y}{\sqrt{0.4 + v_s}} \tag{4.75}$$

If n_1 is the speed of worm in rpm, and d_{m1} is the middle circle diameter in metres, then

$$v_s \,(\mathrm{m/s}) = \frac{\pi d_{m1} n_1}{60 \cos \gamma} \tag{4.76}$$

The value of factor y in **Eq.** 4.75 will depend upon the material combination of the worm and the wheel, and this can be taken from Table **4.15.**

Table 4.15 Factor y for coefficient of friction

Worm material	*Wheel material*	y
Steel, hardened and ground	Cu–Sn Bronze	1
	Al alloy	1.15
	Cast iron	1.25
Steel, heat-treated and not ground	Cu–Sn Bronze, Zn alloy	1.5
	Al alloy	1.73
	Cast iron	1.83

Using Eq. 4.74, if $\eta = 0.5$, then

$$\eta' = 2 - \frac{1}{\eta} = 2 - \frac{1}{0.5} = 0$$

The significance of the above equation is that with the value of η being equal to or less than 0.5, it is not possible for the wheel to drive the worm. This means that the system is irreversible or self-locking, that is, the worm can drive the wheel but the reverse drive is not possible. This property is made **use** of when irreversibility is imperative, but since it is dependent upon μ which is likely to change during service due to various reasons, the irreversibility is not automatically assured even if the above conditions are satisfied and the proper coefficient is initially chosen for the purpose. It is always prudent to have the provision of a brake in such system.

In applications where self-locking should not be there purposely (for example certain crane systems) the lead angle must be so chosen that it is greater than the angle of friction ($\gamma > \rho$). **As** guiding values for rough calculation, the following values may be taken where irreversibility is required

$y \leq 5''$ when antifriction bearing is used

$\leq 6'$ when journal bearing is used

Table **4.16** gives the practical guiding values of μ

Table 4.16 Value of the coefficient of friction μ

Worm		*Mating wheel*		μ
Material	*Condition*	*Material*	*Condition*	
Steel	Heat-treated	Cast iron	Smooth teeth, grease lubrication	0.1
Steel	Heat-treated	Bronze	Untreated teeth, grease lubrication	0.08-0.09
Steel	Heat-treated	Bronze	Machined teeth, oil lubrication	0.06-0.07
Steel	Hardened and ground	Bronze	Machined teeth, oil lubrication	0.05-0.06

Table **4.17** gives the efficiency of worm and wheel system as a function of the lead angle and the coefficient of friction.

Table 4.17 Efficiency of worm-gearing (with worm driving)

Coefficient of friction μ	*Lead angle of worm in degrees*								
	5	10	15	20	25	30	35	40	45
	Efficiency (%)								
0.01	89.7	94.5	96.1	97.0	97.4	97.7	97.9	98.0	98.0
0.02	81.3	89.5	92.6	94.1	95.0	95.5	95.9	96.0	96.1
0.03	74.3	85.0	89.2	91.4	92.7	93.4	93.9	94.1	94.2
0.04	68.4	80.9	86.1	88.8	90.4	91.4	92.0	92.2	92.3
0.05	63.4	77.2	83.1	86.3	88.2	89.4	90.1	90.4	90.5
0.06	59.0	73.8	80.4	84.0	86.1	87.5	88.2	88.6	88.7
0.07	55.2	70.7	77.8	81.7	84.1	85.6	86.4	86.9	86.9
0.08	51.9	67.8	75.4	79.6	82.2	83.8	84.7	85.2	85.2
0.09	48.9	65.2	73.1	77.6	80.3	82.0	83.0	83.5	83.5
0.10	46.3	62.7	70.9	75.6	78.5	80.3	81.4	81.9	81.8

It is evident from the above discussion that the efficiency of a worm gearing varies markedly with lead angle. It follows, therefore, that the large lead angles are desirable for power transmitting sets. However, with increasing lead angle, the root diameter of the worm diminishes correspondingly. This results in reduction in the relative strength of the worm and also makes the face width of the wheel narrower. The choice of lead angle, therefore, cannot be made on the basis of efficiency alone. For proper design, the designer should endeavour to strike a balance between the factors governing the efficiency and strength.

5
Straight Bevel Gears

5.1 Theory of Bevel Gears

We have seen in the chapters dealing with spur and helical gears that these gears are similar in action when two pitch cylinders roll against the surfaces of each other without slipping. Basically, bevel gears are analogous to a friction cane drive when the conical surface of one drives that of the other cone by friction. Since friction cone drive is not attainable in practice, teeth are provided on these cones for positive drive, The pitch cones of bevel gears are analogous to the pitch cylinders of spur and helical gears.

For transmission of power through intersecting axes, the bevel gears are most commonly used. As stated before, the pitch surface of a bevel gear is a (truncated) cone. When two bevel gears mate, their respective pitch cones contact along a common element. The pitch cones, when extended, meet at a common point called the apex. The shaft centre lines also obviously intersect at the apex.

The rolling pitch cones have spherical motion. While in motion, every point in a bevel gear remains at a constant distance from the apex. It is not customary to make the large end, that is, the hack of a bevel gear spherical. It is made conical, this cone being known as the back cone which is tangent to the theoretical sphere at the pitch diameter.

The tooth data of a bevel gear are all given with reference to the large end. While in case of spur and helical gears, the cutting tool represents the teeth on a basic rack as given in Sec. 2.1, the cutting tool of a bevel gear represents the tooth on a basic crown gear as discussed in Sec. 5.2. A crown gear is a bevel gear where the pitch cone angle is 90' and bears the same relation to a bevel gear as a rack does to a spur gear. The tooth form in the bevel gear is slightly modified from a true involute. To ensure practical gear cutting, the basic crown gear have straight sided teeth. The cutting tools having straight cutting edges are inclined to give the desired pressure angle. Strictly speaking, the basic crown gear tooth should be slightly curved to attain the true involute form. Cutting should also conform accordingly to produce this contour. To avoid practical difficulties associated with giving cutters a curved outline, straight teeth are used as indicated before. The teeth produced are said to have "octoid" form. In the generation method, a straight sided tool simulating the crown gear and the blank of the bevel gear cone roll on each other, producing the desired bevel gear.

The basic shape of a bevel gear tooth is almost the same as that of the spur gear. The tooth tapers off as it approaches the apex. The contour of the tooth also varies along its entire length. The angle between the shafts will depend on the conditions of drive. It is usually 90°, but can have other angles also.

The configurations of the straight-sided bevel gears have been shown in Fig. 5.1.

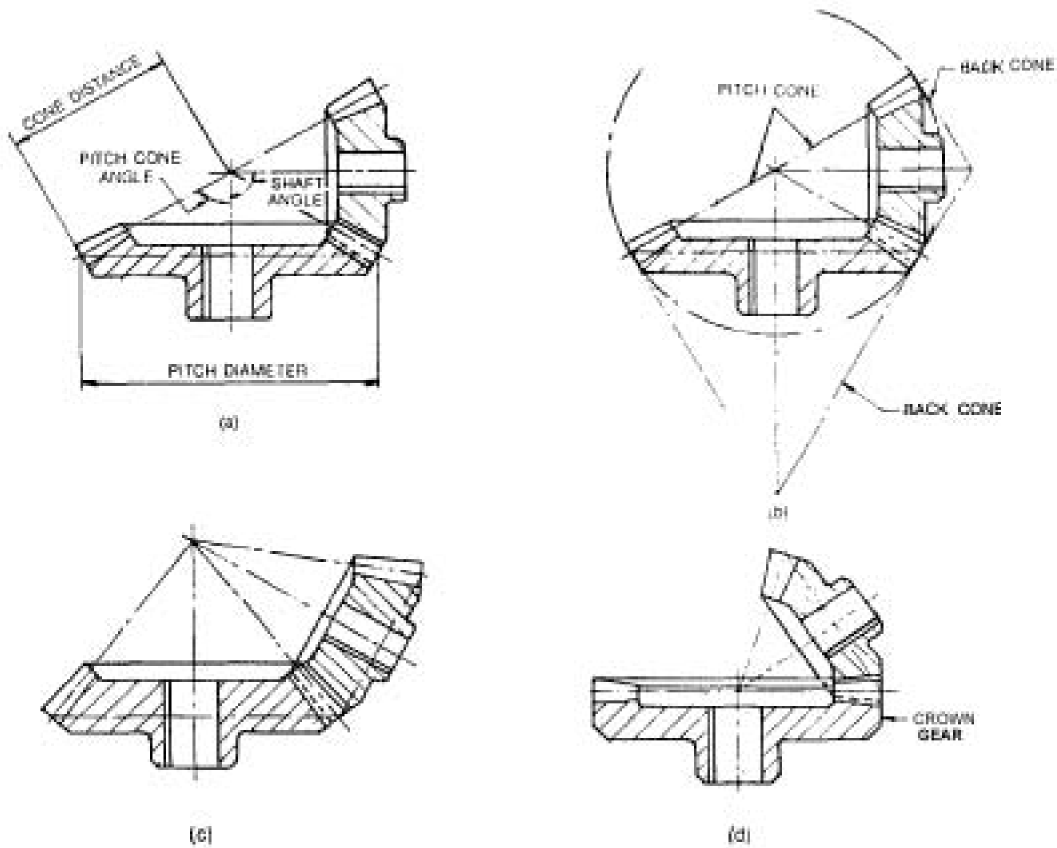


Fig. 5.1 Straight-sided bevel gears

5.2 Bevel Gear Basic Rack and Modules

In IS: 5037, the proportions of the basic rack of a straight-sided bevel gear have been specified. This rack is defined as the profile of the tooth of a crown gear of infinite diameter on a plane at right angles to the tooth surface. For bevel gears having straight teeth, this profile is used as the basis of reference. The tooth proportions have been shown in Fig. 5.2.

Modules

The recommended series of modules are given in Table 5.1. Column 1 is the first choice. Sizes given in brackets under "Choice 3" column are to be avoided.

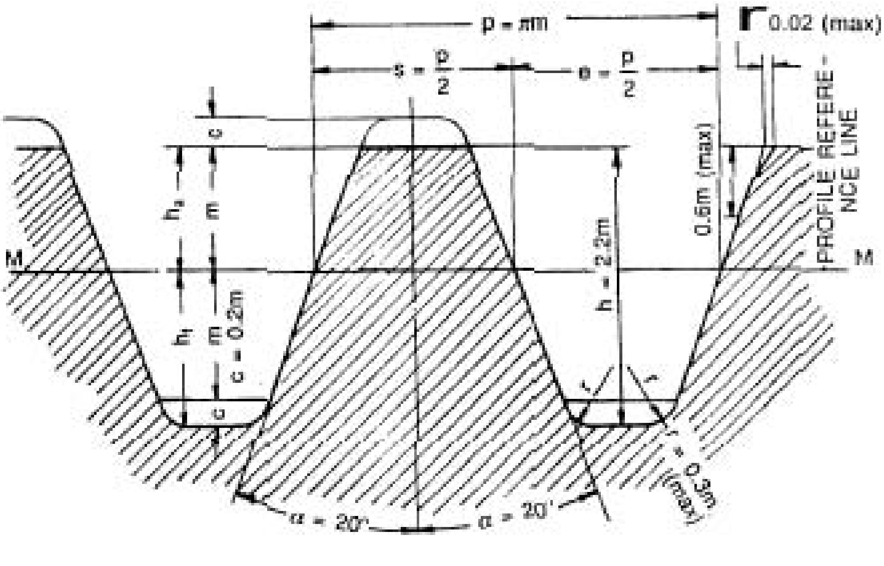


Fig. 5.2 Profile of the basic rack of bevel gear

Table 5.1 Modules for straight sided bevel gears

Preferred	*Choice 2*	*Choice 3*
1	1.125	
1.25	1.375	
1.5	1.75	
2	2.25	
2.5	2.75	
3		(3.25)
	3.5	(3.75)
4	4.5	
5	5.5	(6.5)
6	7	
8	9	
10	11	
12	14	
16	18	
20	22	
25	28	
32	36	
40	45	
50		

When diametral pitches are used, the following values are recommended

Preferred 20, 16, 12, 10, 8, 6,5, 4, 3, 2.5, 2, 1.5, 1.25, 1,0.75, 0.625, 0.5.

Choice2 18, 14, 11, 9, 7, 5.5, 4.5, 3.5, 2.75, 2.25 1.75, 0.875.

Class of Bevel Gears

Both straight sided and curved toothed bevel gears have been categorised into certain classes. This classification will depend mainly upon the circumferential speeds and the transmitted load. For normal transmission of load, bevel gears with speeds above 10 m/s are termed as high-class, precision gears, while commercially produced gears meant for usual industrial purposes with speeds below 6 m/sec belong to the ordinary class. Bevel gears may be cast or milled, but generation methods produce precise and accurate teeth.

5.3 Bevel Gear Terminology and Relations

Bevel gears are cut on conical blanks. The teeth of the bevel gears may be straight or curved. Normally, the axes of the mating pair of gear intersect, but there are also non-intersecting types. The terms which are typical to straight bevel gears are given here. However, the terms which are common to other types of gears and are explained elsewhere, are not repeated here. Some of the parameters of the bevel gear are depicted in Fig. 5.3.

Since the tooth size decreases from the back end towards the apex and the tooth contour also varies accordingly, terms such as addendum, dedendum whole depth, pitch circle diameter, tip circle diameters, etc. are all measured with reference to the large end of the tooth. It is to be specially noted here that if, along with the pitch cones which meet at the apex, other lines such as the addendum line, root line, etc. also meet at the apex when these lines are theoretically extended, then there is a non-uniform clearance between the tip of one tooth and the bottom land of the mating tooth along the length of tooth. This clearance becomes progressively smaller from the back end towards the apex. This has been graphically shown in Fig. 5.3. If uniform clearance is to be maintained, these lines should meet at different points on the axes, that is, only the pitch cones meet at the apex or the point of intersection of the two shafts (when extended) on which the gears are mounted. Other cones, viz. tip cones, root cones, etc. meet their respective gear centre lines at different points along these lines.

As a matter of fact, all these lines and angles meeting at the pitch cone apex represent the old-style design. Modern trend is to make the outer cone of one gear parallel to the root cone of its mating counterpart. This results in constant top clearance and permits a better cutting-tool design and tooth design than the old fashioned one with tapering clearance.

The pitch cones These are cones which roll without slipping when they are in peripheral contact. They are analogous to pitch cylinders of spur and helical gears. The angular velocities of these cones are inversely proportional to the number of teeth of the bevel gears to which they correspond.

The pitch cone angle This is the angle subtended at the apex by the axis and the pitch cone generator. This is usually denoted by δ with subscripts 1 and 2 for pinion and gear respectively as before.

The cone distance This is the length of the pitch cone generator from the pitch circle to the apex, and is denoted by R or R_a.

The back cone This is the cone generated by a line which is perpendicular to the pitch cone generator at a point on the pitch circle. The angle between the back cone generator and the axis of the gear is known as the back cone angle. It is the complement of the pitch cone angle. The back cone radius is denoted by R_r.

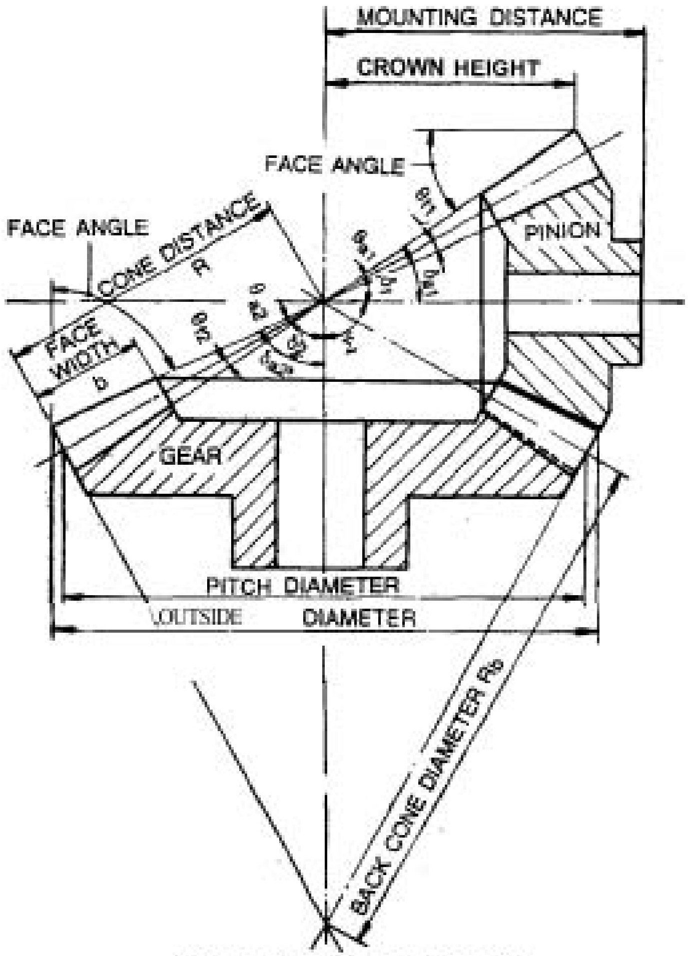


Fig. 5.3 Bevel gear geometry

The blank cone angle This is the angle between the axis and the tip surface of the tooth. The blank cone on which the bevel gear teeth are cut, is analogous to the blank cylinder of a spur or helical gear. This angle is equal to the sum of the pitch cone angle and the addendum angle. The blank cone angle is also known as the face angle.

The shaft angle The angle between the intersecting axes of gears is known as the shaft angle, and is denoted by Σ. It is equal to the sum of the pitch cone angles of the pinion and the wheel. Normally, the shaft angle is 90°, but it can have other values also depending upon the design.

The face width This is the length of the tooth measured along the pitch cone generator, and is denoted by b. Normally, b lies between 8 and 10 m where m is the module of the bevel gear.

Virtual number of teeth As in the case of helical gears, the concept of the virtual number of teeth in case of bevel gears is also quite useful. To be strictly theoretically correct, the tooth profiles of a bevel gear should be developed on a spherical surface. As a true development of a spherical surface into a plane is not possible, an approximation has to be made for which the virtual number of teeth is to be determined. This parameter is defined as the number of teeth which a spur gear would have the radius of which is equal to the back cone radius or distance R_b, and having pitch of the bevel gear. This is called Tredgold's approximation. The number of teeth of this imaginary spur gear is given by

$$z_v = \frac{2\pi R_b}{p} = \frac{z}{\cos\delta} \quad (5.1)$$

where p is the circular pitch, measured at the back end of the teeth.

Obviously, the virtual number of teeth is not necessarily a whole number and in fact, in most cases it is not. The strength calculations of bevel gears are based on the equivalent spur gear concept.

Crown gear The crown gear has already been defined in Sec. 5.1, and its shape illustrated in Fig. 5.1 (d).

In a crown gear, since the pitch cone angle is 90°, the surface of the pitch cone becomes a flat circular plane. The consideration of the crown gear in case of bevel gears is very useful as the dimensional data can be represented on the relevant crown gear. This is especially true in case of spiral or curved toothed bevel gears as we shall see in Chap. 6. In short, the crown gear is the best method to represent the toothing and other characteristics of any kind of bevel gear.

The radius of the crown gear is denoted by the radius of the pitch plane referred to before. Its value is equal to that of the cone distance R of the bevel gear which mates with the crown gear. Referring to Fig. **5.3**, we have

$$\sin\delta = r/R = mz/2R$$

where δ is the pitch cone angle, r pitch circle radius, and z the number of teeth of the mating bevel gear. Since R is also the (pitch) radius of the crown gear, we have

$$2R = mz_c, \text{ where } z_c = \text{Number of teeth of crown gear.}$$

Hence

$$z_c = 2R/m = z/\sin\delta \quad (5.2)$$

5.4 Minimum Number of Undercut-free Teeth

In case of bevel gears, the minimum number of undercut-free teeth is found in a similar way as in the case of spur or helical gears. However, instead of the actual number of teeth, the virtual number of teeth is inserted in the equation to find z_{min}. From Eq. 5.1, we have

$$z_v = \frac{z}{\cos\delta}$$

whence

$$z_{v1} = \frac{z_1}{\cos\delta_1} \text{ and } z_{v2} = \frac{z_2}{\cos\delta_2}$$

Inserting the value **14** for practical limiting number of teeth in case of spur gears as discussed in Sec. **2.10**, we have for bevel gears

$$z_{min\,1} = 14\cos\delta_1 \quad (5.3)$$

$$z_{min\,2} = 14\cos\delta_2 \quad (5.4)$$

where the subscripts 1and 2 stand for pinion and gear as before. The above relation holds good for 20° pressure angle.

5.5 Profile Correction of Bevel Gears

As in the case of spur and helical gears, profile correction is also carried out for bevel gears to avoid the danger to the teeth arising out of undercutting, if conditions warrant such a step.

The relevant equations are similar to those for spur gears. Only, instead of the actual number of teeth, the value of the virtual number of teeth is inserted as shown in the following equation

$$x_1 = \frac{14 - z_{v1}}{17} = \frac{14 - \dfrac{z_1}{\cos \delta_1}}{17} \tag{5.5}$$

Similarly

$$x_2 = \frac{14 - z_{v2}}{17} = \frac{14 - \dfrac{z_2}{\cos \delta_2}}{17} \tag{5.6}$$

However, in case of bevel gear drive, it is desirable to have an S_0-gearing, because in this way the shaft angle remains unaltered. It will be possible to have S_0-gearing if the following condition is satisfied

$$\frac{z_1}{\cos \delta_1} + \frac{z_2}{\cos \delta_2} \geq 2 \times z_{min} \text{ for spur gear} \geq 2 \times 14 \geq 28 \tag{5.7}$$

As before, the above equation holds good for 20° pressure angle gears and the limiting number of teeth is arrived at after considering the practical aspects.

As in the case of spur and helical gears, there are practical limitations to profile correction in bevel gears. Theoretically, correction can be carried out till the teeth become peaked, but since a minimum amount of top land is imperative, the correction is restricted by the following condition

$$\text{Tooth thickness at the tip circle } s_a = 0.25\, m$$

In case of case hardened bevel gear teeth, the tip tooth thickness at the innermost portion of the tooth of the pinion should not be below the following value

$$s_{a1} = 0.4\, m_m$$

where m_m is the value of the module at the middle of the tooth.

5.6 Guidelines for Selection of Dimensions

The bevel gears are sensitive to machining and mounting errors, elastic deformations and particularly to the deflection of shafts. The deflection tends to displace the shafts so that the pitch cones do not meet at the theoretical apex. The errors result in one-sided loading, noisy running, vibrations, jamming of teeth and other undesirable effects. These and other factors have limiting influences on bevel gear dimensions. To alleviate the detrimental effects of misalignment, the teeth of the bevel gears are sometimes generated in such a way that their surfaces are slightly convex in the lengthwise direction.

The designer should keep in mind that since a bevel gear is generally mounted at the end of a shaft, it is especially vulnerable to the effects of bending and deflection leading to misalignment. Besides, due to the wedge-like shape of the teeth, chances of misalignment occur. All these factors necessitate extreme care in mounting and proper bearing selection, though in spite of all these measures, bevel gears do not generally attain the output and the quietness of running of spur and helical gears.

The important dimensions and parameters of bevel gears are summarized in Table 5.2 which should be read along with Fig. 5.3.

In designing a bevel gear drive, the minimum and maximum values of certain dimensional parameters should be maintained for proper running of the drive. These values have been fixed by experience after studying different operational conditions. These are enumerated in the following lines.

The minimum number of teeth of the equivalent spur gear for the pinion is given by

$$z_{v1} = \frac{z_1}{\cos\delta_1} \geq z'_{1\,min} \tag{5.8}$$

where $z'_{1\,min}$ is the smallest number of teeth which a spur pinion should have after taking into account the adequate amount of contact ratio and other factors, such as speed and load. The values have been arrived at after considering that profile correction of proper magnitude has been applied to the spur pinion as and when necessary, and the values are to be taken from Table 2.11 in Sec. 2.16.

Limitations as regards the usable values of the middle module and the face width are as follows

$$\text{Minimum middle module} = m_{m\,(min)} = 2\frac{b}{\lambda} \tag{5.9}$$

Factor λ depends on the condition of the tooth surface and the type of bearing as shown in Table 2.12 (Sec. 2.16) which is applicable here also.

The maximum face width is given by

$$b_{max} = \frac{\lambda}{2} m_m \tag{5.10}$$

It should not exceed the value $R/3$ as indicated in Table 5.2.

Table 5.2 Dimensions of bevel gears

Description	*Pinion*	*Gear*
Number of teeth	z_1	z_2
Pitch circle diameter	$d_1 = z_1 m$	$d_2 = z_2 m$
Transmission ratio	$i = \frac{n_1}{n_2} = \frac{z_2}{z_1}$	
Pitch cone angle (shaft angle is equal to 90°)	$\tan\delta_1 = \frac{d_1}{d_2} = \frac{z_1}{z_2} = \frac{1}{i}$	$\tan\delta_2 = \frac{d_2}{d_1} = \frac{z_2}{z_1} = i$

(Contd)

Table 5.2 (Contd)

Description	Pinion	Gear
Pitch cone angle (shaft angle is not equal to 90°)	$\tan\delta_1 = \dfrac{\sin\Sigma}{1+\cos\Sigma}$	$\delta_2 = \Sigma - \delta_1$
Shaft angle	$\Sigma = \delta_1 + \delta_2$	
Tip circle diameter	$d_{a1} = d_1 + 2m\cos\delta_1$	$d_{a2} = d_2 + 2m\cos\delta_2$
Middle circle diameter (subscript m for middle)	$d_{m1} = d_1 - b\sin\delta_1$	$d_{m2} = d_2 - b\cos\delta_2$
Face width	$b_{max} \le \dfrac{R}{3}$	
Cone distance	$R = \dfrac{d_1}{2\sin\delta_1} = \dfrac{d_2}{2\sin\delta_2}$	
Virtual number of teeth (subscript v for virtual or equivalent)	$z_{v1} = \dfrac{z_1}{\cos\delta_1}$	$z_{v2} = \dfrac{z_2}{\cos\delta_2}$
Middle module (subscript m for middle)	$m_m = \dfrac{d_{m1}}{z_1} = \dfrac{d_{m2}}{z_2}$	
Top clearance	$c = 0.2\,m$	
Whole depth	$h = 2m + 0.2m = 2.2m$	
Addendum	$h_{a1} = h_{a2} = m$	
Dedendum	$h_{f1} = h_{f2} = 1.2\,m$	
Addendum angle	$\tan\theta_{a1} = \tan\theta_{a2} = m/R$	
Dedendum angle	$\tan\theta_{f1} = \tan\theta_{f2} = 1.2\,m/R$	
Blank cone angle or Face angle	$\delta_{a1} = \delta_1 + \theta_{a1}$	$\delta_{a2} = \delta_2 + \theta_{a2}$
Crown height	$CH_1 = \dfrac{d_2}{2} - m\sin\delta_1$	$CH_2 = \dfrac{d_1}{2} - m\sin\delta_2$
Back cone distance	$R_{b1} = R\tan\delta_1$	$R_{b2} = R\tan\delta_2$

5.7 Force Analysis for Bevel Gears

For force analysis of a pair of mating bevel gears, it is assumed that the total force F_N acts on the pitch point P at the middle of the tooth width. The resultant, however, actually occurs somewhere between the midpoint and the back end of the tooth, but the error due to the above assumption is marginal.

The mean tooth force F_N is resolved into three mutually perpendicular components — the tangential force or transmitted load F_t, the radial force F_r, and the axial force F_a (Fig. 5.4). From geometry, we can arrive at the values of these forces for the bevel pinion and the gear as illustrated in Fig. 5.5.

$$\text{Tangential force } F_{t1} = F_{t2} = F_t = F_N \cos \alpha \tag{5.11}$$

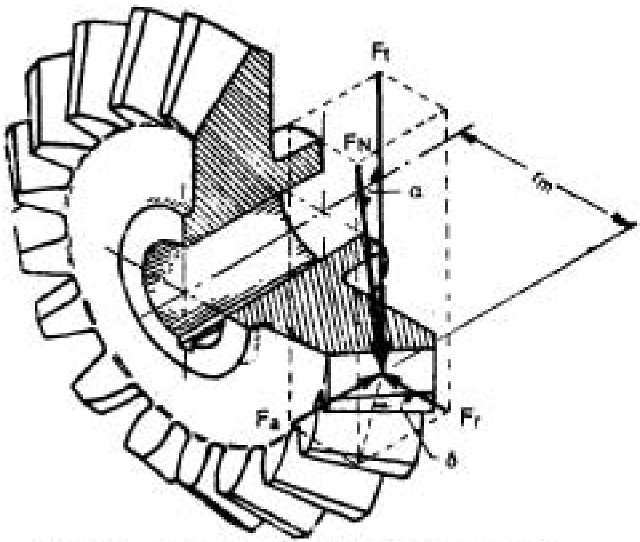


Fig. 5.4 Forces acting on bevel gear tooth

This force is calculated from the torque as in the case of other types of gear drives, and is given by

$$F_t = \frac{2T_1}{d_{m1}} = \frac{T_1}{r_{m1}} \tag{5.12}$$

where T_1 = Pinion torque in N m = $9550\, P_1/n_1$

P_1 = Pinion power in kW

n_1 = Pinion speed in rpm

$$\text{Radial force } F_{r1} = F_t \tan \alpha \cos \delta_1 \tag{5.13}$$

$$F_{r2} = F_t \tan \alpha \cos \delta_2 \tag{5.14}$$

$$\text{Axial force } F_{a1} = F_t \tan \alpha \sin \delta_1$$

$$F_{a2} = F_t \tan \alpha \sin \delta_2 \tag{5.15}$$

If the shaft angle is 90°, then the following relations hold good

$$F_{a1} = F_{r2} \tag{5.16}$$

$$F_{a2} = F_{r1} \tag{5.17}$$

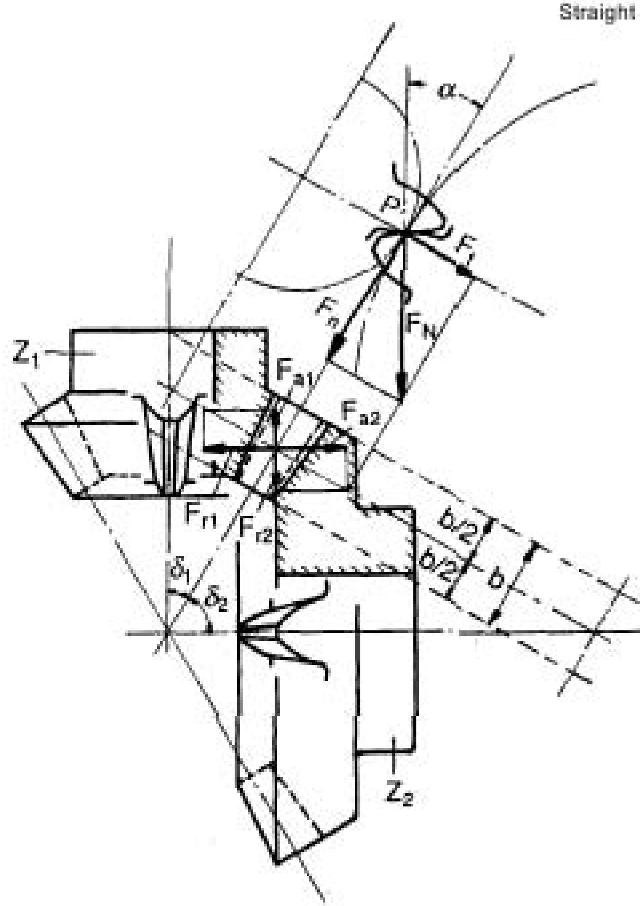


Fig. 5.5 **Distribution** of tooth forces on bevel gears

Based on Die Tragfaehigkeit der Zahnraeder, Thomas and Charchut, 7th Edition, 1971, Fig. No. 51 p. 134. Carl Hanser Verlag, Munich

The above relations are in absolute values and do not take into account the respective algebraic signs.

5.8 Bevel Gear Bearing Loads

For the determination of bearing loads in case of bevel gears in mesh, it is convenient to analyse first the forces acting on the bearings in two mutually perpendicular planes and then add the partial bearing loads vectorially to arrive at the resultant load on each bearing. The method is explained below.

The three forces which act on a bevel gear tooth are F_t, F_r and F_a, as explained in Sec. 5.7. Referring to Fig. 5.6, we have in the $X-Z$ plane

$$F_{BIx}\, L = F_t\,(L_1 + L) \quad \text{whence } F_{BIx} = F_t\,(L_1 + L)/L \tag{5.18}$$

Similarly

$$F_{BIIx} = F_t\, L_1/L \tag{5.19}$$

In $Y-Z$ plane

$$F_{BIy} = \frac{F_r(L_1+L)-F_a r_m}{L} \tag{5.20}$$

$$F_{BIIy} = \frac{F_r L_1 - F_a r_m}{L} \tag{5.21}$$

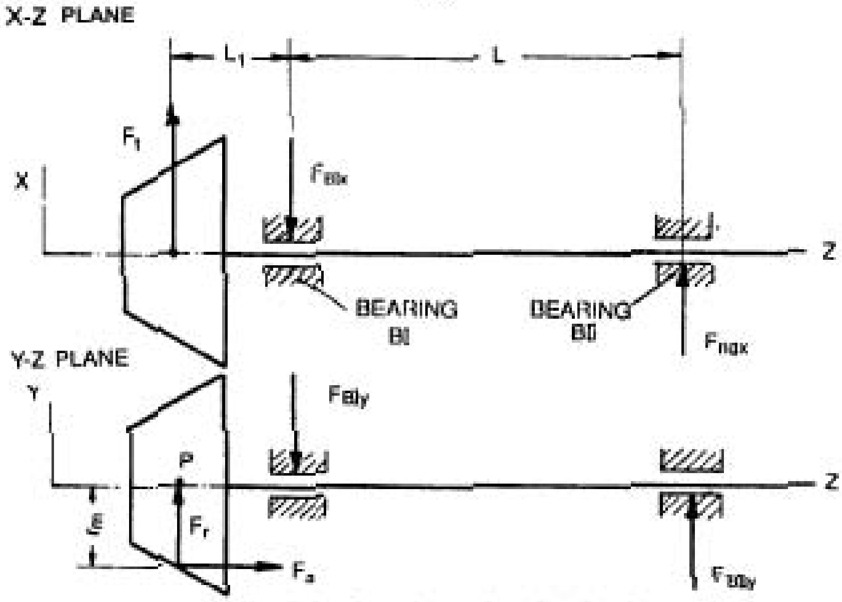


Fig. 5.6 Bevel gear bearing loads

The resultant values of the bearing loads on bearings BI and BII are

$$F_{BI} = \sqrt{(F_{BIx})^2+(F_{BIY})^2} \tag{5.22}$$

$$F_{BII} = \sqrt{(F_{BIIx})^2+(F_{BIIY})^2} \tag{5.23}$$

Bending Moments

The shaft will be subjected to bending besides torsion and the axial thrust F_a by which the bearing will be loaded. For shaft calculation, all these parameters are to be taken into account in the usual way as given in standard books on mechanics. Normally, F_a can be neglected without any significant effect in final calculation. The bending moments (B) are calculated for the points P and for the mid-point of BI. They are given by

$$B_p = F_a r_m \tag{5.24}$$

$$B_{BI} = F_{BII} L \tag{5.25}$$

Example 5.1 *Given:* P = 14 kW, n = 1000 rpm, z = 25, m = 3 mm, Σ = 90°, α = 20°, δ = 21° 40′, distance between bearings L = 100 mm, overhung distance L_1 = 50 mm.

To find the bearing loads and bending moments.

Solution: $T = 9550\frac{P}{n} = 9550\frac{14}{1000} = 133.7\,\text{Nm}$, $d = mz = 3 \times 25 = 75$ mm,

$R = d/2\sin\delta = 101.57$ mm, $b = R/3 = 34$ mm, $d_m = d - b\sin\delta = 62$ mm, $r_m = 31$

$F_t = T/r_m = 133.7 \times 1000/31 = 4313$ N, $F_r = F_t\tan 20^\circ\cos 21^\circ 40' = 1459$ N, $F_a = F_t$ $\tan 20^\circ \sin 21^\circ 40' = 580$ N

$$F_{BIx} = \frac{4313\,(50 + 110)}{110} = 6273\ \text{N}$$

$$F_{BIIx} = \frac{4313 \times 50}{110} = 1960\ \text{N}$$

$$F_{BIy} = \frac{1459(50 + 110) - 580 \times 31}{110} = 1959\,\text{N}$$

$$F_{BIIy} = \frac{1459 \times 50 - 580 \times 31}{110} = 500\text{N}$$

Using Eqs 5.22 and 5.23, $F_{BI} = 6572$ N and $F_{BII} = 2023$ N. Also, $B_P = -17.98$ Nm, and $B_{BI} = 222.53$ Nm. As moments act in opposite sense they are of different sings.

5.9 Bending Stress Calculations

The strength calculations in case of bevel gears, can be made in a similar way as in the case of spur gears. The basic design methods are the same but to perform such calculations, the values of bevel gears are replaced by equivalent values of spur gears as explained in Sec. 5.3. The design procedures, therefore, are based on the concept that the load carrying capacity of a bevel gear is equal to that of an equivalent spur gear having the same tooth width. Moreover, since the teeth are of unequal depth and thickness along the face width, the calculations are based on the value at the middle of the teeth.

Recalling the relations given in Table 5.2 (Sec. 5.6), we have for the pinion

$$z_{v1} = \frac{z_1}{\cos\delta_1}, \quad R = \frac{d_1}{2\sin\delta_1},$$

$$d_m = d_1 - b\sin\delta_1 = d_1 - \frac{b d_1}{2R} = d_1\left(1 - \frac{b}{2R}\right)$$

$$m_t = \frac{d_{m1}}{z_1} = \left(1 - \frac{b}{2R}\right)m = 0.8\,m$$

Equivalent diameter at the middle

$$d_{m1v} = \frac{d_{m1}}{\cos\delta_1}$$

Expressions for the above values for the gear can be similarly obtained by substituting the subscript 1 by 2.

For strength calculations, the following relations are necessary

Equivalent transmission ratio $= i_v = \dfrac{z_2 \cos\delta_1}{z_1 \cos\delta_2} = i\,\dfrac{\sin(90^\circ - \delta_1)}{\cos \delta_1} = i \tan\delta_2 = i^2$

Circumferential force at the middle is given by

$$F_{tm} = \frac{2T_1}{d_{m1}} \times 1000$$

where the torque T_1 is in N m, F_{tm} is in N, and d_{m1} is in mm.

For beam strength calculation, a similar (but modified to suit bevel gears) set of equations is used as in the case of spur gears. As before, for calculating the bending and other stresses, the force is taken to act at the tip corner of the gear tooth. The bending stress is first found and then checked against the allowable stress. When referred to the middle circle, the expression for the bending stress is given by

$$\sigma_{b_1} = \frac{F_{tm}}{b\,m_m} q_{k1} q_{e1} \tag{5.26}$$

$$\sigma_{b_2} = \frac{F_{tm}}{b\,m_m} q_{k_2} q_{e2} \tag{5.27}$$

The values of q_r can be found from Fig. 2.49 corresponding to the number of teeth of the equivalent spur gears Z_{v1} and Z_{v2}, as given in the abscissa of the figure. For medium and coarse quality bevel gears which are mostly used for normal drive, the value of q_r can be taken as unity.

The allowable bending stress will depend upon the material. If the endurance limit σ_r of the material is known, then the allowable stress σ_{bp} can be found by using the formula

$$\sigma_{bp} = \frac{\sigma_r}{2 \text{ to } 3} \tag{5.28}$$

It can also be directly taken from the values given in Appendix E.

To find a suitable value for module m, we can proceed as follows.

Taking $b = \dfrac{R}{3}$, $m_m = \dfrac{R_m}{R} m$, where R_m is cone distance up to the mid-point where the module is m_m

Therefore

$$m_m = \frac{m}{R}\left(R - \frac{b}{2}\right) = \frac{m}{R}\left(R - \frac{R}{6}\right) = \frac{5}{6} m$$

$$d_{m1} = z_1 m_m = z_1 \frac{5}{6} m$$

Als

$$F_{tm} = \frac{2T_1}{d_{m1}} \times 1000 = \frac{2T_1 6}{5 z_1 m} \times 1000$$

$$R = \frac{d_1}{2\sin\delta_1} = \frac{z_1 m}{2\sin\delta_1} \quad \text{and} \quad b = \frac{R}{3} = \frac{z_1 m}{6\sin\delta_1}$$

Putting the above value in Eq. 5.26, we have

$$\sigma_{b1} = \frac{12000T_1}{5z_1m} \times \frac{6}{5m} \times \frac{3}{z_1m} \times 2\sin\delta_1 qk_1 q_{e1}$$

Solving for module and putting the expression for permissible value of stress, we get

$$m = 3\sqrt{\frac{17.28T_1 \sin\delta_1\ 1000}{z_1^2\sigma_{bp}}}\ q_{k1}\ q_{e1} \quad \text{(mm)} \tag{5.29}$$

For rough calculation to determine the module, we take the average values of qk_1 and q_{e1} to be 2.2 and 0.9 respectively. Hence

$$m \approx 3\sqrt{\frac{34T_1 \sin\delta_1\ 1000}{z_1^2\sigma_{bp}}} \quad \text{(mm)} \tag{5.30}$$

5.10 Contact Stress Calculations

In general, for contact stress calculations in case of bevel gears, it is sufficient to check the stress at the pitch point P to ensure the Hertzian pressure at the pitch point is within the allowable limit. Equations used for calculating contact stress in case of bevel gears are similar to those given in Chap. 2 on spur gears. Thus, the contact pressure at the pitch point in case of a pair of bevel gears with any shaft angle is given by

$$P_p = \sqrt{0.35\,\frac{F_m E}{b\,d_{m1v}}\,\frac{u_v+1}{u_v}\,\frac{1}{\cos^2\alpha\tan\alpha_w}} \tag{5.31}$$

Here

$$u_v = \frac{z_{v2}}{z_{v1}} = \frac{d_{m2v}}{d_{m1v}} = \frac{d_{m2}}{\cos\delta_2} \Big/ \frac{d_{m1}}{\cos\delta_1} = \frac{d_{m2}}{d_{m1}}\,\frac{\cos\delta_1}{\cos\delta_2} = \frac{z_2m_m}{z_1m_m} \times \frac{\cos\delta_1}{\cos\delta_2} = u\,\frac{\cos\delta_1}{\cos\delta_2}$$

For 90° shaft angle

$$\frac{\cos\delta_1}{\cos\delta_2} = \frac{\cos(90^\circ-\delta_2)}{\cos\delta_2} = \frac{\sin\delta_2}{\cos\delta_2} = \tan\delta_2 = u$$

Therefore

$$u_v = u^2$$

It may be recalled here that

$$u = \frac{\text{Number of gear teeth}}{\text{Number of pinion teeth}} = \frac{z_2}{z_1} \geq 1$$

$$i = \text{Transmission ratio} = \frac{\text{Speed of driving member}}{\text{Speed of driven member}}$$

Since normally the pinion is the driving member, $i = n_1/n_2 = z_2/z_1 = u$

In the case, as in the case of step-up gear-set, the gear is the driving member, the transmission ratio is smaller than 1 because of the reversed functions, n_1 stands for the speed of the gear and is less than n_2, the speed of the pinion.

Again

$$d_{m1,v} = \frac{d_{m1}}{\cos\delta_1} = d_{m1}\frac{\sqrt{u^2+1}}{u}$$

Inserting these values in Eq **5.31**

$$P_p = \sqrt{0.35\frac{F_{tm}E}{bd_{m1}}\frac{\sqrt{u^2+1}}{u}\frac{1}{\cos^2\alpha\tan\alpha_w}} \qquad (5.32)$$

Putting material coefficient

$$y_m = \sqrt{0.35E} = \sqrt{0.35\frac{2E_1E_2}{E_1+E_2}} \qquad (5.33)$$

where E_1 and E_2 are the modulii of elasticity of the pinion and the gear materials, and putting the pitch point coefficient

$$y_p = \sqrt{\frac{1}{\cos^2\alpha\tan a_w}} \qquad (5.34)$$

and inserting these values in **Eq. 5.32** yields the following relation

$$p_p = y_m\,y_p\sqrt{\frac{F_{tm}}{bd_{m1}}\frac{\sqrt{u^2+1}}{u}} \qquad (5.35)$$

The value of the pitch point coefficient y_p can be taken as **1.76** for straight sided bevel gears in normal cases. For material coefficient y_m, see Table **2.17** (Sec. **2.23**).
Recalling equation is given in Sec. 5.9, we have

$$d_{m1} = z_1\frac{5}{6}m,\ b = \frac{z_1 m}{6\sin\delta_1}\ \text{and}\ \sin\delta_1 = \frac{z_1 m}{2R}$$

With shaft angle = 90°, $R = \sqrt{r_1^2 + r_2^2} = \frac{1}{2}\sqrt{d_1^2 + d_2^2} = \frac{1}{2}z_1 m\sqrt{u^2+1}$

By inserting the values, we get

$$\sin\delta_1 = \frac{1}{\sqrt{u^2+1}}\ \text{and}\ b = \frac{z_1 m}{6}\sqrt{u^2+1}$$

Putting all the above values in Eq. **5.35**, we have

$$p_p = y_m\,y_p\sqrt{\frac{17.28\,T_1}{z_1^3 m^3}\times\frac{1000}{u}}\ (\text{N/mm}^2) \qquad (5.36)$$

In the above equation, F_{tm} has been replaced by the relation

$$F_{tm}\,(\text{N}) = \frac{2T_1\,(\text{Nm})}{d_{m1}(\text{mm})}\times 1000$$

By transposing and inserting the expression for the allowable value of contact pressure p_{cp}, and solving for the module, we get

$$m = \frac{25.85}{z_1}\sqrt[3]{\frac{T_1}{u}\left(\frac{y_m y_p}{p_{cp}}\right)^2} \qquad (5.37)$$

In the example that follows, the relevant equations given in this section as well as in Sec. 5.9 will be used to solve a bevel gear problem.

Example 5.2 *Given:* Shaft angle = 90°, nominal input power = 6 kW, i = 3, n_1 = 750 rpm, z_1 = 20, α = 20°. Tooth surface should be hardened. Prime mover is electric motor.

To calculate the design data for a pair of bevel gears confirming to the above specifications. Take impact factor = 1.75 for continuous duty with considerable impact. Gears are to be mounted on anti-friction bearings in a gear box.

Solution: The following data are taken

Material of pinion 40 Cr 4
Material of gear 45 C 8

The materials are flame or induction hardened, with the following strength data taken from Appendix E

$$\sigma_{bp1} = 200 \text{ N/mm}^2,\ \sigma_{bp2} = 180 \text{ N/mm}^2$$

Taking a factor of safety of 1.5, we have

$$\text{Allowable contact pressure } (p_{cp}) = \frac{\text{Surface fatigue strength } (p_{sc})}{1.5}$$

Therefore

$$p_{cp1} = 1620/1.5 = 1080 \text{ N/mm}^2,\ p_{cp2} = 1640/1.5 = 1093 \text{ N/mm}^2,$$

$z_2 = i,\ z_1 = 3 \times 20 = 60$. To have hunting tooth action, the value of z_2 is taken to be 61.

Hence $i = z_2/z_1 = 61/20 = 3.05$
Since it is a step-down arrangement, $u = z_2/z_1 = i = 3.05$.
Therefore

$$n_2 = 750/3.05 \approx 246 \text{ rpm},\ \tan\delta_1 = z_1/z_2 = 20/61$$

whence

$$\delta_1 = 18°9'9.74'',\ \delta_2 = 90 - \delta_1 = 71°50'50.26''$$

$$\text{Nominal torque on the pinion shaft} = 9550 \times \frac{P_1}{n_1} = 9550 \times \frac{6}{750} = 76.4 \text{ N m}$$

Taking impact factor into account, the effective torque T_1 = 76.4 x 1.75 = 133.7 N m. Using **Eq.** 5.30, we have

$$m = 3\sqrt{\frac{34\,T_1 \sin\delta_1\ 1000}{z_1^2\ \sigma_{bp1}}} = 3\sqrt{\frac{34 \times 133.7 \times \sin 18°9'9.74'' \times 1000}{20^2 \times 200}} = 2.6$$

Module selected (m) = 3 mm

$$d_1 = z_1 m = 20 \times 3 = 60 \text{ mm},\ d_2 = z_2 m = 61 \times 3 = 183 \text{ mm}$$

$$R = d_1/2\sin\delta_1 = 60/2 \times \sin 18°9'9.74'' = 96.293 \text{ mm}$$

$$b = R/3 = 32 \text{ mm},\ b = 30 \text{ mm (taken)}$$

$$d_{m1} = d_1 - b\sin\delta_1 = 60 - 30 \sin\delta_1 = 50.65 \text{ mm},\ m_m = d_{m1}/z_1 = 2.53 \text{ mm}$$

Check for m_m **and** b: Referrings to Eqs 5.9 and 5.10 (Sec. 5.6) and taking $\lambda = 25$ from Table 2.12, we have

$$m_m(\min) = 2\frac{b}{\lambda} = 2\frac{30}{25} = 2.4 < 2.53\,\text{mm}$$

$$b_w = \frac{a}{2}m_m = \frac{25}{2} \times 2.53 = 31.63 > 30\,\text{mm}$$

The accepted values are, therefore, within the allowable limits for m_m and b.

$$z_{v1} = \frac{z_1}{\cos\delta_1} = \frac{20}{\cos 18°\,9'9.74''} = 21.048$$

$$z_{v2} = \frac{z_2}{\cos\delta_2} = \frac{20}{\cos 71°\,50'50.26''} = 195.8$$

$$u_v = u^2 = z_{v2}/z_{v1} = 9.3025$$

$$d_{m1v} = \frac{z_1 m_m}{\cos\delta_1} = z_{v1}m_m = 21.048 \times 2.53 = 53.25\,\text{mm}$$

$$d_{m2v} = \frac{z_2 m_m}{\cos\delta_2} = z_{v2}m_m = 185.8 \times 2.53 = 495.37\,\text{mm}$$

$$\text{Velocity}\,(v) = d_m\ \pi\ n_1/60000 = 50.65 \times 3.14 \times 750/60000 = 2\,\text{m/s}$$

$$F_{tm} = 2T_1/d_{m1} = 2 \times 133.7 \times 1000/50.65 = 5279\,\text{N}$$

Since standards as regards tolerance *vis-a-vis* velocity are not available for bevel gears, we shall use Table 2.15 which is valid for spur and helical gears.

Quality of tolerance = 10 (taken)

For quality 10, q_e can be taken as 1. From Fig. 2.49, we get

$$q_{k1} = 2.85 \text{ for } z_{v1} = 21.048$$

Figure 2.49 gives the values of q_k up to $z_v = 100$. By extending the curves of Fig. 2.49 and interpolating, the values of q_k for z_v above 100 can be obtained. Thus

$$q_{k2} \approx 2.15 \text{ for } z_{v2} \approx 195.8$$

$$\sigma_{b1} = \frac{F_{tm}}{bm_m}q_{k1}\,q_e = \frac{5279}{30 \times 2.53} \times 2.85 = 198\,\text{N/mm}^2$$

$$\sigma_{b2} = \frac{F_{tm}}{b\,m_m}q_{k2}\,q_e = \frac{5279}{30 \times 2.53} \times 2.15 = 150\,\text{N/mm}^2$$

(taking $q_e = 1$ in both th se)

Since both the above values lies within the allowable values given earlier, σ_{bp1} and σ_{bp2}, the system is safe against failure by bending

$$p_p = y_m y_p \sqrt{\frac{F_{tm}}{bd_{m1}}\frac{\sqrt{u^2+1}}{u}} = 269 \times 1.76\sqrt{\frac{5279}{30 \times 50.65} \times \frac{\sqrt{3.05^2+1}}{3.05}} = 905\,\text{N/mm}^2$$

Hence, the system is safe against surface stress also since the above value is less than the allowable contact pressure given earlier.

5.11 Blanks and Mountings for Bevel Gears

Since the solids on whose surfaces the pitch cones of a pair of bevel gears are in rolling contact, the blanks on which the teeth of bevel gears are cut, are also obviously conical in shape.

For any type of gear, the ultimate quality of the gear in the finished form is largely dependent on the design and accuracy of the gear blank. In general, for bevel gears and for other types of gears as well, the blank should be so designed and treated that the localised stresses are avoided. Care should be taken to avoid deflections within the blank. The designer should provide adequate amount of metal under the tooth root to ensure proper support. This should be at least as much as the height of the tooth and this metal depth should be maintained throughout the length of the tooth.

The other important criteria for proper blank design, are the ease of machining and appropriate hardening conditions in case of hardened gears. Another factor of vital importance is the provision of a suitable locating surface on the back end of the blank. This surface should be very carefully machined or ground, because checking and mounting will be done with reference to this surface. (See mounting distance in Fig. 5.3.)

For mounting, the shaft angle of the axes of the bevel gears must be exactly the same as the design shaft angle. Though very small deviations do take place from the ideal condition in actual practice, care should be taken to restrict these deviations to as small a value as possible, as otherwise serious running difficulties will be encountered. The axes must intersect. While mounting the bevel gears, the mounting distances a5 given in the drawing must be rigidly observed so that the gears are mounted at the correct distances from the cone apex.

As for bearings, the journal bearings are not considered suitable. Anti-friction bearings give better results. To minimise the detrimental effect of deflection, bearings should be properly spaced. The spacing will depend upon the stiffness of the shafts carrying the gears and the orientation of mounting, that is, whether the gears are straddle mounted or overhung mounted.

Mountings should be rigid so that the displacements of the gears under service conditions are kept within allowable limits. To minimise the detrimental effects of misalignment, proper alignment of the gears should be ensured. Besides, mountings should be accurately machined, keys should be properly fitted and couplings accurately mounted.

As a result of experience and tests carried out by the reputed manufacturers, the following permissible deviations may be taken as guidelines: for gears of 150 to 400 mm diameter, the lift or depression of gear at the centre of the face width should not exceed 0.08 mm, the axial movement of the pinion should be restricted to 0.08 mm in either direction, and that of the gear to 0.25 mm for gear ratios greater than one.

5.12 Efficiency of Bevel Gear Drive

For all practical purposes, the efficiency of straight bevel gear drive and of spiral bevel gear drive can be found by using the formulae used for spur and helical gear drive. These formulae, however, are to be used with certain modifications as explained below.

In both the cases, the virtual number of teeth are to be used instead of the actual number of teeth. The relations are given by

$$z_v = \frac{z}{\cos\delta} \text{ in case of straight bevel gear}$$

$$= \frac{z}{\cos\delta \cos^3 \beta_m} \text{ in case of spiral bevel gear}$$

where β_m = The middle spiral angle of the spiral bevel gear.

Using Eq. 2.11, the modified expression for the efficiency of straight bevel gear is given by

$$\eta = 1 - f\mu \left(\frac{\cos\delta_1}{z_1} + \frac{\cos\delta_2}{z_2} \right) \tag{5.38}$$

Using Eq. 3.71, we get the following value for the spiral bevel gear

$$\eta = 1 - 0.8 \cos\beta_m f \mu \left(\frac{\cos\delta_1 \cos^3\beta_m}{z_1} + \frac{\cos\delta_2 \cos^3\beta_m}{z_2} \right) \tag{5.39}$$

The different factors in the above formulae have the same meanings and values as given previously in the relevant sections (Secs 2.6, 2.29 and 3.18).

6
Spiral Bevel Gears

6.1 General Classification of Spiral Bevel Gears

Spiral bevel gear is a term generally used in case of bevel gears that have teeth curved longitudinally along the length of the teeth. Though, strictly speaking, bevel gears like hypoid gears and a few others which resemble spiral bevels, do not fall in this category, nevertheless since "spiral bevel gear" is a loose term, we will include in this class the varieties of bevel gears which do not have straight teeth. A class of bevel gears having straight teeth but are tangent to a circle concentric to the centre of the gear (i.e. the teeth do not pass through the apex when extended but are obliquely placed on the conical blank) is also sometimes referred to as spiral bevel gears. These gears are also known as "skew bevel gears" or "oblique spiral bevel gears".

The curved-toothed bevel gears which are normally encountered in practice will be discussed in this chapter. The main advantage of these gears over the straight-toothed varieties lies in the fact that as more teeth are in contact at the same time because of the curved-shaped contour of the teeth, a smoother meshing action between the mating pair is ensured. In view of the shape of the tooth, the curved toothed bevel gears have special properties which are summarised below.

1. Longer tooth-engagement time is effected diie to simultaneous meshing of a number of teeth.
2. Greater contact ratio can be achieved.
3. The meshing action is gradual and progressive over the whole length of the gear teeth.
4. The noise level is considerably small.
5. The flank and the root strengths of teeth are greater.
6. The minimum number of teeth to avoid undercutting is reduced, thus allowing a design with greater reduction ratio for the same space requirements, if necessary.
7. In general comparatively higher transmission ratio is achievable.
8. These gears are not unduly sensitive to fluctuating types of loads. Hence, these gears can take care of misalignment in mounting and bearing systems.
9. Above all, spiral bevel gears have greater load carrying capacity than the straight -toothed bevel gears, other parameters remaining the same.

Broadly, these gears have similar advantages over the straight-toothed bevel gears as the helical gears have over the spur gears.

Depending on the type chosen, spiral bevel gears may have constant tooth height along the width or the height may decrease progressively from periphery towards the centre. Moreover, the shape of the tooth will depend on the manufacturing method employed and on the profile of the cutter used.

The curved toothed bevel gears can be broadly grouped into seven categories discussed here.

1. The spiral bevel gears that have teeth shaped like the arc of a circle along the length are patented by the Gleason Works of Rochester, USA. They are generally used for speeds greater than **5** m/s or 1000 rpm, though for special application where smoothness and quietness are the main design criteria, they may also be run at lower speeds. The spiral angle is normally 35°. The tooth height decreases progressively towards the centre.

2. Zerol bevel gears have also been developed by the Gleason Works. When the spiral angle is zero, the spiral bevel gear is known as zerol bevel gear. Zerol gears are used mostly in high-precision instruments where it is often necessary to have almost zero backlash. The teeth of zerol gears can be accurately finish-ground and therefore they are ideally suitable where very hard tooth surfaces must be accurately finished.

3. Hypoid gears are the special spiral bevel gears which are mounted on non-intersecting, cross-axis shafts, having an offset between the two shaft axes. Among others, hypoid gears offer two distinct advantages.

(*i*) Because the pinion shaft or drive shaft occupies a lower position than the gear shaft, hypoid gear pairs have been adopted by many automobile manufacturers for use in the differential for the rear axle drive of the car. This is due to the fact that a lowered drive shaft permits a lower-floored body of the vehicle. The centre of gravity of the system is thereby lowered which brings greater stability as a consequence.

(*ii*) In industrial applications, where there is a sufficient offset between the two shafts, the shafts may pass one another with enough clearance, thus permitting the use of a compact straddle mounting on the gear and the pinion. Gang drive is thus possible so that several hypoid pinions can be arranged on a single shaft with drives to several machines from a single input shaft.

The surfaces of the pitch solids of these gears are hyperboloids. If the offset is zero, a pair of hypoid gears becomes an ordinary spiral bevel gear pair.

4. Bevel gears with Archimedian spiral have tooth-traces that follow an Archimedian spiral. These gears are not much used in industries.

5. Spiral bevel gears with involute tooth trace are used in case of the Palloid type of spiral bevel gears developed by a German manufacturer, Klingelnberg. The height of tooth remains nearly constant along the tooth width.

6. Eloid spiral bevel gears, developed by a Swiss manufacturer, Oerlikon, have epicycloids or hypocycloids as spirals. The height of tooth is constant.

7. Kurvex toothed gears, generated by special cutters, are similar to the Gleason system, except that the tooth height remains constant throughout. The longitudinal contour of the tooth along its length is an arc of a circle as in the Gleason system.

Besides the above mentioned categories, certain other types which are usually known by their trade names, also come under the general category of spiral bevel gears. These are-coniflex, beveloid, spiroid, planoid, helicon and other types.

The different. types of bevel gears are illustrated in Fig. **6.1.** These gears are represented in a plane, that is, they are the crown gears of the respective types of gears. The crown gear has already been defined and described in Sec. **5.3.**

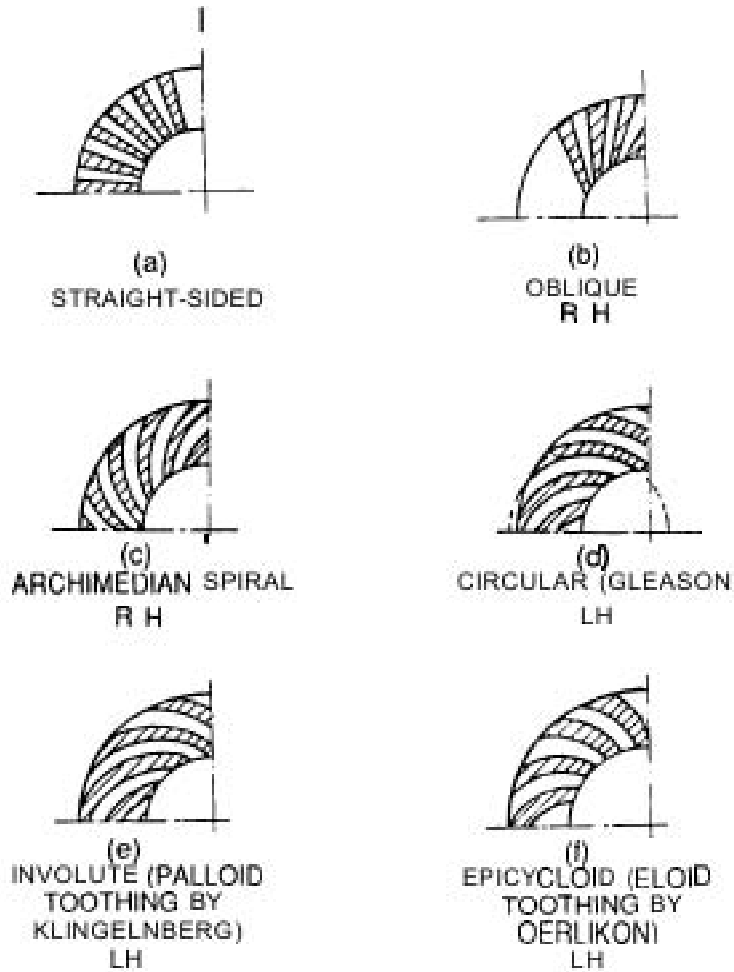


Fig. 6.1 Types of straight-sided and spiral bevel gears

The spiral bevel gears are also categorised according to the method of manufacture.

1. Generation by hobbing in part with cutter head: Gleason and Arcoid systems of toothing are produced by this method. In these systems, the tooth curve is a circular arc and the tooth height is tapered from back towards the apex. The generating machine is fitted with a circular cutter head which carries straight-sided cutter at the front side. The teeth of Kurvex gears are also manufactured by this system of generation, but here special cutters with two cutter heads are used. The teeth of these gears are of constant height throughout the length.
2. Generation by continuous spiral hobbing with cutter head: The Oerlikon gears belong to this category. The tooth is of epicycloid curve or hypocycloid and is of constant height. The generation is effected continuously by the cutter head fitted with a series of cutters.
3. Generation by continuous spiral hobbing with conical or cylindrical hob: The Klingelnberg

gears are produced by this system. The tooth curve is involute and the tooth height is nearly constant. The generation is done by a spiral type cutter or a bevel gear type conical hob cutter.

6.2 Spiral Bevel Gear Geometry and Basic Relations

The basic terminology of spiral bevel gears is the same as in the case of the straight-sided bevel gears. The relevant terms have been defined in Sec. 5.3. The terms as well as the relations which are pertinent to spiral bevel gears only, or are modified because of the spiral angle, will be defined and discussed in this Section.

Spiral Angle

Two spiral bevel gears in mesh are shown in Fig. 6.2. The spiral bevel dimensional parameters have been depicted on the sectional views of the gears as well as on the crown gear.

Referring to the geometry of the pitch plane of the crown gear, the spiral angle β_m is defined as the angle which is subtended at the point of intersection of the tooth spiral and the middle circle of radius R_m, and is contained between the tangent to the curve at the point of intersection and the radial through that point. The spiral angle along with other parameters, such as the face advance, face width and circular pitch are shown in a simplified manner in Fig. 6.3.

The spiral angle as defined above is called the middle spiral angle or simply spiral angle in common usage. The spiral angle, however, varies at different cone distances. The spiral angles at the tip and the root circles are denoted by symbols β_a and β_i respectively. At any cone distance R', the spiral angle β' for Gleason spiral bevel gears can be found from Eq. 6.1

$$\sin\beta' = \frac{R_m}{R'}\left(\sin\beta_m + \frac{R'^2 - R_m^2}{2R_m r_c}\right) \tag{6.1}$$

where r_c is the radius of curvature of the mean spiral curve as shown in Fig. 6.2. Though the spiral angle varies according to design considerations, its usual value is 35°.

Since the teeth are curved, a considerable amount of overlap results when two such gears mate. Contact of more than one pair of teeth at all time is assured. As emphasised in Sec. 6.1, the tooth engagement is gradual and continuous because of the spiral teeth which is also conducive to smooth and quiet running. Moreover, for the same size, the spiral bevels have greater load carrying capacity than the straight sided ones.

When other design factors permit, the spiral angle should be so selected that a face contact ratio of at least 1.25 is assured. Maximum smoothness of drive, however, is attained when the face contact ratio is between 1.50 and 2.0.

Hand of Spiral

When viewed from the front, that is, from apex, if the tooth makes a clockwise spiral from the base winding towards the apex, then it is designated as a right-handed spiral. The reverse is true for the left-handed spiral. In a meshing pair, the hands of the spirals of the component members are always opposite. In gear technology, it is customary to identify a combination by the hand of spiral of the pinion which is normally the driving member. It may be mentioned here that the hand of spiral plays no part as far as the smoothness or quietness of operation and the efficiency of the gear-set are concerned.

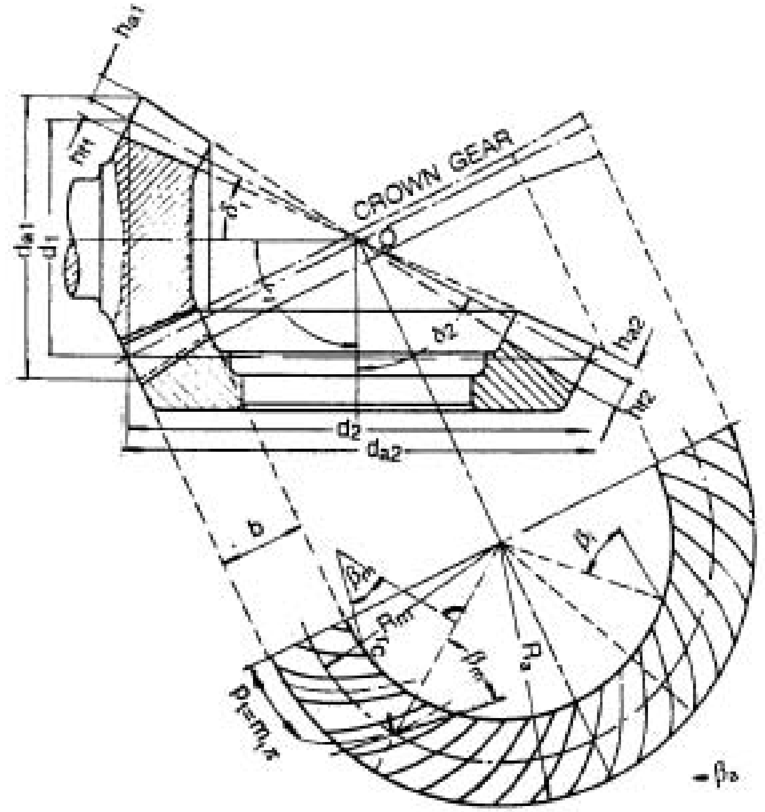


Fig. 6.2 Spiral bevel gear geometry

Based on Maschinenelemente, Niemann, vol. II, 1965 edition, Fig. No. 133/2, p. 133. Springer Verlag. Heidelberg

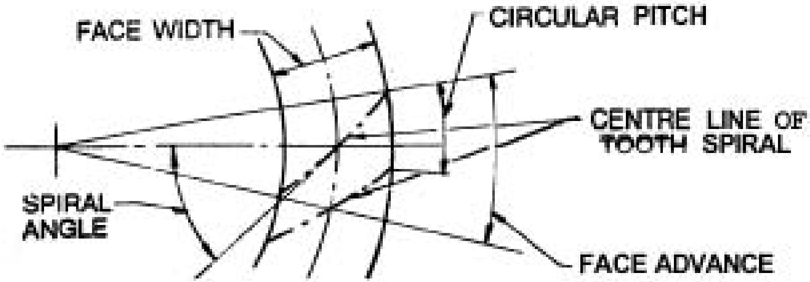


Fig. 6.3 Spiral angle of spiral bevel gear

Direction of Rotation

The direction of rotation is determined as viewed from the back, that is, not from the apex. Accordingly, the shaft along with its mounted gear is said to rotate in clockwise or anti-clockwise direction.

Selection of Direction of Spiral and Rotation

The selection of hand of spiral and direction of rotation will depend on several factors, e.g. type and magnitude of load, mounting conditions, effect of backlash, etc. There is no choice on a reversible drive but the following guidelines may be given for an ordinary drive, which will determine the proper selection.

First, the following rules are to be kept in mind

(i) An LH pinion, driving clockwise, tends to move axially away from the cone centre.
(ii) An RH pinion, driving clockwise, tends to move axially towards the cone centre.
(iii) An LH pinion, driving anti-clockwise, tends to move axially towards the cone centre.
(iv) An RH pinion, driving anti-clockwise, tends to move axially away from the cone centre.

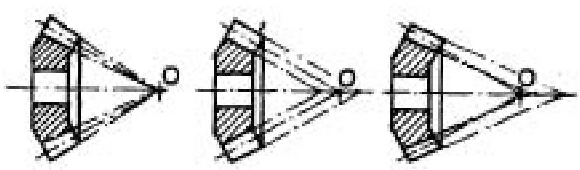
The resultant effect of the thrust loads should be carefully considered before final selection is made. If the mounting condition is such that there is adequate end-play in the pinion shaft, then an RH pinion, driving clockwise, will have a tendency to reduce the backlash and the teeth of pinion and gear may wedge together. An LH pinion in such condition will introduce additional backlash. Usually, when the gear ratio, pressure angle and spiral angle permit, the hand of spiral should be so selected as to allow the axial thrust which will tend to move both the pinion and the gear out of mesh. If not, then only the pinion should be allowed to tend to move out of mesh.

Shapes of Bevel Gear Cones

Comparative shapes of cones of different spiral bevel gears have been shown in Fig. 6.4. Point O in each case represents the point of intersection of the axes of the mating pair. The relative difference of positions of the pitch cone, the blank cone and the root cone in case of the Gleason, Oerlikon and the Klingelnberg systems can be easily seen.

Hand of Spiral, Direction of Rotation and their Relation to Forces

In case of spiral bevel gears, including hypoid gears, the spiral angle introduces additional force components when compared to straight bevel gears. In a helical gear, an axial thrust results because of the helix angle as we have seen in Chap. 3 on helical gears. But this axial thrust is a function of the helix angle alone. In case of spiral bevel gears, however, the spiral angle gives rise to both axial and radial components.



(a) GLEASON (b) OERLIKON ELOID (c) KLINGELNBERG PALLOID

Fig. 6.4 Centres of cones of bevel gears

Based on Maschinenelemente, Niemann, vol. II, 1965 edition, Fig. No. 137/1, p. 137. Springer Verlag, Heidelberg

It can be seen from Fig. 3.7 that the direction of the axial thrust depends upon the hand of helix, the direction of rotation, relative position of the driver and the driven gear, and whether the gear is the driving or the driven member. In case of spiral bevel gears, in addition to the above factors, the pitch cone angle is also a determining factor. All these factors determine whether the axial component will make the gear move away or towards the cone centre.

As far as the direction of the resultant axial component is concerned, the fundamental difference between the straight and zerol bevel gears on one side, and the spiral and hypoid gears on the other is that in the former case, the axial components always tend to force the pinion and the gear out of mesh, while in the latter case, the direction to which the thrust force acts may be either towards or away from the cone centre, depending upon the algebraic sign of the calculated resultant force. Conventionally, the axial force which tends to move the two components out of mesh is regarded as positive and the other one as negative. It follows, therefore, that in case of spiral bevels and hypoids, since the resultant axial component may, as a consequence of both the pressure angle and the spiral angle, act in either direction, the choice of the direction and magnitude of the spiral angle is a prime consideration of the designer. The component gears may move away, creating unnecessarily a large backlash, or they may move towards each other into a tight mesh which may result in jamming or seizure. Since proper running of spiral bevel and hypoid gears necessitates the right kind of support against a possible axial displacement of either member of the gear-set, meticulous design calculations of thrust forces are imperative for the selection of the proper kind of bearings and other allied factors. Figure 6.5 illustrates the relevant relations between the hand of spiral, direction of rotation, and the axial thrust forces.

Resolution of Forces

The main forces acting on spiral bevel gear teeth are

(*i*) The tangential tooth load F_t

(*ii*) The axial thrust F_a

(*iii*) The radial separating force F_r

All the above forces are referred to the mid-point of tooth

The Tangential tooth load This load is given by

$$F_{t1}\ (\text{N}) = \frac{2T_1}{d_{m1}} \times 1000\ (d_{m1} \text{ in mm})$$

Where F_{t1} = Tangential tooth load on pinion, and is equal in magnitude to the tangential tooth loads on gear F_{t2} in case of spiral bevel gears. The tangential tooth loads on pinion and gear are different in case of hypoid gears.

$$T_1\,(\text{Nm}) = 9550\,\frac{P_1}{n_1} \times K = \text{Maximum torque on the pinion}$$

Here, P_1 is the nominal motor power in kW, and n_1 is the motor speed in rpm.

Factor K takes care of the effect of additional operational and impact forces. Selection of K will depend on the service conditions and experience. Values given in Table 2.16 (Sec. 2.22) for ϕ can be taken for K too for all practical purposes.

***The** axial thrust* The axial forces acting on the gear components can be computed from Table 6.1. If the resultant axial force is of positive sign, then the force is directed away from the cone apex. Negative sign indicates movement towards the apex. The relation between the force acting on a spiral bevel gear and its proper algebraic sign for the forces are shown in Fig. 6.5(b).

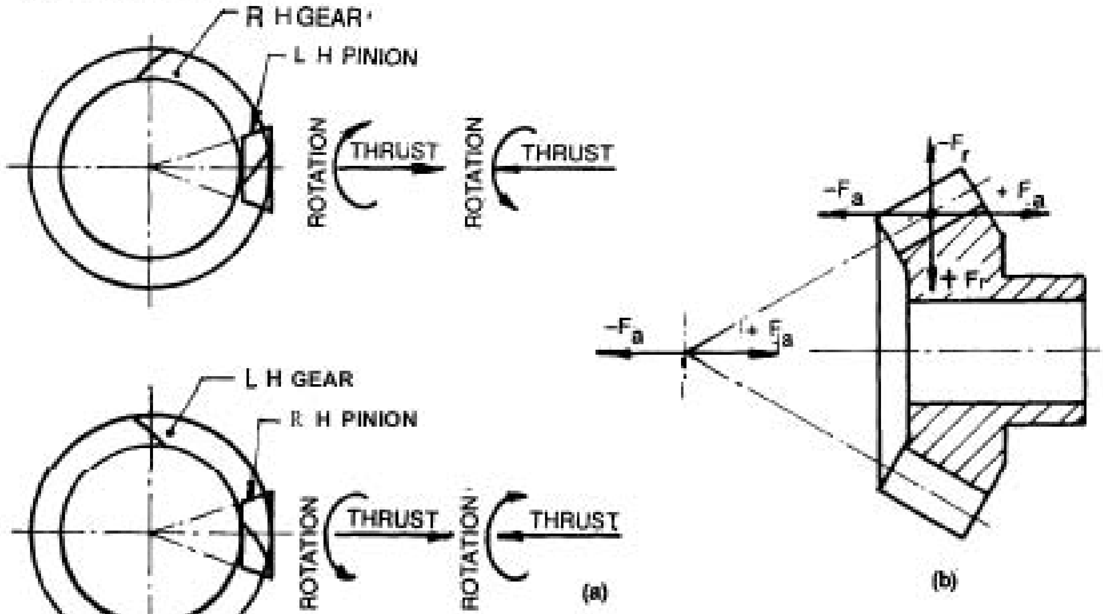


Fig. 6.5 Direction of forces in spiral bevel gear

The radial separating force The radial separating force acting on the gear components can be computed from Table **6.2.**

If the resultant radial separating force is of positive sign, then the force is directed towards the gear axis, that is, away from the mating member. Negative sign indicates a direction away from the gear axis, that is, towards the mating member.

Table 6.1 Formulae for the determination of axial forces acting on bevel gears

Driver		
Hand of spiral	*Direction of rotaton*	*Magnitude of axial thrust*
Right	Clockwise	Driver $F_a = \frac{F_t}{\cos\beta}(\tan\alpha\sin\delta - \sin\beta\cos\delta)$
Left	Anti-clockwise	Driven $F_a = \frac{F_t}{\cos\beta}(\tan\alpha\sin\delta + \sin\beta\cos\delta)$
Right	Anti-clockwise	Driver $F_a = \frac{F_t}{\cos\beta}(\tan\alpha\sin\delta + \sin\beta\cos\delta)$
Left	Clockwise	Driven $F_a = \frac{F_t}{\cos\beta}(\tan\alpha\sin\delta - \sin\beta\cos\delta)$

Table 6.2 Formulae for the determination of radial separating force on bevel gears

Hand of spiral (Driver)	Direction of rotation	Magnitude of separating force
Right	Clockwise	Driver $F_r = \frac{F_t}{\cos\beta}(\tan\alpha\cos\delta + \sin\beta\sin\delta)$
Left	Anti-clockwise	Driven $F_r = \frac{F_t}{\cos\beta}(\tan\alpha\cos\delta - \sin\beta\cdot\sin\delta)$
Right	Anti-clockwise	Driver $F_r = \frac{F_t}{\cos\beta}(\tan\alpha\cos\delta - \sin\beta\sin\delta)$
Left	Clockwise	Driven $? = \frac{F_t}{\cos\beta}(\tan\alpha\cos\delta + \sin\beta\sin\delta)$

In each of the above cases, while using the above formulae given in the tables for the determination of different forces, care should be taken to ensure that the relevant values of parameters, corresponding to the particular member in question are inserted. That is, proper subscripts are attached to the forces and angles, e.g. subscript 1 for pinion and 2 for gear or 1 for the driving member and 2 for the driven member, as the case may be. The normal pressure angle (α) is the pressure angle on the driving side of the tooth.

Direction for Use of Tables 6.1 and 6.2

In a condition where the driver is of RH spiral and the direction of rotation is anti-clockwise and the driver may be pinion or gear, the driven member is obviously of **LH** spiral and rotates in clockwise direction.

From Table **6.1**, complying with the above arrangement of drive, we have

$$\text{Axial force of driver} = F_a = \frac{F_t}{\cos\beta}(\tan\alpha\sin\delta + \sin\beta\cos\delta)$$

Taking the pinion to be the driver and assigning the subscript 1 to the pinion and 2 to gear, we have

$$F_{a1} = \frac{F_{t1}}{\cos\beta}(\tan\alpha\sin\delta_1 + \sin\beta\cos\delta_1)$$

$$F_{a2} = \frac{F_{t2}}{\cos\beta}(\tan a\sin\delta_2 - \sin\beta\cos 6,)$$

The tangential forces are the same. Hence, $F_{t1} = F_{t2} = F_t$. F_t can be calculated from the given data, such as power and speed, as per the usual formulae. **As** an example, take $\alpha = 20°$, $\beta = 35°$, $\Sigma = \delta_1 + \delta_2 = 90°$ and $\delta_1 = 30°$, $\delta_2 = 60°$.

From the above equations, F_{a1} is obviously positive. The axial force of the driven (gear) member is

$$F_{a2} = \frac{F_t}{\cos 35^\circ} (\tan 20^\circ \sin 60^\circ - \sin 35^\circ \cos 60^\circ) = F_t \times 0.03468$$

Since F_{a2} is also positive, both the driver and the driven member tend to move away from the apex. The same equations, used in the above example, are also valid for a driver having LH spiral and clockwise rotation mating with a driven gear having RH spiral and anti-clockwise rotation. Table 6.2 for radial forces can be similarly used for calculation and checking.

***Stress** Calculations and **Power** Rating*

Since the curved toothed bevel gears are produced by different types of machines and since these gears also have different geometrical configurations depending on the manufacturing processes involved, it is advisable to perform calculations according to the instructions of the manufacturers of the relevant machines and cutting processes.

As far as the strength calculations are concerned, the load carrying capacity of the spiral bevel gears, in general, can be taken to be around 15 to 25% higher than that of the corresponding straight-sided bevel gears.

In Sec. 6.1, we have seen that there exist various systems according to which the spiral bevel gears are manufactured. Each system has its own geometrical characteristics arising out of its own peculiar generating mechanism, by means of which the spiral bevel gears belonging to that particular system are produced. Hence, the tooth design will vary accordingly. Gear calculations and dimensional parameters based on one system will not be valid for those conforming to some other system. Besides, a designer may have his own idiosyncrasies as regards the design criteria and procedures for a particular system of spiral bevel gear. It is, therefore, always prudent to have a dialogue with the supplier before the designer undertakes to fix the relevant parameters and makes the necessary manufacturing drawings. This way, a fruitful result will be ensured because the supp[illegible] due to the lack of proper equipments and manufacturing facilities, may or may not be in a position to make the gear if the drawings and calculations are made beforehand without prior consultation.

In many industrial organisations which lack the facilities for producing spiral bevel gears in their own shops, but which need these gears for replacement as spare parts for their machines or gear boxes, the usual practice is to off-load these items to firms specialised in the manufacture of spiral bevel gears. The design offices of such organisations make manufacturing drawings of these gears for all other body portions except the teeth portions, which are shown as solid, truncated conical blanks with sufficient machining allowances. The firm which gets the order is free to cut the teeth portions according to its own system, provided it fulfils the customer's requirements and pre-conditions as regards power, torque, strength properties, reduction ratio, dime[illegible]ional and space constraints, speed, guaranteed life, inspection procedures and all other releva[illegible] parameters. The gears, thus produced and delivered, should fit in the machine without difficulty and should run smoothly during their operational life.

6.3 Gleason System of Spiral Bevel Gears

One of the pioneers of the spiral bevel gear technology is the Gleason Works of Rochester, USA. The Gleason system was originally developed in the FPS units. Though metric Gleason system

has also been developed, most of the spiral bevels using the Gleason system are still DP gears. Hence, in this section, the FPS units are used for convenience.

Table 6.3 Parameters for Gleason spiral bevel gears

(All dimensions are in inch units)

Description	*Pinion*	*Gear*
Number of teeth	z_1	z_2
Shaft angle	$\Sigma = 90$	
Addendum	h_{a1} = Working depth $- h_{a2}$	$h_{a2} = \frac{1}{P}\left[0.46 + 0.39\left(\frac{z_1}{z_2}\right)^2\right]$
Dedendum	$h_{f1} = h - h_{a1}$	$h_{f2} = h - h_{a2}$
Working depth	$\frac{1.700}{P}$	
Whole depth	$h = \frac{1.888}{P}$	
Clearance	$c = \frac{0.188}{P}$	
Pitch circle diameter	$d_1 = \frac{z_1}{P}$	$d_2 = \frac{z_2}{P}$
Pitch cone angle	$\delta_1 = \tan^{-1}\frac{z_1}{z_2}$	$\delta_2 = 90° - \delta_1$
Cone distance	$R = \frac{d_2}{2\sin\delta_2}$	
Circular pitch	$p = \frac{\pi}{P}$	
Tip circle diameter	$d_{a1} = d_1 + 2h_{a1}\cos\delta_1$	$d_{a2} = d_2 + 2h_{a2}\cos\delta_2$
Dedendum angle	$\theta_{f1} = \tan^{-1}\frac{h_{f1}}{R}$	$\theta_{f2} = \tan^{-1}\frac{h_{f2}}{R}$
Face angle of blank	$\delta_{a1} = \delta_1 + \theta_{f2}$	$\delta_{a2} = \delta_2 + \theta_{f1}$

Table 6.4 Minimum number of teeth to avoid undercutting

i	$z_{1\,min}$	$z_{2\,min}$	i	$z_{1\,min}$	$z_{2\,min}$
1	17	17	1.43	14	20
1.12	16	18	1.69	13	22
1.26	15	19	2.16	12	26

As stated in the beginning of this chapter, the spiral-curve of a Gleason bevel gear is an arc of a circle. The gear parameters and relations which are given here are as per the 1952 Revision of Gleason Works. These data pertain to spiral bevel gears for general industrial purposes. The teeth are manufactured by the generation process. The validity of the data is for gears with 12 DP and coarser. The Gleason Works have listed 5 categories of gears which do not conform to the standard proportions given here. These may be considered as special designs and include spiral bevels for automotive rear-axle drives, gears of finer pitch than 12, ratios having lesser number of teeth than 12 in the pinion, and some other special types.

The main emphasis of Gleason design is on practical operating requirements. According to the present day Gleason design, the face cone element of a blank is made parallel to the root cone element of the mating gear. The purpose of this is to ensure constant clearance throughout the length of the tooth from back to front. This also permits the use of larger edge radii on the cutters without fillet interference at the small end. The pressure angle for the system is 20° and the spiral angle is 35°.

Spiral Angle of Gleason System

As stated before, the spiral angle is 35° which is most commonly used. Calculations for equal stress in pinion and gear have been based and determined using this angle. However, for other spiral angles from 20° to 45°, the same tooth proportions can be used. Special proportions are required for angles below 20°. High spiral angle is conducive to greater face contact ratio, smoother and quieter operation, but it affects the ensuing thrust loads in service.

The common data which are required for the Gleason system are given in Tables 6.3 and 6.4. To keep uniformity with metric gears, the symbols and subscripts used are the same as in the case of metric gears, though inch units are used.

The unit of diametral pitch P is inch^{-1}.

6.4 Zerol and Hypoid Gears

Zerol Bevel Gears

When the spiral angle of a spiral bevel gear is designed to have a value of zero, it is termed as a zerol bevel gear. As indicated earlier, this is a patented item of the Gleason Works.

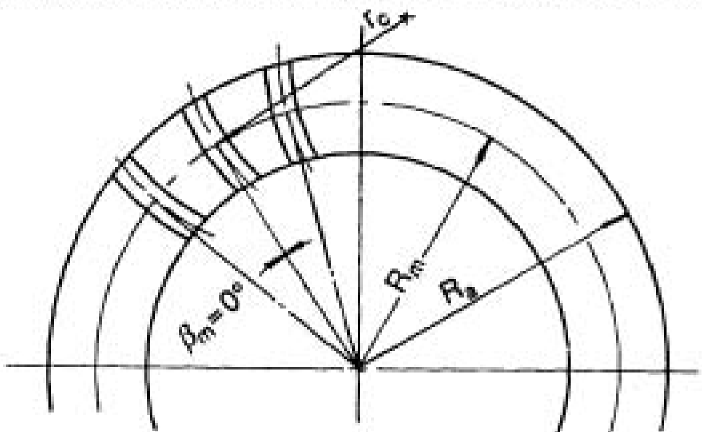


Fig. 6.6 Zerol bevel gear

The zerol bevel gears have curved teeth the orientation of which is in the same general direction as the straight teeth of the straight-sided bevel gears. The zerol gears can have ground teeth. Hence, when hardened gears having extreme accuracy and precision are required, such as those used in aircraft industries, these gears are recommended in preference to the straight-sided bevel gears.

The same machines which are used for producing spiral bevel gears can be made to produce zerol bevel gears. As far as the tooth action and the end thrust are concerned, they have the same effect as the straight-sided ones.

The zerol gear geometry is such that the face cone parameters do not pass through the pitch cone apex. They are more or less parallel to the root cone elements of the mating gear. The reason for such configuration is the same as that adopted in case of certain types of designs of straight bevels, namely, to have uniform tooth clearance along the length of the teeth.

Zerol gears can be used up to a circumferential velocity of around 5 m/s. At higher speeds, they tend to be noisy.

When extreme smoothness of tooth action and quietness of drive are imperative, these gears are highly recommended by the gear designers. Journal bearings can be used for mountings, though anti-friction bearings give better results. However, plain bearings offer compactness of design and are less expensive, and this is one of the reasons why straight and zerol bevels are extensively used in differentials. Other types of bevel gears require more elaborate mountings to compensate for the different types of forces which are produced thereby.

Design calculations involved in zerol gears are comparatively simple, and are similar to those of straight-sided bevel gears. The machine settings for cutting these gears are also easy. Zerol bevels are generally recommended when the loads are light. They are also satisfactory for high static loads and when the wear of the tooth surfaces is not a critical factor. The basic pressure angle is 20°, though when avoidance of undercutting is necessary, pressure angles of 22.5° and 25° are also used. The face width is generally limited to one-fourth of the cone distance.

Hypoid Gears

The hypoid gears resemble the spiral bevel gears in general appearance. The shape of the tooth is similar to that of the spiral bevel. One notable difference is that in case of a spiral bevel, the pitch solid is a cone whereas it is a hyperboloid of revolution in case of a hypoid gear.

The axis of the hypoid pinion may be offset above or below the axis of the gear. This is shown in Fig. 6.7. Besides the advantages mentioned in Sec. 6.1, the hypoid gears offer the following special advantages.

1. So far as smooth and quiet running is concerned, hypoid gears are more suitable than ordinary spiral bevel gears.
2. Hypoid gears can easily produce large speed reductions. Transmission ratios of 60:1 and higher are quite common in industrial applications using these gears.
3. The hypoid drive is compact and the pinion strength is high. When compared to the straight or zerol bevels, the hypoid drive makes it possible to reduce the overall size of the installation with a smaller number of teeth in the pinion.

Generally, the hypoid pinion is designed to have a larger **spiral** angle than the gear, thereby making the pinion diameter bigger which in turn results in a pinion stronger than the corresponding spiral bevel pinion. The increased pinion diameter permits the use of comparatively high speed ratios, at the same time allowing the pinion-body to have enough material left

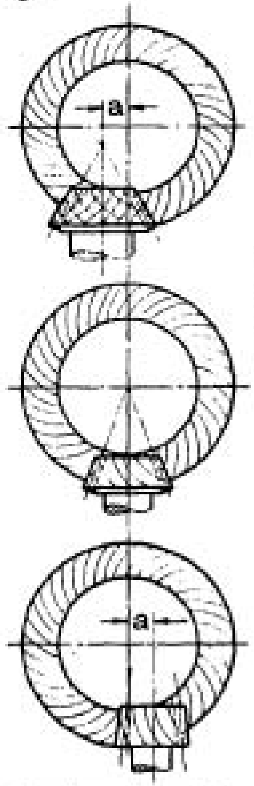


Fig. 6.7 Orientations of hypoid gear
Based on Maschinenelemente, Niemann, Vol. II, 1965 edition, Fig. No. 144, p. 144. Springer Verlag, Heidelberg

after the requisite hole has been bored to accept the shaft of adequate size commensurate with the torque it is designed to deliver.

In a hypoid drive, there are limitations as to the magnitude of the shaft offset. Normally, the offset should not exceed 40% of the equivalent bevel gear back cone distance. For heavy duty equipments, this should be nearer 20%. Oil tight case with anti-friction bearings should be used as housing mountings, and thrust bearings must be provided.

Because of the peculiar motion between the two members comprising a hypoid drive, the meshing action is subjected to a considerable sliding lengthwise along the teeth. This high degree of sliding gives rise to a greater energy loss. In this respect, a hypoid gear pair is analogous to other types of non-planer gears, such as worm and worm wheel drive. High localised temperatures are generated which may result in welding of metals at the hot spots. The extreme pressure (EP) lubricants inhibit this welding tendency and also sustain high pressures, (see also Sec. 8.6).

It has been found that at low ratios, the hypoid gears are comparatively more efficient. In general, the efficiency of a hypoid gear-set is given by

$$\eta = \frac{\cos\alpha + \mu \tan\beta_2}{\cos\alpha + \mu \tan\beta_1} \times 100\% \qquad (6.2)$$

where β_1 and β_2 are the spiral angles of the pinion and the gear respectively, α is the normal pressure angle and μ is the coefficient of friction. Sliding action across the tooth surfaces entails loss of efficiency. This is a common feature of all non-intersecting gear sets.

6.5 Klingelnberg Palloid System of Spiral Bevel Gears

In Klingelnberg Palloid system, the teeth of the spiral bevel gear have involute-shaped spirals along their lengths. The tooth height remains nearly constant throughout the length of the tooth. These spiral bevel gears are easily mounted and are not affected by bearing inaccuracies. A long life can be expected for these gears if properly designed and maintained. Using the same symbols and subscripts as before, the following relations of parameters are relevant to the system.

To avoid a high spiral angle and the resulting high axial forces, the following relation between the transverse module and the normal module is usually maintained.

$$\frac{m_t}{m_n} \geq 1.5$$

For $\Sigma = 90^\circ$, $\cot \delta_1 = i$, and for $\Sigma \neq 90^\circ$, $\cot \delta_1 = \dfrac{1 + \cos \Sigma}{\sin \Sigma}$ $(\Sigma = \delta_1 + \delta_2)$

$$R_a = \frac{d_2}{2 \sin \delta_2},\ b \leq 10 m_n = \frac{R_a}{3.25 \text{ to } 3.75},\ \cos \beta_i = \frac{m_n z}{d_m},\ R_i = R_a - b$$

Base circle radius of the involute spiral $= z_2 m_n / 2 \sin \delta_2$, $p_t = \pi m_t$; Middle transverse module $m_{tm} = d_m / z$; Middle transverse – pitch $p_{tm} = \pi m_{tm}$; Number of teeth of crown gear $z_c = z_2 / \sin \delta_2$

$$d_1 = m_t z_1,\ d_{m1} = d_1 - b \sin \delta_1,\ d_2 = m_t z_2,\ h_{a1} = m_n (1 + x_1),\ h_{a2} = 2m_n - h_{a1}$$

The correction factor (x_1) is to be inserted from Table 6.5.

$$d_{a1} = d_1 + 2 h_{a1} \cos \delta_1,\ d_{a2} = d_2 + 2 h_{a2} \cos \delta_2$$

Table 6.5 Values of $(1 + x_1)$ for $\alpha = 20^\circ$

z_2	z_1 8	9	10	11	12	13	14	15	16
22	1.27	1.23	1.16	1.04	1.00	1.00	1.00	1.00	1.00
24	1.26	1.22	1.16	1.06	1.00	1.00	1.00	1.00	1.00
26	1.25	1.22	1.16	1.07	1.00	1.00	1.00	1.00	1.00
28	1.25	1.22	1.17	1.08	1.00	1.00	1.00	1.00	1.00
30	1.25	1.22	1.17	1.09	1.00	1.00	1.00	1.00	1.00
35	1.26	1.23	1.18	1.11	1.00	1.00	1.00	1.00	1.00
40	1.28	1.24	1.20	1.13	1.02	1.00	1.00	1.00	1.00
45	1.29	1.25	1.21	1.15	1.05	1.00	1.00	1.00	1.00
50	1.30	1.26	1.23	1.16	1.07	1.00	1.00	1.00	1.00
60	1.33	1.30	1.26	1.20	1.11	1.03	1.00	1.00	1.00
70	1.36	1.32	1.29	1.24	1.16	1.08	1.00	1.00	1.00

Angle correction In Klingelnberg Palloid system, an "angle correction" is carried out. By means of this angle correction of the pitch cone angles δ_1 and δ_2, better tooth action and undercutting can be attained. The magnitude of this correction is small and in this section, simplified formulae have been used, disregarding the angle correction. The reader may refer to more advanced treatise on Klingelnberg system or to the company norms for more information.

A simplified and somewhat modified version of a n example based on DIN 3990 and involving Klingelnberg Palloid system of spiral bevel gears is given in Example 6.1.

Example6.1: In a Palloid system of spiral bevel drive set, having hardened gears, the following data are given :

$z_1 = 9$, $z_2 = 28$, $u = z_2/z_1 = 28/9 = 3.11$, $m_t = 4.875$ mm, $\alpha_n = 20°$,
$\beta_m = 33° 20'$, $\Sigma = 90°$, $x_1 = +0.75$, $x_2 = -0.75$, $b = 20$ mm,
working depth of teeth, h_w = 170 m, , whole depth of tooth, h = 1888 m, ,
cutter addendum, h_{a0} = 1038 m, , nominal torque, T_1 = 140 Nm,
service factor = 125.

To calculate the relevant dimensional parameters of the gear set and select suitable materials for the same.

Solution :

$$d_1 = m_t z_1 = 4.875 \times 9 = 43.875 \text{ mm}$$

$$d_2 = m_t z_2 = 4.875 \times 28 = 136.50 \text{ mm}$$

$$\cot \delta_1 = i = u = 3.11, \text{ whence } \delta_1 = 17° 49'$$

$$\delta_2 = 90° - \delta_1 = 90° - 17°49' = 72°11'$$

$$d_{m1} = d_1 - b \sin \delta_1 = 43.875 - 20 \times 0.30597 = 37.755 \text{ mm}$$

$$d_{m2} = d_2 - b \sin \delta_2 = 136.50 - 20 \times 0.95204 = 117.46 \text{ mm}$$

$$R_a = \frac{d_1}{2 \sin \delta_1} = 7168 \text{ mm}, \quad R_m = R_a - \frac{b}{2} = 6168$$

Virtual numbers of tooth are given by

$$z_{v1} = \frac{z_1}{\cos \delta_1 \cos^3 \beta_m} = \frac{9}{\cos 17° 49' \cos^3 33° 20'} = 16.21$$

$$z_{v2} = \frac{z_2}{\cos \delta_2 \cos^3 \beta_m} = \frac{28}{\cos 72° 11' \cos^3 33° 20'} = 156.91$$

Module of virtual toothing

$$m_v = m_{nm} = m_t \cos \beta_m \frac{R_m}{R_a} = 3.50 \text{ mm}$$

The correction factor given in Table 6.5 takes care of the undercutting aspect only. Here, we have a case of S_0–gearing with $x = \pm 0.75$. The correction factors of the virtual toothing are given by

$$x_{v1} = \frac{x_1}{\cos \beta_m} = \frac{+0.75}{\cos 33° 20'} = +0.898$$

$$x_{v2} = \frac{x_2}{\cos \beta_m} = \frac{-0.75}{\cos 33° 20'} = -0.898$$

Pitch circle diameters of the virtual gears

$$d_{v1} = z_{v1}\, m_n = 16.21 \times 3.5 = 56.73 \text{ mm}$$
$$d_{v2} = z_{v2}\, m_n = 156.91 \times 3.5 = 549.18 \text{ mm}$$

Addendum:

$$h_{a1} = \frac{h_w}{2} + x_1 m_t = \frac{170\, m_t}{2} + x_1 m_t = \left(\frac{1.70}{2} + 0.75\right) 4.875 = 7.80 \text{ mm}$$

$$h_{am1} = h_{a1} \frac{R_m}{_k} = 7.8\, \frac{6168}{7168} = 6.71 \text{ mm}$$

$$h_{a2} = \frac{h_w}{2} + x_2 m_t = \frac{170\, m_t}{2} + x_2 m_t = \left(\frac{1.70}{2} - 0.75\right) 4.875 = 0.4875 \text{ mm}$$

$$h_{am2} = h_{a2} \frac{R_m}{_k} = 0.4875\, \frac{6168}{7168} = 0.419 \text{ mm}$$

Dedendum:

$$h_{f1} = h_{ca} - x_1 m_t = (1038 - 0.75)\, 4.875 = 140 \text{ mm}$$

$$h_{fm1} = h_{f1} \frac{R_m}{R_a} = 1.40\, \frac{6168}{71.68} = 1.20 \text{ mm}$$

$$h_{f2} = h_{ca} - x_2 m_t = (1038 + 0.75)\, 4.875 = 8.71 \text{ mm}$$

$$h_{fm2} = h_{f2} \frac{R_m}{R_a} = 8.71\, \frac{61.68}{71.68} = 7.49 \text{ mm}$$

The outside diameter and the root diameter of the virtual gears can now be calculated thus:

$$d_{av1} = d_{v1} + 2h_{am1} = 56.73 + 2 \times 6.71 = 70.15 \text{ mm}$$
$$d_{fv1} = d_{v1} - 2h_{fm1} = 56.73 - 2 \times 120 = 54.33 \text{ mm}$$
$$d_{av2} = d_{v2} + 2h_{am2} = 549.18 + 2 \times 0.419 = 550.02 \text{ mm}$$
$$d_{fv2} = d_{v2} - 2h_{fm2} = 549.18 - 2 \times 7.49 = 534.2 \text{ mm}$$

Now, the tooth profile in this particular example is not as per the standerd reference profile. Hence, the values of form factor, q_k, given in Fig. 2.49 cannot be used here. Analytical determination of these values is extremely complicated. DIN 3990, therefore, recommends that on the basis of the dimensional data of the virtual toothing calculated so far, that is, using the value of z_v, m_n, α_n, x_n and other parameters, and taking fillet radius at the root, $r_v = 0.25 m_v = 0.875$ mm, reasonably appropriate sketches of a virtual tooth of the pinion (S – plus) and of the gear (S – minus) should be drawn as per some suitably enlarged scale. (In this connection, see Fig. 2.47 in Sec. 2.25.)

Having drawn such figures of the virtual toothing of pinion and gear in this particular case, the following data are obtained by actual physical measurement on the drawings by means of suitable drawing instruments:

$$h_{qv1} = 7.50 \text{ mm}, \quad S_{qv1} = 8.25 \text{ mm}, \quad \alpha'_{av1} = 39°40'$$
$$h_{qv2} = 7.00 \text{ mm}, \quad S_{qv2} = 7.75 \text{ mm}, \quad \alpha'_{av2} = 19°\,50'$$

Analogous to Eq. 2.99 in Sec. 2.25, we have the following expression for the form factor of the virtual toothing

$$q_{kv} = \frac{6\, m_v\, h_{gv} \cos \alpha'_{av}}{S_{av}^2 \cos \alpha_t}$$

Inserting the relevant values, we get

$$q_{kv1} = \frac{6 \times 3.5 \times 7.5 \times \cos 39^\circ 40'}{8.25^2 \times \cos 20^\circ} = 1.89$$

$$q_{kv2} = \frac{6 \times 3.5 \times 7.0 \times \cos 19^\circ 50'}{7.75^2 \times \cos 20^\circ} = 2.45$$

The value of the contact ratio factor, q_e, is taken to be 1 for both pinion and gear. The tangential force or the transmitted load at the pitch circle is given by

$$F_{tm}\,(\mathrm{N}) = \frac{2T_1\,(\mathrm{Nm}) \times \text{Service factor}}{d_{m1}\,(\mathrm{mm})} \times 1000 = \frac{2 \times 140 \times 125 \times 1000}{37.755} = 9270$$

The bending stresses are given by

$$\sigma_{b1} = \frac{F_{tm}}{b m_{mn}}\, q_{k1}\, q_e = \frac{9270}{20 \times 3.5} \times 189 \times 1 = 250\ \mathrm{N/mm^2} \quad \text{(See Eqs 5.26 and 5.27)}$$

$$\sigma_{b2} = \frac{9270}{20 \times 3.5} \times 2.45 \times 1 = 324\ \mathrm{N/mm^2}$$

For calculation of contact stresses, the values of material coefficient, y_m, and of pitch point coefficient, y_p, are required. From Table 2.17 y_m = 269 for gears made of steel. The value of y_p can be taken from the graph given in DIN 3990, corresponding to the spiral angle and correction factors. In this case, y_p = 1.53 (from the above graph). In the absence of such graph, somewhat higher value can be assumed for a safe design. The contact stress is given by

$$p_t = y_m\, y_p \sqrt{\frac{F_{tm}}{b\, d_{m1}} \frac{\sqrt{u^2 + 1}}{u}} \quad \text{(see Eq. 5.35)}$$

$$= 269 \times 1.53 \sqrt{\frac{9270}{20 \times 37.755} \frac{\sqrt{3.11^2 + 1}}{3.11}} = 1478\ \mathrm{N/mm^2}$$

The bending and the contact stresses produced in this case are rather on the higher side, and suitable common materials to cope with such conditions are not covered by the gear materials given in Appendix E. Hence, in this case, materials for the pinion and the gear can be selected after consulting product manuals of firms manufacturing high alloy and special purpose steels. These manuals usually contain enough information regarding strength properties, heat treatment and other data to facilitate proper selection of the materials to suit the service conditions.

6.6 Kurvex System of Spiral Bevel Gears

This system can afford to have an economical method of manufacture of spiral bevel gears. As indicated earlier, the Kurvex gear has teeth which are curved in the form of circular arc and the tooth height is constant along the length of the tooth, covering the face width.

As in the case of other spiral bevel gears, the geometrical parameters of the gear are referred to on the crown gear. The reference profile is the tooth profile of middle normal section, and the calculations are based on the normal module at the middle of the tooth, m_{nm}. The axial force increases with increasing spiral angle.

The dimensional parameters of an uncorrected Kurvex gearing system are summarised in Table 6.6. These parameters have been pictorially represented in Figs 6.8 and 6.9. The two gears have been shown in mesh in Fig. 6.10.

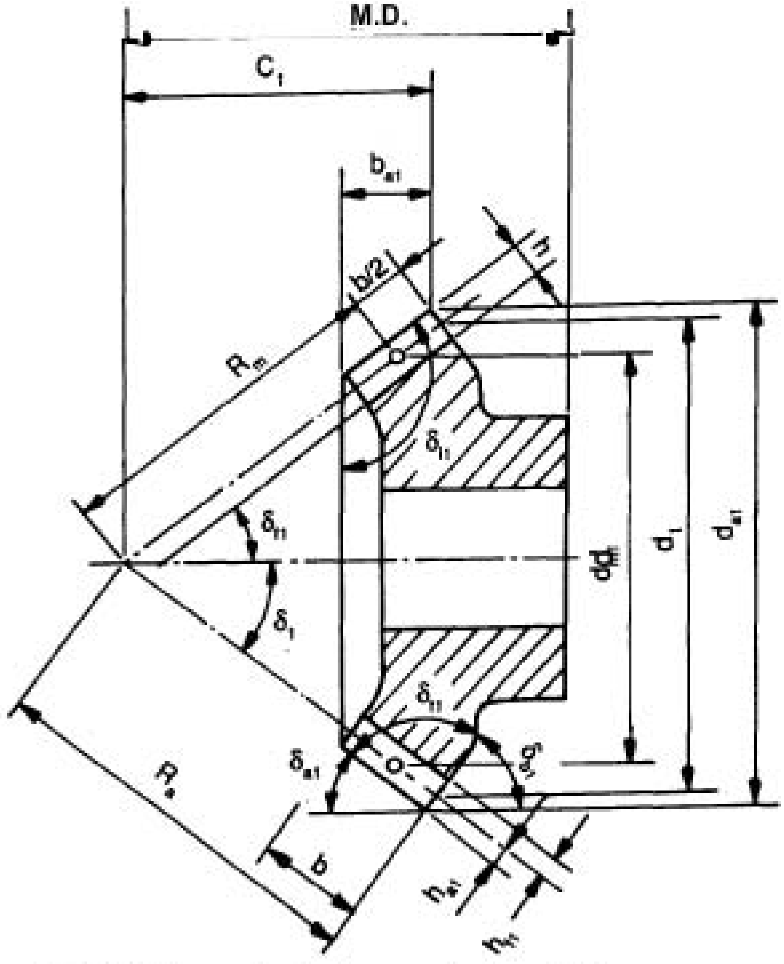


Fig. 6.8 Geometry of a **Kurvex spiral bevel pinion**

Based on Zahnraeder. Zirpke, 11th edition, 1980, Fig. No. 181 p. 283, VEB Fachbuchverlag, Leipzig

Table 6.6 Dimensional parameters of Kurvex spiral bevel gears

Description	*Pinion*	*Gear*
Normal pressure angle	$\alpha = 20^\circ$	
Middle spiral angle	Range : $\beta_m = 25^\circ$ to 40° Usual value : $\beta_m = 35^\circ$	
Number of teeth	$z_1 \geq 12$	z_2
Sum of teeth	$z_1 + z_2 \geq 34$	
Transmission ratio	$i = \frac{n_1}{n_2}$	
Tooth ratio	$u = \frac{z_2}{z_1} \geq 1$	
Pitch cone angle For shaft angle $\Sigma = 90^\circ$	$\tan\delta_1 = \frac{z_1}{z_2}$	
For shaft angle $\Sigma < 90^\circ$	$\tan\delta_1 = \dfrac{\sin\Sigma}{\frac{z_2}{z_1} + \cos\Sigma}$	$\delta_2 = \Sigma - \delta_1 < 90^\circ$
For shaft angle $\Sigma > 90^\circ$	$\tan\delta_1 = \dfrac{\sin(180^\circ - \Sigma)}{\frac{z_2}{z_1} - \cos(180^\circ - \Sigma)}$	
Shaft angle	$\Sigma = \delta_1 + \delta_2$	
Middle pitch cone distance	$R_m = \dfrac{m_{nm} z_1}{2\cos\beta_m \sin\delta_1}$	
Face width	$b \leq \frac{R_m}{3} = \frac{2}{7} R_e$	
Outer pitch cone distance	$R_e = R_m + \frac{b}{2}$	
Outer transverse module	$m_{te} = \dfrac{R_e m_{nm}}{R_m \cos\beta_m}$	
Pitch circle diameter	$d_1 = z_1 m_{te}$	$d_2 = z_2 m_{te}$
Top clearance	$c = 0.2\, m_{nm}$	
Addendum	$h_a = h_f - c = 1.15\, m_{nm} - 0.2\, m_{nm}$	
Dedendum	$h_f = 1.15 m_{nm}$	

Description	Pinion	Gear
Tooth height	$h = h_a + h_f = 2.1\, m_{nm}$	
Tip circle diameter	$d_{a1} = d_1 + 2\, h_a \cos \delta_1$	$d_{a2} = d_2 + 2\, h_a \cos \delta_2$
Blank cone angle	$\delta_{a1} = \delta_1$	$\delta_{a2} = 4$
Root cone angle	$\delta_{f1} = \delta_1$	$\delta_{f2} = \delta_2$
Back cone angle	$\delta_{a1} = \delta_1$	$\delta_{a2} = \delta_1$
Tip cone angle	$\delta_{i1} = \delta_{i1} - 90^\circ$	
Inner cone angle	$\delta_{i1} = 90^\circ + \delta_1$	$\delta_{i2} = 90^\circ + \delta_2$
Crown height	$C_1 = R_a \cos \delta_1 - h_a \sin \delta_1$	$C_2 = R_a \cos \delta_2 - h_a \sin \delta_2$
Tooth width in axial direction	$b_{a1} = b \cos \delta_1$	$b_{a2} = b \cos \delta_2$

Calculation of Kurvex Gearing

The procedure involved in the calculation of a Kurvex gearing system is explained and illustrated in Example **6.1.**

Example 6.2: Given: $\Sigma = 90^\circ$, $\alpha = 20^\circ$, $\beta_m = 35^\circ$, $m_{nm} = 4$ mm, $z_1 = 20$, $z_2 = 26$, $n_1 = 500$ rpm (Direction of rotation : anti-clockwise), Nominal motor power, $P_1 = 2.4$ kW, Hand of spiral : pinion-RH, gear-LH, Continuous drive and overhung bearings.

To find the dimensions of gears, the relevant forces and suitable materials.

Solution:

$$u = \frac{26}{20} = 1.3,\quad n_2 = \frac{n_1}{u} = \frac{500}{1.3} = 385 \text{ rpm (Direction of rotation: clockwise)}$$

$$\tan \delta_1 = \frac{z_1}{z_2} = \frac{20}{26} = 0.76923 \text{ whence } \delta_1 = 37^\circ 34' 7''$$

$$\text{and } \delta_2 = \Sigma - \delta_1 = 90^\circ - 37^\circ 34' 7'' = 52^\circ 25' 53''$$

$$R_m = \frac{m_{nm} z_1}{2 \cos \beta_m \sin \delta_1} = \frac{4 \times 20}{2 \cos 35^\circ \sin 37^\circ 34' 7''} = 80.09 \text{ mm}$$

$$b \leq \frac{R_m}{3} = \frac{80.09}{3} = 26.7 \approx 26 \text{ mm (taken)}$$

$$R_a = R_m + \frac{b}{2} = 80.09 + \frac{26}{2} = 93.09 \text{ mm}$$

$$m_{to} = \frac{R_a m_{nm}}{R_m \cos \beta_m} = \frac{93.09 \times 4}{80.09 \cos 35^\circ} = 5.676 \text{ mm}$$

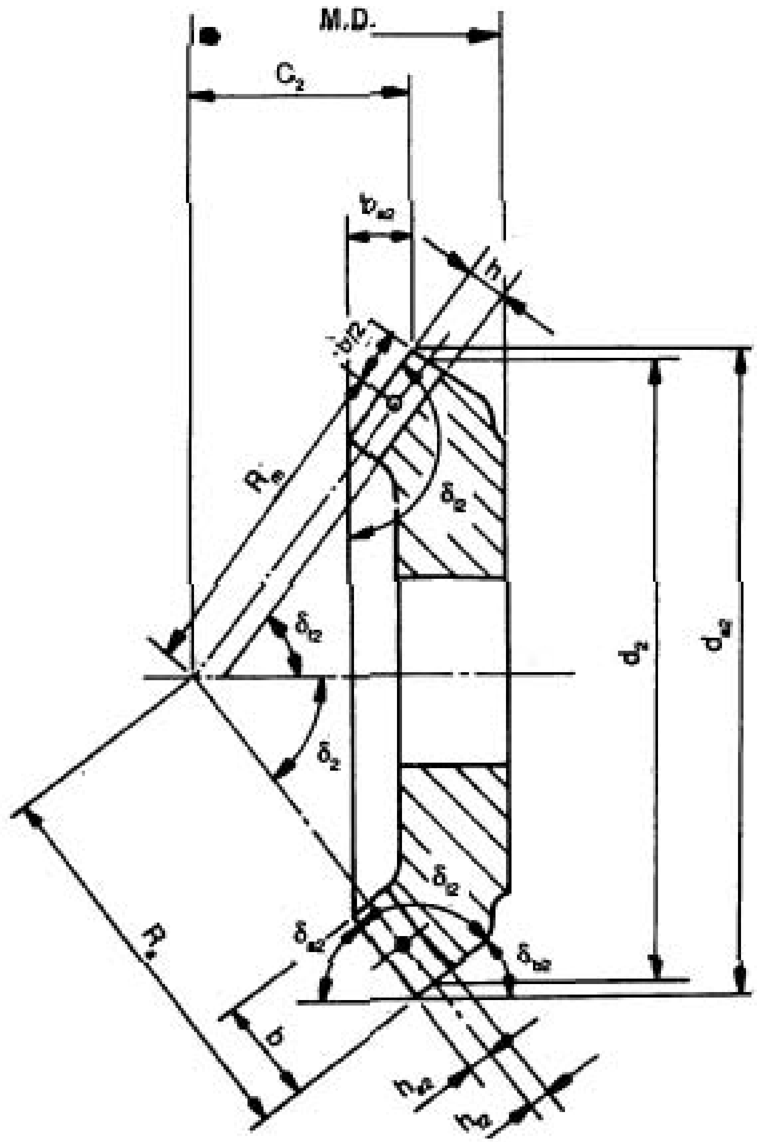


Fig. 6.9 Geometry of a Kurvex spiral bevel gear
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 181 p. 283, VEB Fachbuchverlag, Leipzig

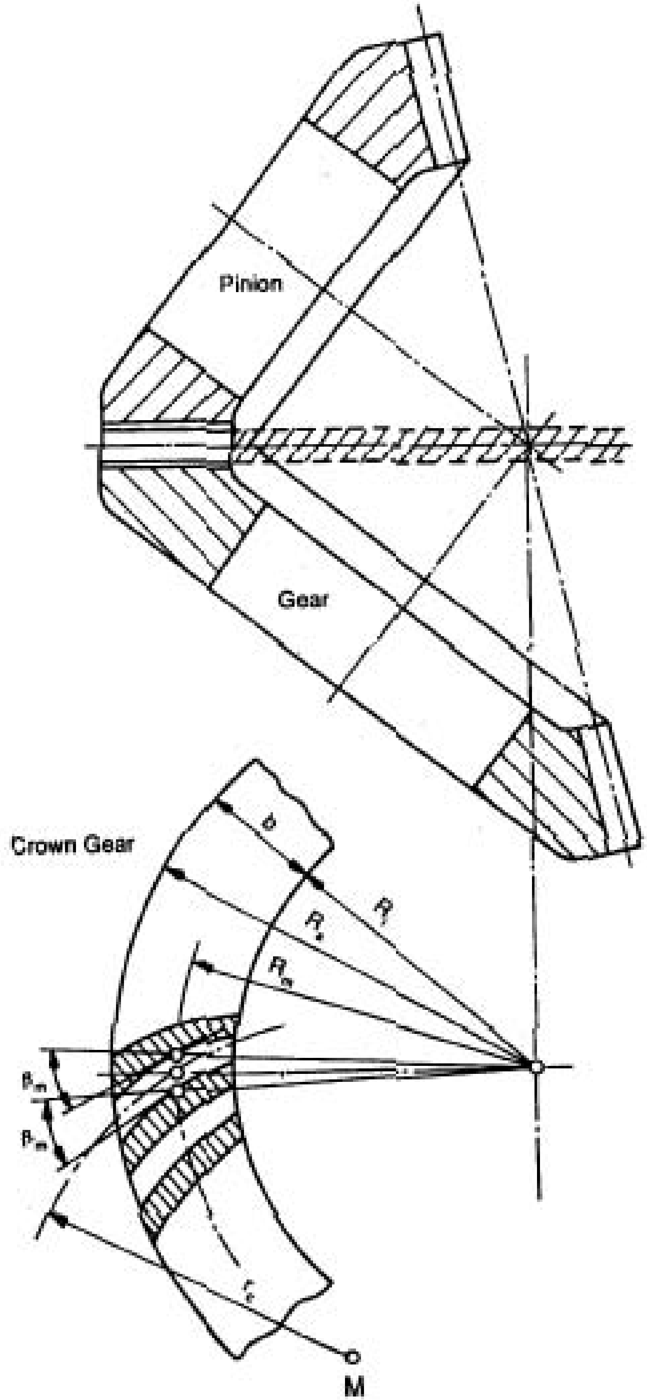


Fig. 6.10 Pair of Kurvex spiral bevel gears in mesh

Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 173 p. 277, VEB Fachbuchverlag, Leipzig

$d_1 = z_1\, m = 20 \times 5.676 = 113.52$ mm

$d_2 = z_2 m_{te} = 26 \times 5.676 = 147.58$ mm

$c = 0.2 m_{nm} = 0.2 \times 4 = 0.8$mm

$h_f = 1.15 m_{nm} = 1.15 \times 4 = 4.60$ mm

$h_a = h_f - c = 4.60 - 0.80 = 3.80$ mm

$h = 2.1\, m_{nm} = 8.40$ mm

$\delta_{a1} = \delta_{f1} = 6_1 = 37° 34' 7''$

$\delta_{a2} = \delta_{f2} = \delta_2 = 52° 25' 53''$

$d_{a1} = d_1 + 2h_a \cos\delta_1 = 113.52 + 2 \times 3.8 \cos 37°34' 7'' = 119.54$ **mm**

$d_{a2} = d_2 + 2h_a \cos 6_1 = 147.58 + 2 \times 3.8 \cos 52° 25' 53'' = 152.21$mm

$b_{a1} = b \cos 6_1 = 26 \cos 37° 34' 7'' = 20.61$mm

$b_{a2} = b\cos 6_1 = 26 \cos 52° 25' 53'' = 15.85$ mm

$C_1 = R_a \cos\delta_1 - h_a \sin 6_1 = 93.09 \cos 37° 34' 7'' - 3.8 \sin 37° 34' 7'' = 7147$ mm

$C_2 = R_a \cos\delta_2 - h_a \sin\delta_2 = 93.09 \cos 52° 25' 53'' - 3.8 \sin 52° 25' 53'' = 53.75$ mm

$\delta_{b1} = \delta_2 = 52° 25' 53''$

$\delta_{b2} = \delta_1 = 37° 34' 7''$

$\delta_{f1} = \delta_{f2} = 90'$

$\delta_{i1} = 90° + \delta_1 = 90' + 37° 34' 7'' = 127° 34' 7''$

$\delta_{i9} = 90° + \delta_2 = 90° + 52° 25' 53'' = 142° 25' 53''$

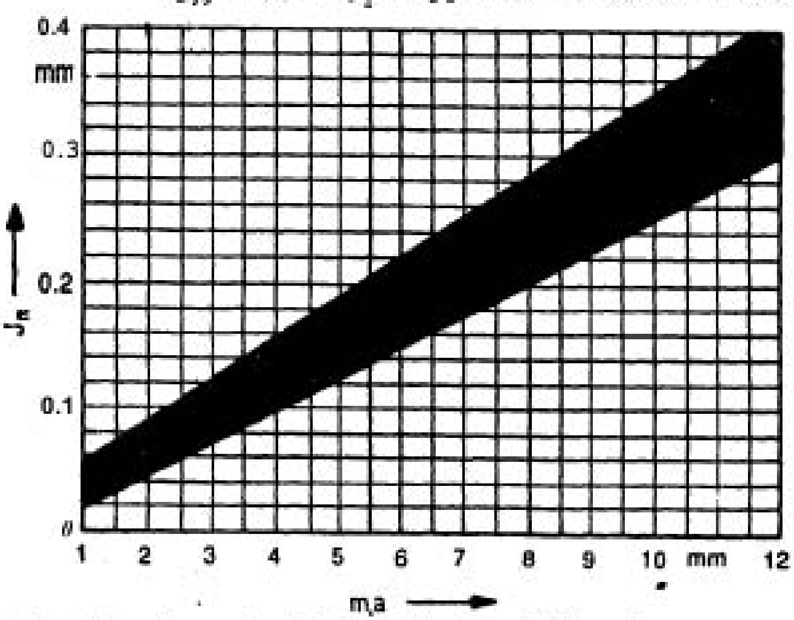


Fig. 6.11 Normal backlash for Kurvex spiral bevel gears

Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 162 p. 272, VEB Fachbuchverlag, Leipzig

For proper running of the gear pair, suitable backlash is to be calculated and indicated in the drawing. Referring to Sec. 2.8, two types of backlashes are to be provided.

These are normal j_n, and the torsional backlash j_t. From Fig. 6.11, we find the normal backlash corresponding to $m_{m} = 5.676$ to be

$$j_n = 0.14 \text{ to } 0.21 \text{ mm} = 0.18 \text{ (taken)}$$

The torsional backlash is given by:

$$j_t = \frac{j_n}{\cos\beta_m \cos\alpha} = \frac{0.18}{\cos 35^\circ \cos 20^\circ} = 0.23 \text{ mm}$$

To determine the maximum torque on the pinion shaft, we recall the following formula

$$T_1 \text{ (Nm)} = 9550 \times \frac{P_1 \text{ (kW)}}{n_1 \text{ (rpm)}} = 9550 \times \frac{2.4}{500} = 45.84$$

The above value of torque is to be multiplied by a service factor K, the values of which have been determined by experimental measurements and experience. For electrical drive, the following guiding values may be taken

Full load, steady: 1.25
Full load, light impact: 1.50
Full load, heavy impact: 1.75

∴ '' he maximum torque $T_1 = 45.84 \times 1.25 = 57.30$ Nm

As stated earlier, the selection of the factor K will depend on the service conditions and experience. From Table 2.16 (Sec. 2.22), we get the values of ϕ whichean be taken for K for all practical purposes. In this case, for steady impact-free full load and having unhardened gear material, which means full vulnerability to wear, the value of ϕ (or K) is found to be around 1.25.

The middle pitch circle diameter of the pinion is given by

$$d_{m1} = z_1 \frac{m_{nm}}{\cos\beta_m} = \frac{20 \times 4}{\cos 35^\circ} = 97.66 \text{ mm}$$

Tangential tooth load at the middle pitch circle is

$$F_{t1} = \frac{2000 T_1}{d_{m1}} = \frac{2000 \times 57.30}{97.66} = 1173 \text{ N}$$

'' he tangential forces are the same. Hence

$$F_{t1} = F_{t2} = F_t$$

Referring to Table 6.1 and 6.2, we now calculate the axial and radial forces. Since the pinion (driver) has R.H. spiral and rotates in anti-clockwise direction, while the driven gear has L.H. spiral and rotates in clockwise direction, the axial forces are as follows (see Table 6.1):

$$F_{a1} = \frac{F_t}{\cos\beta_m} (\tan\alpha \sin 6, + \sin\beta_m \cos\delta_1)$$

$$= \frac{1173}{\cos 35^\circ} (\tan 20^\circ \sin 37^\circ 34' 7'' + \sin 35^\circ \cos 37^\circ 34' 7'')$$

$$= 969 \text{ N}$$

$$F_{a2} = \frac{F_t}{\cos\beta_m}(\tan\alpha \sin\delta_1 - \sin\beta_m \cos\delta_2)$$
$$= \frac{1173}{\cos 35^\circ}(\tan 20^\circ \sin 52^\circ 25' 53'' - \sin 35^\circ \cos 52^\circ 25' 53'')$$
$$= -88\ \text{N}$$

Consulting Fig. 6.5 (b), we find that the pinion tends to move away from the cone apex, while the gear is directed towards the apex.

The radial forces are as follows (see Table 6.2):

$$F_{r1} = \frac{F_t}{\cos\beta_m}(\tan\alpha \cos\delta_1 - \sin\beta_m \sin\delta_1)$$
$$= \frac{1173}{\cos 35^\circ}(\tan 20^\circ \cos 37^\circ 34' 7'' - \sin 35^\circ \sin 37^\circ 34' 7'')$$
$$= -88\ \text{N}$$

$$F_{r2} = \frac{F_t}{\cos\beta_m}(\tan\alpha \cos\delta_1 + \sin\beta_m \sin\delta_2)$$
$$= \frac{1173}{\cos 35^\circ}(\tan 20^\circ \cos 52^\circ 25' 53'' + \sin 35^\circ \sin 52^\circ 25' 53'')$$
$$= 969\ \text{N}$$

Hence, we see that for shaft angle $\Sigma = 90^\circ$

and

$$F_{a1} = F_{r2} = 969\ \text{N}$$
$$F_{a2} = F_{r1} = -88\ \text{N}$$

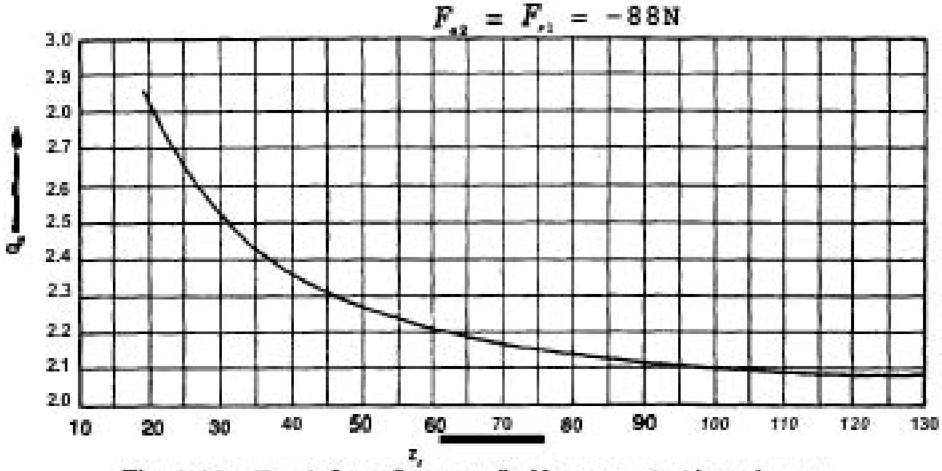


Fig. 6.12 Tooth form factor q_k for Kurvex spiral bevel gears

Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 184 p. 290, VEB Fachbuchverlag, Leipzig

Virtual number of teeth are found from the following expressions

$$z_{v1} = \frac{z_1}{\cos^3\beta_m \cos\delta_1} = \frac{20}{\cos^3 35^\circ \cos 37^\circ 34' 7''} = 45.9$$

$$z_{v2} = \frac{z_2}{\cos^3\beta_m \cos\delta_2} = \frac{26}{\cos^3 35^\circ \cos 52^\circ 25' 53''} = 77.6$$

For uncorrected gear drive, the form factors are obtained from Fig. 6.12 as follows

$$\text{For } z_{v1} = 45.9, q_{k1} = 2.30$$

$$\text{For } z_{v2} = 77.6, q_{k2} = 2.14$$

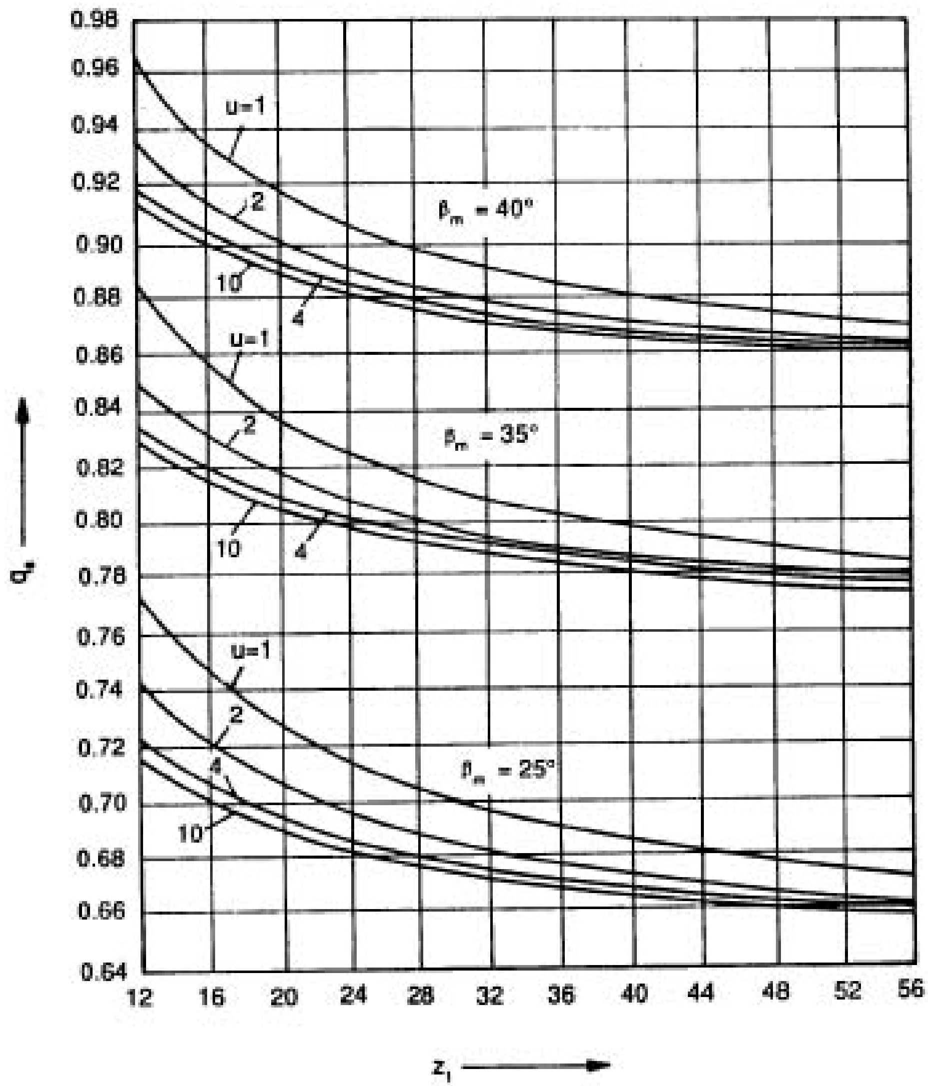


Fig. 6.13 Transverse contact ratio factor q_e for Kurvex spiral bevel gears
Based on Zahnraeder, Zirpke, 11th edition. 1980, Fig. No. 186. p. 291, VEB Fachbuchverlag, Leipzig

The overlap or the transverse contact ratio factor, q_e, is given in Fig. 6.13 for uncorrected gears. By interpolation, we get the following values

For $\beta_m = 35°$, $z_1 = 20$ and u = 1.3, we have

$$q_e = 0.83$$

For overhung bearings, a bearing factor, q_b, of 1.1 is taken. Finally, we arrive at the bending stresses of the pinion and gear from the following equations

$$\sigma_{b1} = \frac{F_t}{b m_{ta}} q_{k1} q_e q_b$$

$$= \frac{1173}{26 \times 4} \times 2.30 \times 0.83 \times 1.1 = 23.7 \text{ N/mm}^2$$

$$\sigma_{b2} = \frac{F_t}{b m_{ta}} q_{k2}\ q_e\ q_b$$

$$= \frac{1173}{26 \times 4} \times 2.14 \times 0.83 \times 1.1 = 22.1 \text{ N/mm}^2$$

It has been emphasized before that for unhardened gears, the contact stress is usually the deciding design criterion. We shall now calculate the surface stresses developed on the teeth of the gears.

From Table 2.17, Sec. 2.23, we get the material coefficient for steel on steel

$$y_m = 269 \sqrt{\text{N/mm}^2}$$

The pitch point coeffcicient is a function of the spiral angle. For $\beta_m = 35°$

$$y_p = 1.51$$

In case of spiral bevel gears, the tooth length factor, y_L, takes care of the different magnitudes of load on the m... teeth during action, especially when the length of contact of the meshing teeth is minimum. This factor is calculated as follows

$$\frac{b}{m_{ta}} = \frac{26}{5.676} = 4.6$$

Corresponding to this value and $\beta_m = 35°$, we find the value of the face contact ratio, CR_{FA}, to be 1.18 from Fig. 6.14. The transverse contact ratio, CR_T, is given by

$$CR_T = \frac{1}{q_e} = \frac{1}{0.83} = 1.2$$

Finally, we get the value of y_L from Fig. 6.15 corresponding to the above values of CR_{FA} and CR_T.

$$Y_L = 0.92$$

The contact stress developed is given by

$$p_c = y_m\ y_P\ y_L \sqrt{\frac{F_t}{b\ d_{m1}} \frac{\sqrt{u^2+1}}{u}}$$

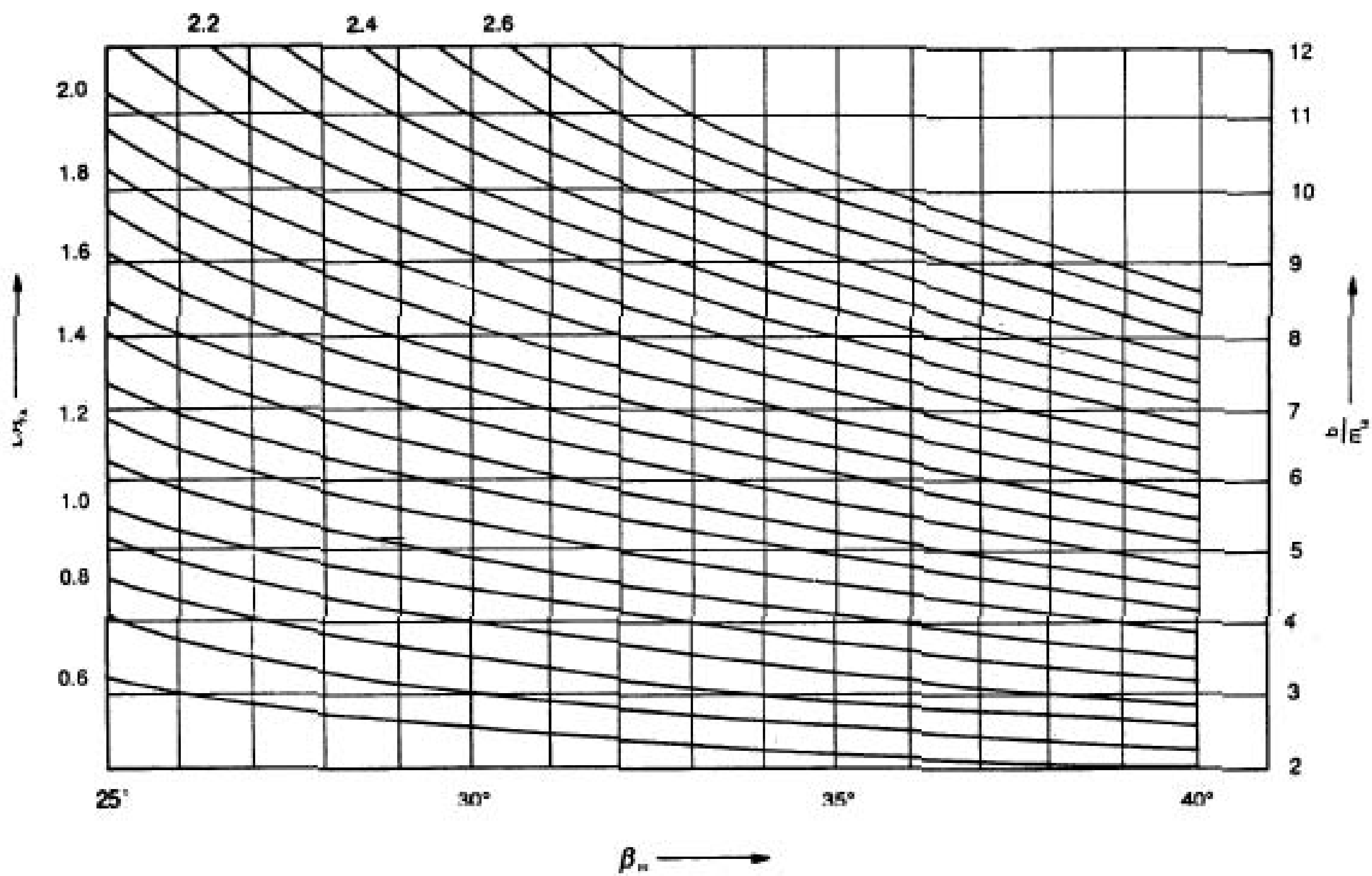


Fig. 6.14 Face contact ratio CR_{FA} for Kurvex spiral bevel gears
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 188, p. 292, VEB Fachbuchverlag, Leipzig

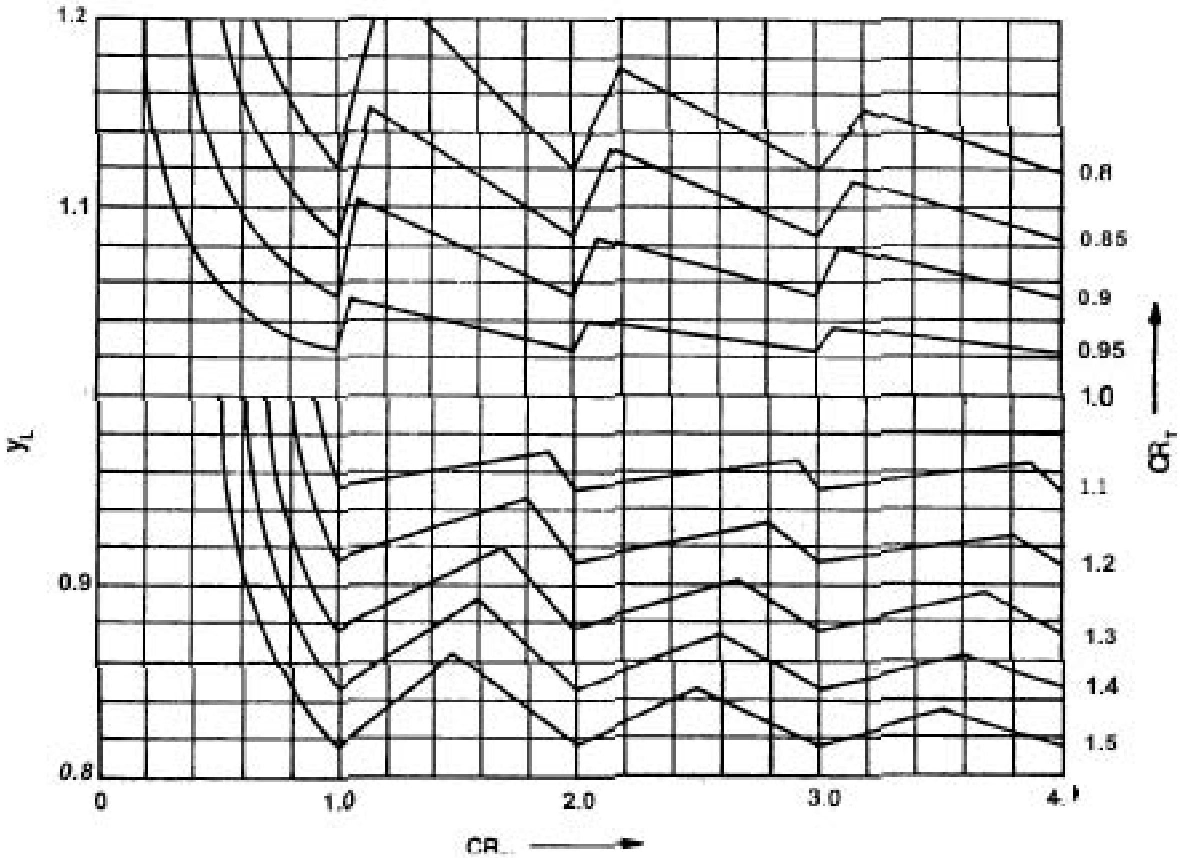


Fig. 6.15 Tooth Length factor y_L for Kurvex spiral bevel gears
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 171 p. 276, VEB Fachbuchverlag, Leipzig

$$= 269 \times 151 \times 0.92 \sqrt{\frac{1173}{26 \times 97.66} \cdot \frac{\sqrt{1.3^2 + 1}}{1.3}}$$

$$= 285 \text{ N/mm}^2$$

As material, we tentatively select the following

For pinion : Fe 690 (formerly: St 70)

For gear : Fe 490 (formerly: St 50)

From Appendix E, we get the following strength values

For Fe 690: yield strength = 410 N/mm²

endurance limit = 370 N/mm²

For Fe 490: yield strength = 290 N/mm²
endurance limit = 255 N/mm²

Using Eq. 2.01, we calulate the permissible bending stress

$$\sigma_{bp1} = \frac{\sigma_{e1}}{2} = \frac{370}{2} = 185\ \text{N/mm}^2 > \sigma_{b1} = 23.7\ \text{N/mm}^2$$

$$\sigma_{bp2} = \frac{\sigma_{e2}}{2} = \frac{255}{2} = 128\ \text{N/mm}^2 > \sigma_{b2} = 22.1\ \text{N/mm}^2$$

For the calulation of allowable contact pressure, the material with lesser yield strength is the deciding criterion

$$p_{cp} = \sigma_{Y2} \cdot C_L = 290 \times 1 = 290\ \text{N/mm}^2 > p_c = 285\ \text{N/mm}^2$$

Here, C_L is the life factor whose value is taken as 1 for continuous drive. From above calculations, we find the materials selected will serve the purpose for this gear-drive.

As stated earlier, Figs 6.8 and 6.9 show drawings for Kurvex pinion and gear. After making the necessary calculations, values of the relevant parameters are to be entered as in the above figures, which should also contain gear data tables. Besides these data, the shop drawings should also contain mounting distance (M.D. in the drawings), mounting instructions, and other usual technical information.

6.7 Arcoid System of Spiral Bevel Gears

The Arcoid system of spiral bevel gears uses a toothing whose lengthwise configuration is a spiral in the form of a circular arc and, unlike the Kurvex system, the tooth height is tapered from back towards the apex. It is similar to the Gleason system and, as mentioned in Sec. 6.1, the toothings of the two systems are generated by the same method.

The Arcoid spiral bevel gears are categorised for use in two types of drives, namely:

1. Drive for automobile transmission, known as System I, with $z_1 < 12$, and
2. Drive for machine tools and general engineering purposes, known as System II, with $z_1 \geq 12$.

Since System II is more in use commercially, this system only will be discussed in this section.

Table 6.7 Dimensional parameters of Arcoid System II spiral bevel gears

Description	*Pinion*	*Gear*
Middle spiral angle	Range: β_m = 35° to 40° Usual value : β_m = 35°	
Top clearance	$c = 0.25 m_{mn}$	
Addendum	$h_{a1} = h_{f2} - c$	$h_{a2} = h_{f1} - c$
Dedendum	$h_{f1} = \frac{R_1}{R_m} m_{mn} (1.25 - x)$	$h_{f2} = \frac{R_2}{R_m} m_{mn} (1.25 + x)$
Tooth height	$h = h_{a1} + h_{f1} = h_{a2} + h_{f2}$	

(Contd)

Table 6.7 *(Contd)*

Description	*Pinion*	*Gear*
Tip circle diameter	$d_{a1} = d_1 + 2h_{a1} \cos \delta_1$	$d_{a2} = d_2 + 2h_{a2} \cos \delta_2$
Addendum angle	$\theta_{a1} = \theta_{f2}$	$\theta_{a2} = \theta_{f1}$
Dedendum angle	$\tan \theta_{f1} = \dfrac{h_{f1}}{R_a}$	$\tan \theta_{f2} = \dfrac{h_{f2}}{R_a}$
Blank cone angle	$\delta_{a1} = \delta_1 + \theta_{a1}$	$\delta_{a2} = \delta_2 - \theta_{a2}$
Root cone angle	$\delta_{f1} = \delta_1 - \theta_{f1}$	$\delta_{f2} = \delta_2 - \theta_{f2}$
Tip cone angle	$\delta_{t1} = 90° - \theta_{a1}$	$\delta_{t2} = 90° - \theta_{a2}$
Inner cone angle	$\delta_{i1} = 90° + \delta_{a1}$	$\delta_{i2} = 90° + \delta_{a2}$
Crown height	$C_1 = R_a \cos \delta_1 - h_{a1} \sin \delta_1$	$C_2 = R_a \cos \delta_2 - h_{a2} \sin \delta_2$
Tooth width in axial direction	$b_{a1} = b \dfrac{\cos\delta_{a1}}{\cos\theta_{a1}}$	$b_{a1} = b \dfrac{\cos\delta_{a2}}{\cos\theta_{a1}}$

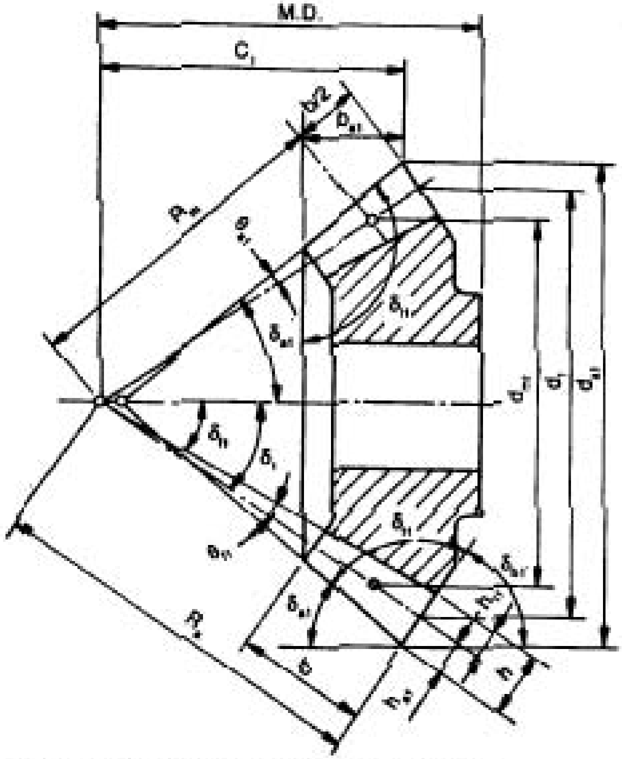


Fig. 6.16 Geometry of an Arcoid spiral bevel pinion

Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 158 p. 264, VEB Fachbuchverlag, Leipzig

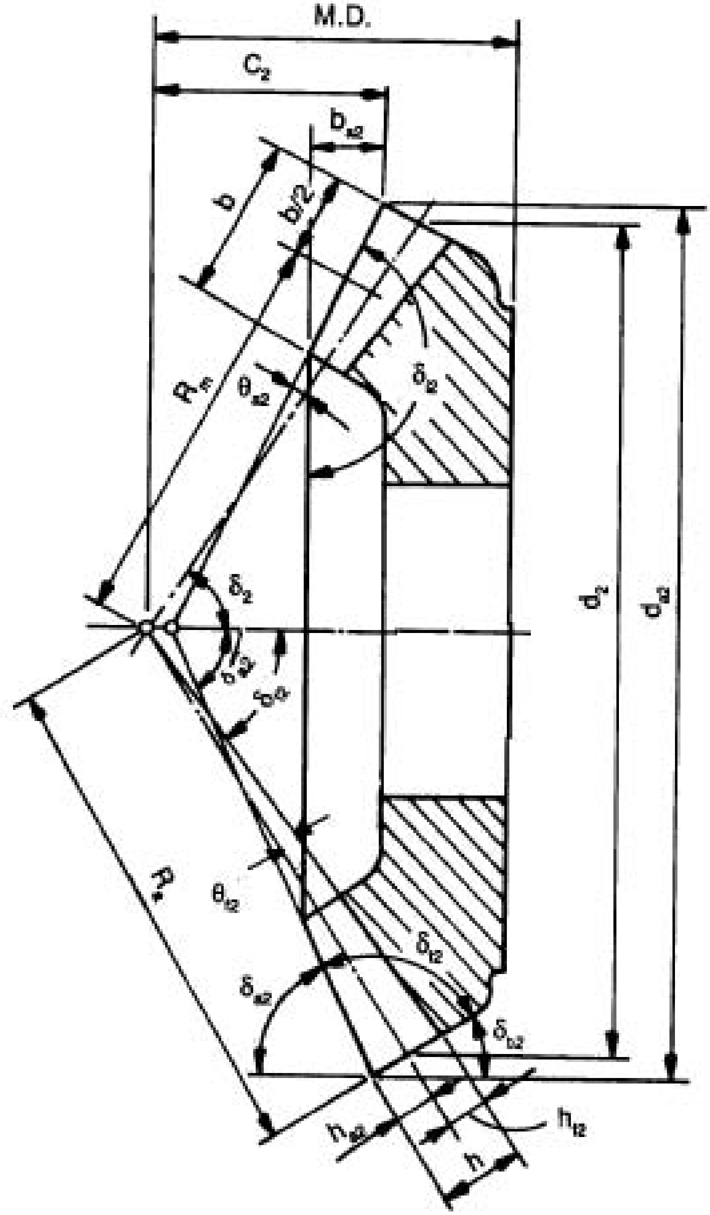


Fig. 6.17 Geometry of an Arcoid spiral bevel gear
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 158 p. 264, VEB Fachbuchverlag, Leipzig

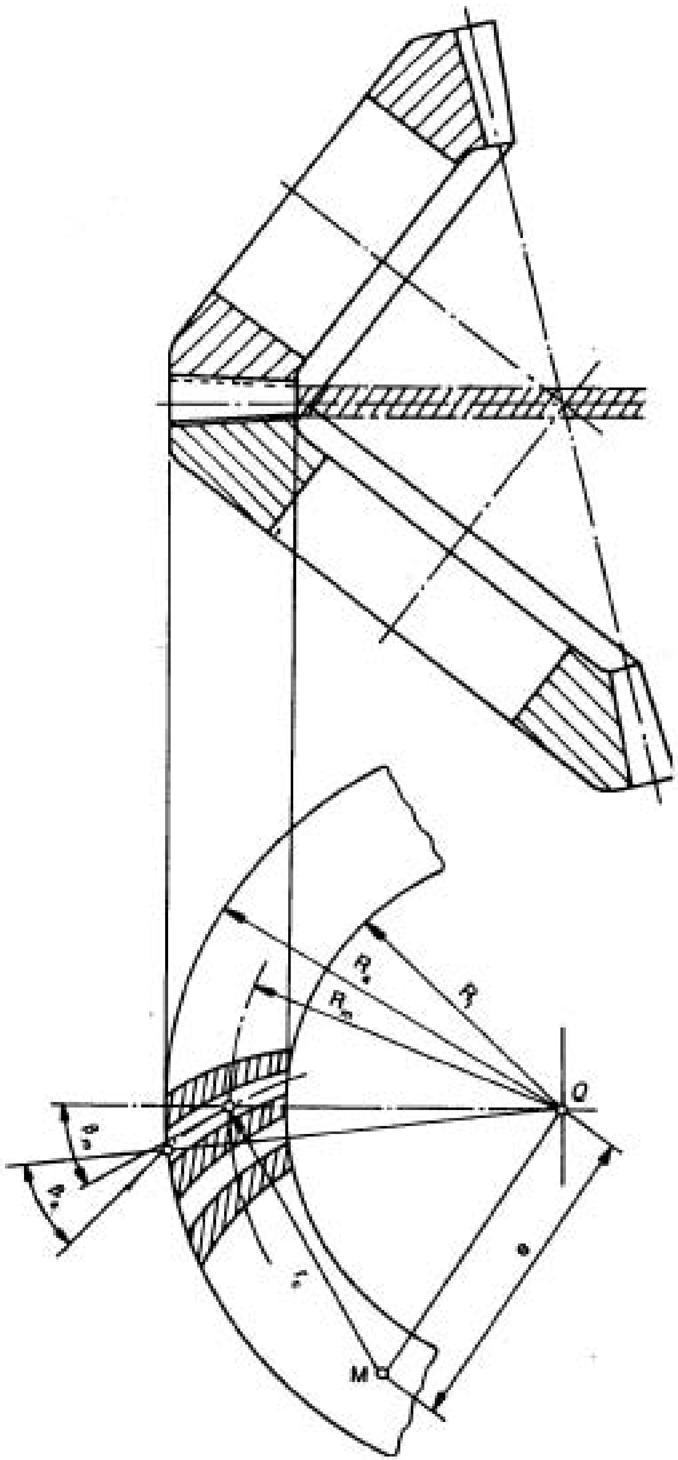


Fig. 6.18 Pair of Arcoid spiral bevel gears in mesh
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 156, p. 261, VEB Fachbuchverlag, Leipzig

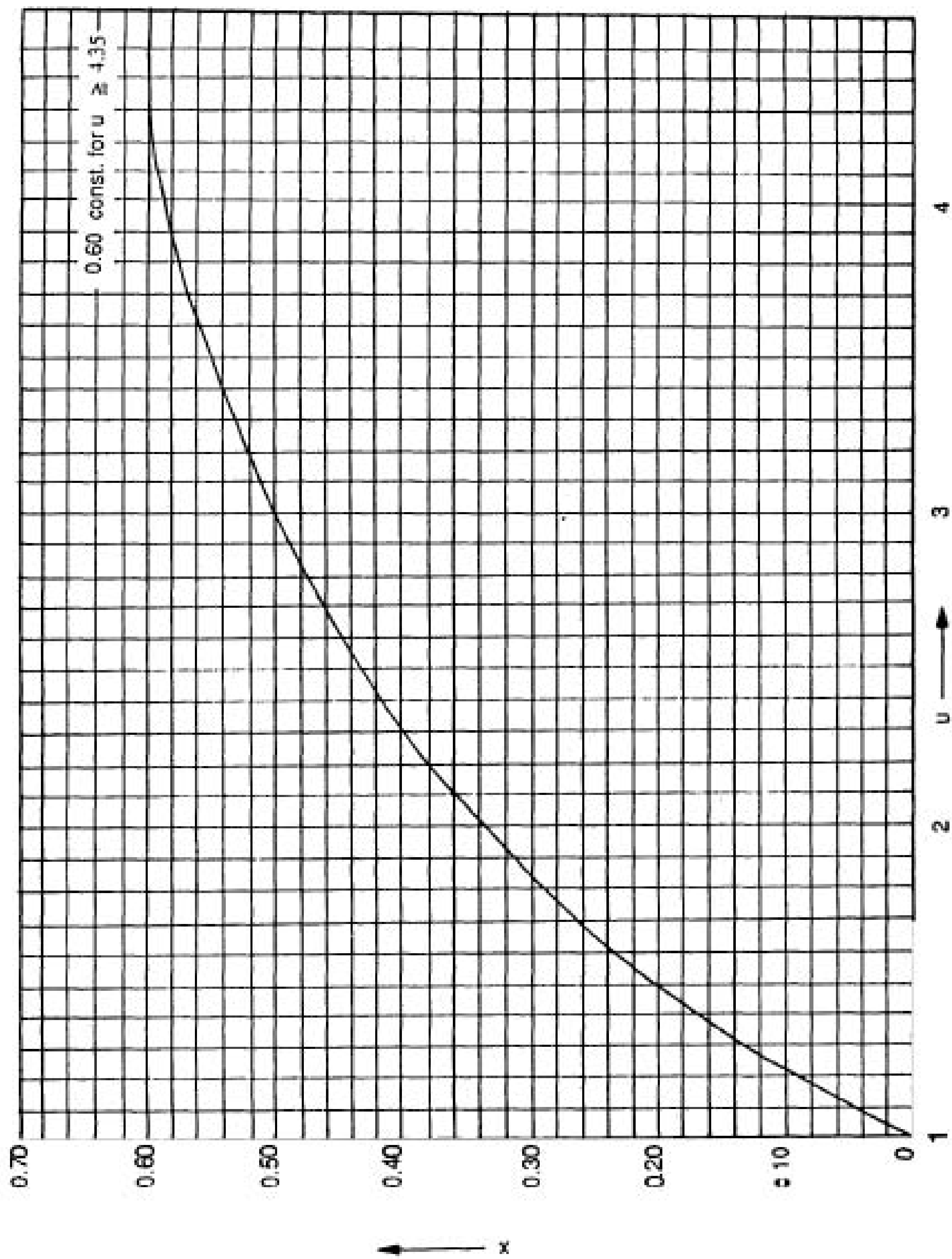


Fig. 6.19 Relation of correction factor *x* vis-a-vis transmission ratio *u*
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 161, p. 272, VEB Fachbuchverlag, Leipzig

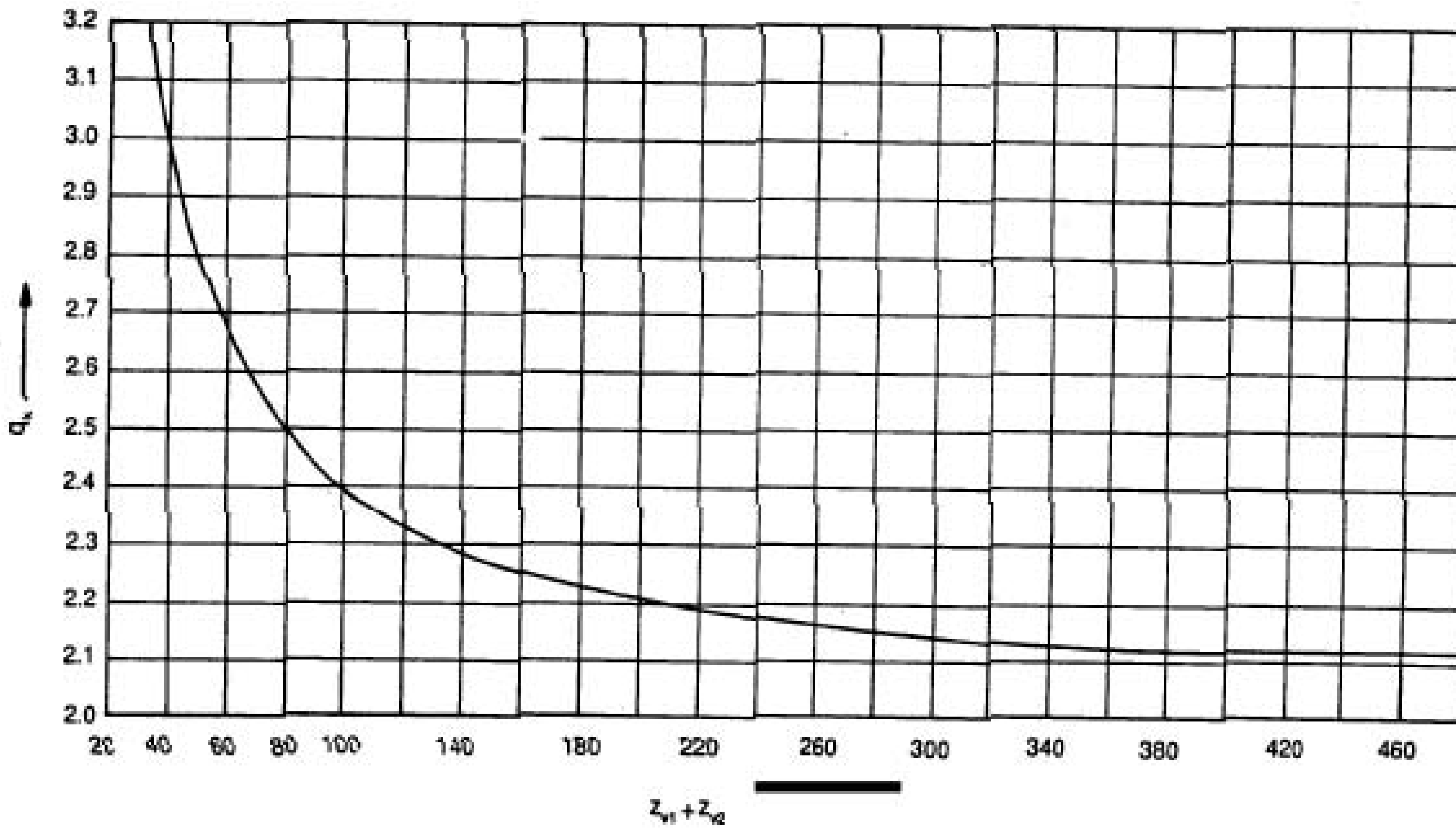


Fig. 6.20 Tooth form factor, q_k, for Arcoid spiral bevel gears
Based on Zahnraeder. Zirpke, 11th edition, 1980, Fig. No. 182, p. 289, VEB Fachbuchverlag, Leipzig

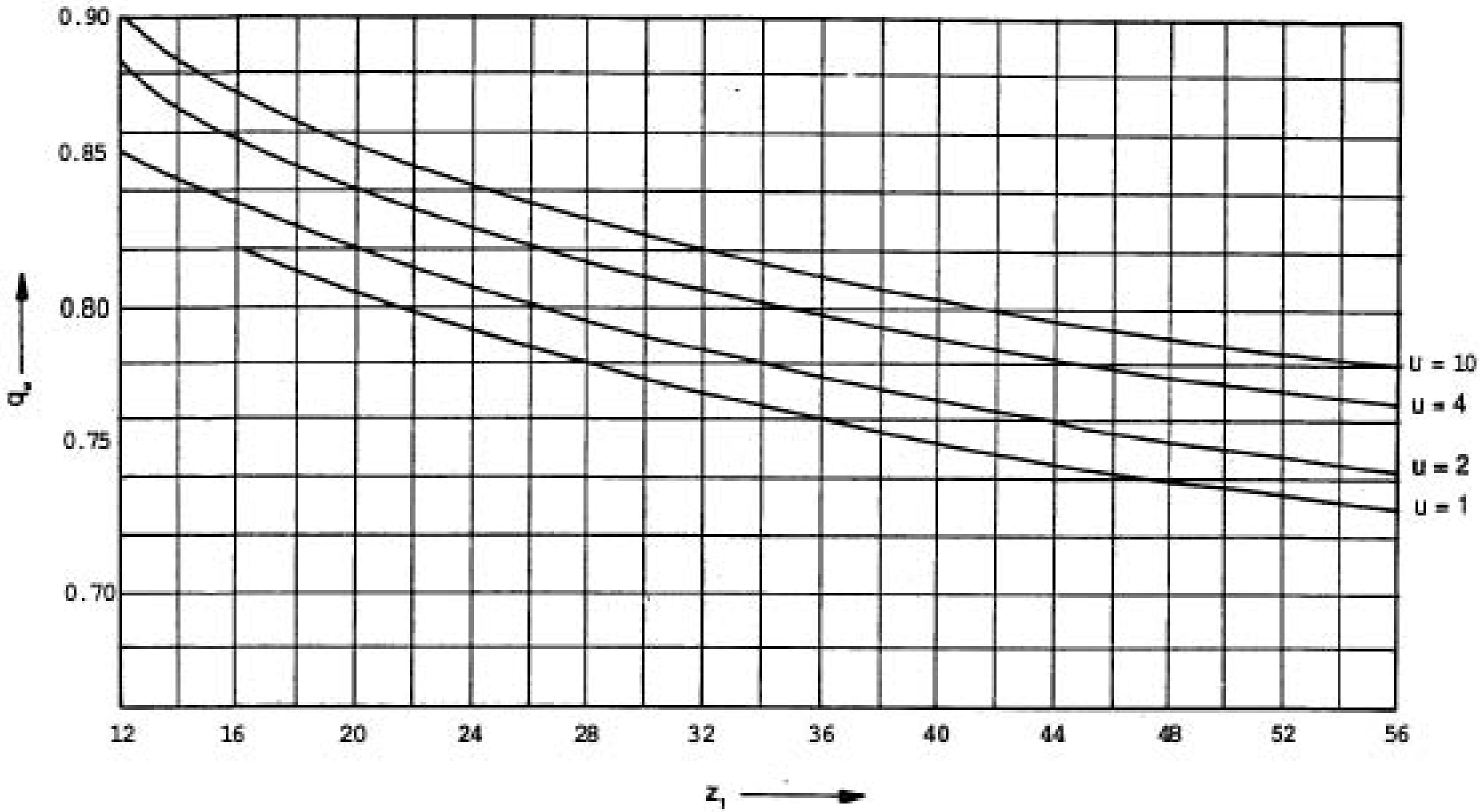


Fig. 6.21 Transverse contact ratio factor, q_e for Arcoid spiral bevel gears
Based on Zahnraeder. Zirpke, 11th edition, 1980, Fig. No. 185, p. 291, VEB Fachbuchverlag. Leipzig

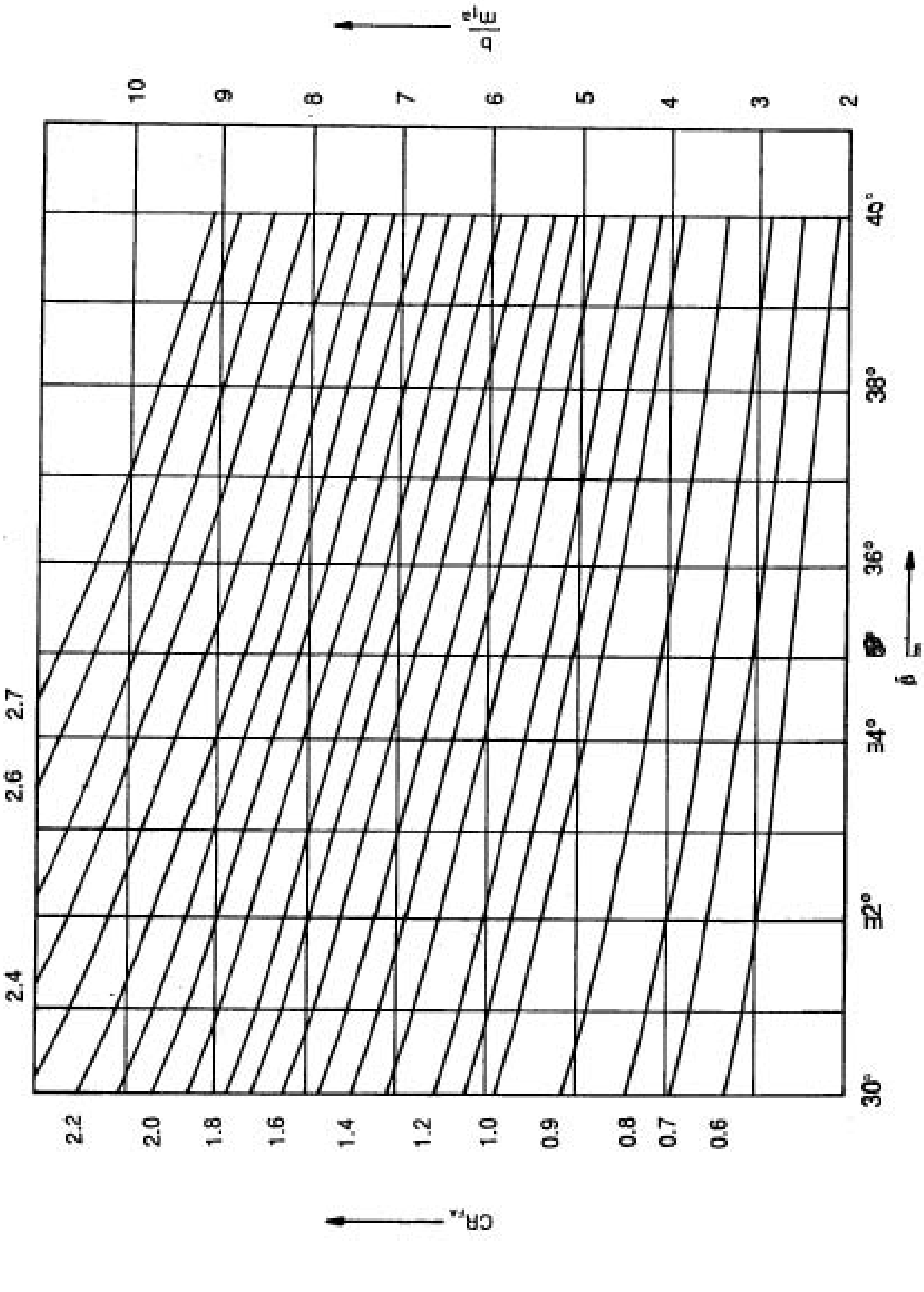


Fig. 6.22 Face contact ratio, CR_{FM}, for Arcoid spiral bevel gears
Based on Zahnraeder, Zirpke, 11th edition, 1980, Fig. No. 170, p. 275, VEB Fachbuchverlag, Leipzig

The dimensional parameters of the gearing system conforming to Arcoid System II are given in Table 6.7. The parameters which are common to both this system and Kurvex system, given in Table 6.6, are not repeated here. Only those data which differ from Kurvex toothing and are peculiar to the Arcoid toothing are given in Table 6.7. These are to be read in conjunction with Figs 6.16, 6.17 and 6.18.

In Arcoid System II, if the transmission ratio $u = 1$, then the gears are not corrected. Otherwise, S_o-gearing is generally resorted to. The correction factor, x (equal in magnitude, but positive for the pinion and negative for the gear), is a function of the transmission ratio, and is to be taken from Fig. 6.19. The basis on which the values of the correction factor have been fixed is the equalization of bending stresses at the tooth roots of the pinion and the gear, so that the form factors, $q_{k1} = q_{k2} = q_k$. The value of q_k with respect to the sum of the virtual number of teeth, z_{v1} and z_{v2}, is shown in Fig. 6.20. The strength and other calculation procedures for the Arcoid gearing can be made in the same manner as in the case of Kurvex gearing illustrated in Example 6.2. The same formulae are valid, but care should be taken so that the proper parameters, relevant to Arcoid gearing only, are inserted in the expressions at places where the data relating to the two systems differ. Figures 6.11 and 6.15 are common to both the systems. Figures 6.21 and 6.22 show the transverse contact ratio factor and the face contact ratio respectively, for gearing conforming to the Arcoid System II.

7 Miscellaneous Gearings

7.1 Gear Train

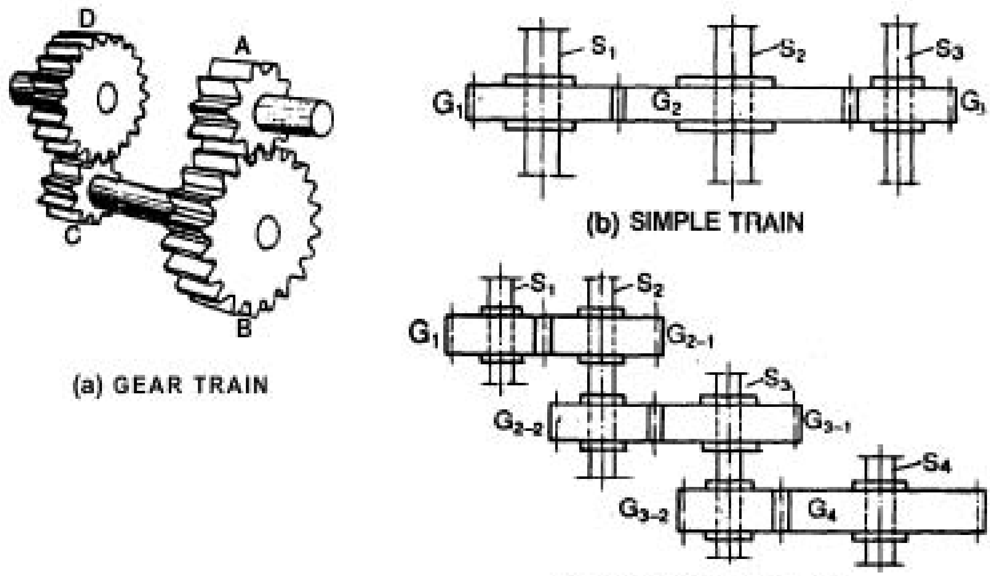
A gear train consists of a combination of two or more gears, mounted on rotating shafts, to transmit torque or power and also to act as a speed reducer or increaser, generally as a reducer in common industrial applications. If a large reduction is envisaged, it can be attained by using two gears only. Obviously, in that case one gear has to be enormously big, thereby creating problems of space and other difficulties. Such a design, therefore is precluded. To obtain the desired speed ratio, a train normally consists of several smaller gears requiring considerably less space. Such a train may include any one type of gears or a combination of different types of gears, e.g. spur, helical, bevel, as well as worm and worm-wheel.

Two types of gear trains are generally used—(i) the ordinary gear train [Fig. 7.1(a)] in which all gears rotate on fixed axes relative to a single frame of reference, and (ii) the epicyclic gear train in which at least one gear axis rotates relative to the frame in addition to the gear's rotation about its own axis. The second system, also called the planetary gear train, is treated in Sec. 7.2.

An ordinary gear train can be simple or compound. In a simple train, each shaft carries one gear only as shown in Fig. 7.1(b). Gear G_1, mounted on the shaft S_1, is the driver and G_1 drives G_2 which in turn drives G_3. Gear G_2 is the idler which serves the purpose of changing the direction of rotation, without having any effect on the gear ratio of the train. The idler also serves to bridge the gap between the effective gears, thereby affording a chance to reduce the diameters of those gears. In a compound train, shown in Fig. 7.1(c), some of the shafts carry two or more gears. Here, large speed ratio can be obtained in a small and compact form. As shown in Fig. 7.1(a), the driving and the driven shafts may be arranged to be coaxial, if necessary.

In any train, the ultimate speed ratio, (i) is given by

$$i = \frac{\text{Rotational speed of first driving gear}}{\text{Rotational speed of last driven gear}}$$



(a) GEAR TRAIN

(b) SIMPLE TRAIN

(c) COMPOUND TRAIN

Fig. 7.1 Gear train

The train value (e) is the reciprocal of i, and is given by

$$e = \frac{\text{Product of number of teeth of driving gears}}{\text{Product of number of teeth of driven gears}}$$

hence $i = \frac{1}{e} = \frac{n_F}{n_L}$, where n_F and n_L are the speed (rpm) of the first and the last gear of the train respectively. Since $d = mz$, the number of teeth can be replaced by the pitch diameters in the above relation. For the train in Fig. 7.1(b), we have $e = (z_1/-z_2) \times (-z_2/z_3) = z_1/z_3$, and $i = z_3/z_1 = n_1/n_3$, where z_1, z_2 ... and n_1, n_2 ..., are the number of teeth and speeds in rpm of gears G_1, G_2,.. ,the negative sign indicating rotation in opposite sense. Similarly for the compound train in Fig. 7.1(c), we have

$$e = \frac{z_1}{-z_{2\text{-}1}} \times \frac{-z_{2\text{-}2}}{z_{3\text{-}1}} \times \frac{z_{3\text{-}2}}{-z_4}$$

In reduction gear units, commonly known as "gearboxes" in industrial and commercial usage, the selection criteria mainly taken into consideration are: The overall speed reduction ratio, the maximum allowable speed reduction in any one stage, the space requirements, the values of speed reduction from stage to stage which are usually in geometrical progression, and the gear tooth modules which must necessarily be of progressively increasing values from stage to stage commensurate with torques as they increase from the input end to the output end in a reduction unit.

7.2 Planetary Gears

An epicyclic or planetary gear train consists of one or more rotating gears revolving around a central gear. The basic components of such a gear train are shown in Figs 7.2 and 7.3. The train usually consists of a central gear or sun gear (S), one or more gears surrounding the sun gear called planet gears (P), a member with one or more arms to which the planet gears are mounted called arm (A), or planet carrier or spider, and an annular gear or ring gear (R) which is concentric to the sun gear.

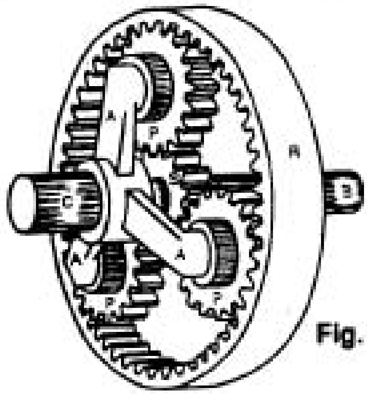


Fig. 7.2 Pictorial view of planetary gear system

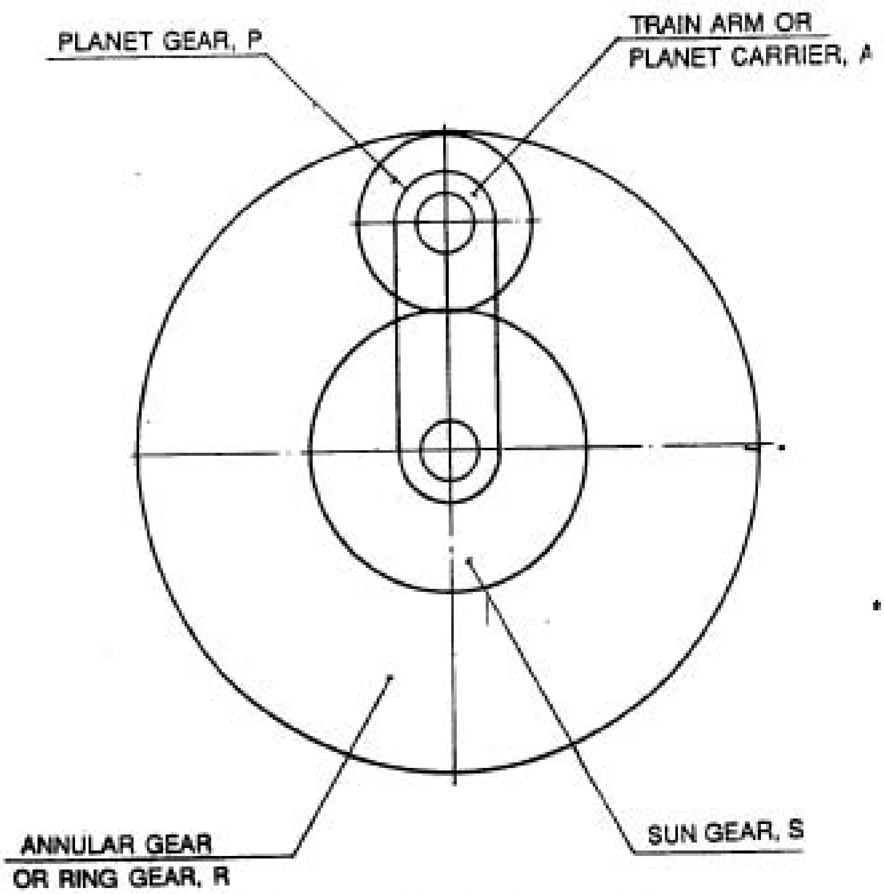


Fig. 7.3 Components of planetary gear train

The aspects which characterise a planetary gear system are its compactness, co-axial arrangement of driving and driven shafts, large speed reduction possibilities vis-a vis its overall size, possibilities of a number of combinations of driving and driven inputs and outputs, large torque conversion possibilities, and different possibilities of orientation of drives.

A unique feature of this type of gearing is that it permits some of the gear axes to rotate with respect to others. How this characteristic is incorporated in the system has been explained in Fig. 7.4 (a-d). Consider the movements shown in Fig. 7.4 (a). Here planet P is freely attached to the arm *A*. Assuming that there is a gap between the sun gear *S* and the planet gear P, as *A* rotates about *S*, a point a on the periphery of P will always point downwards as shown because *P* is freely mounted on *A*. This is analogous to the motion of a passenger-carrying gondola in a merry-go-round. It can be easily seen that in this case, planet P makes no revolution about its own axis. In other words, if we designate the angular speeds of the arm as n_A, of planet as n_p and of sun as n_s then for one rotation of the arm,

$$n_A = 1, \text{and } n_p = 0$$

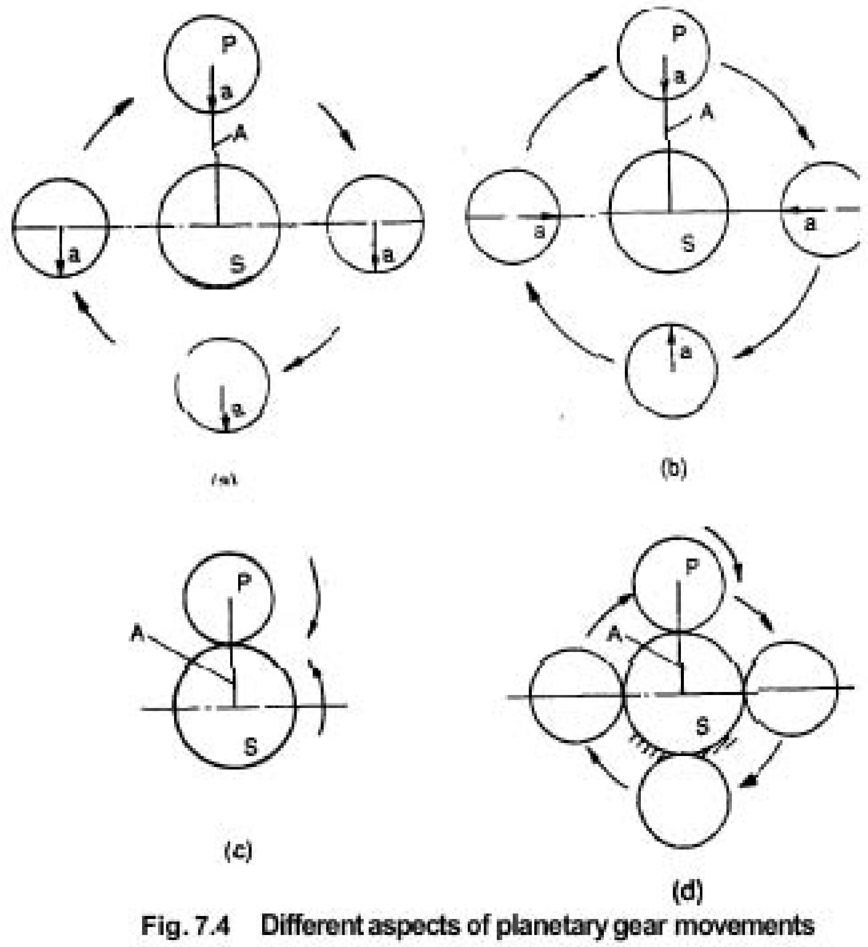


Fig. 7.4 Different aspects of planetary gear movements

Consider now Fig. 7.4(b) where P is locked to A and they both rotate en bloc about S. In this case point a is always directed towards S. The effect of this is that the axis of P on the plane of this paper makes a complete 360° rotation around the centre of the system, that is, P makes one rota-tion. This is analogous to the motion of the moon around the earth (locked motion).

Therefore $$n_A = 1, \text{and } n_p = 1$$

Coming to Fig. 7.4(c) both P and S are free to rotate, A is fixed and P drives S. This is a simple 2-gear train the reduction ratio of which is given by

$$\frac{n_p}{n_s} = \frac{z_S}{z_P} = \frac{\text{Pcd of } S}{\text{Pcd of } P} \qquad (z = \text{Number of teeth})$$

i.e. for each revolution of S, planet P makes n_p/n_s revolutions.

Let us now consider the case represented by Fig. 7.4(d). Here, sun S is fixed, planet P is free to rotate on the arm A, and A rotates around S. In this arrangement, for one rotation of A around S, planet P can be seen to execute two rotational motions: (i) it makes one revolution as its axis makes one revolution which is shown in Fig. 7.4(b), and (ii) it also rolls around S with the same effect as in Fig. 7.4(c).

Therefore, the end result for the rotation of P is the summation of (i) and (ii). For one rotation of the planet carrier A, the number of revolutions made by the planet $P = 1 + n_p/n_s$.

This is the fundamental principle on which the planetary gearing system is based.

A planetary gear train comes in various combinations. In fact the number of ways in which the gears may be arranged in this system can be of an infinite variety. Planet gears may form a compound system, i.e. the system may have more than one gears on the same shaft and these gears may be made to mesh with other sun or internal gears, thereby making it a complex system. By choosing a suitable combination, speed ratio of 10000 : 1 are easily obtained.

In a simple planetary gear train consisting of the central sun gear, the planet carrier and the internal ring gear, any one of the above three elements can be made to be the fixed member. Any of the two remaining elements can be used as the input or the output component for power transmission. With a single planetary gear train, there are thus six combinations of speed ratios possible. Because of its flexibility for attaining various speed ratios and torque conversion, ability to transmit comparatively high power in limited space and reduced weight, the planetary system is much used in aircrafts, hoists, machine tools, automobile differentials, servo-mechanisms, and automatic transmission gear boxes of cars. In a system of planetary gear train, it can happen that no gear is fixed such as in the case of the bevel planetary gear train, in an automobile differential.

Calculation of Reduction Ratio

The ultimate reduction ratio in an epicyclic gear train can be found by the following methods which are described here in detail.

Tabulation Method

This is one of the commonly used methods of solving planetary gear problems. This method is convenient because one of its advantages lies in the fact that the ensuing table gives a complete picture of the angular motions of the different rotary components at a glance and, unlike the algebraic method (described later) which gives only the final ratio, any intermediate ratios can also be easily found from the tabulation method. It is actually a summation process in which the system is first considered to rotate en bloc, thereby having no relative motion between the different members comprising the system, and then the system is considered as an ordinary gear train as if all the gears are free to rotate about their own axes, disregarding the arm. The result is then added and presented in a tabular form. The following examples will clarify the tabulation method.

Referring to Fig. 7.5(a), the method is applied in steps enumerated below.

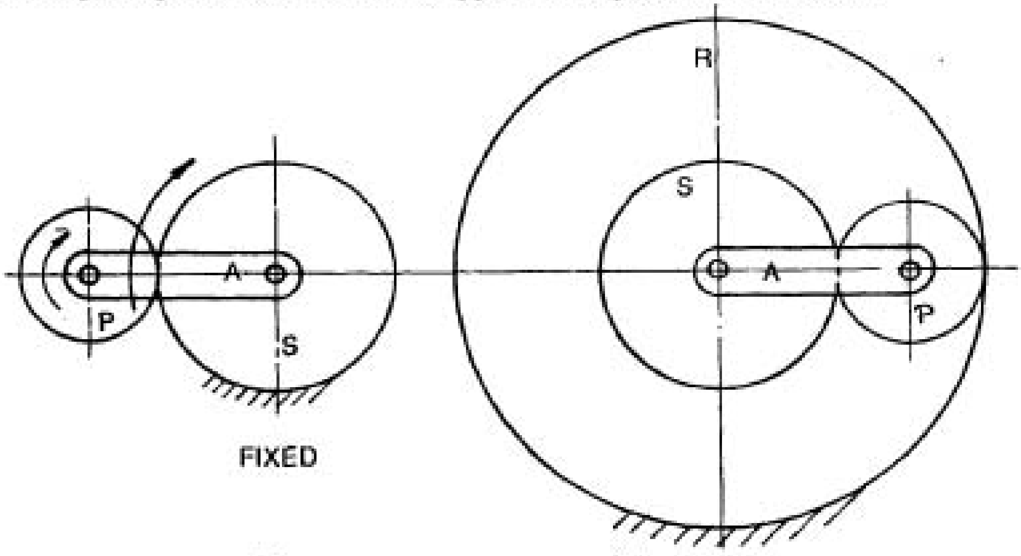


Fig. 7.5 Externaland internalplanetary systems

Step 1. Unlock the sun gear so that it is free to rotate on its shaft. Lock the sun and the planet to the arm. It is obvious that now there cannot be any relative motion among the members.

Step 2. Rotate the whole system en bloc once about the centre of the sun gear, taking the clockwiserotation as positive. It can be seen that each individual member rotates once about its own individual axis.

Step 3. Unlock the gears from the arm. Disregard the arm and consider the system as an ordinary gear train. Give the sun gear one counter-clockwise (negative) rotation.

We can now sum up the total number of revolutions made by each component of the planetary system so that the absolute values are obtained. This is represented in the following tabular form.

Condition of members	*Sun*	*Revolution of Planet*	*Arm*
1. All members locked to one another, the whole system rotates en bloc	+ 1	+ 1	+ 1
2. Arm fixed, sun rotates planet	− 1	$+\frac{Z_S}{Z_P}$	0
Resultant rotation	0	$1+\frac{Z_S}{Z_P}$	+ 1

This tallies with the result previously obtained.

Referring to Fig. 7.5(b), a table is made as before with the following results:

	Condition of members	Revolution of Ring	Sun	Planet	Arm
1.	All members lacked	+1	+1	+1	+1
2.	Arm fixed, ring gear given one negative rotation	−1	$\left(-\frac{z_R}{z_P}\times\frac{z_P}{z_S}\right)\times\frac{1}{-1} = \frac{z_R}{z_S}$	$-\frac{z_R}{z_P}$	0
	Resultant rotation	0	$1+\frac{z_R}{z_S}$	$1-\frac{z_R}{z_P}$	+1

Example 7.1: *Given:* $z_R = 100, z_S = 60, z_P = 20$.

To find the speed of sun gear if the arm makes 9 rpm. The ring gear is fixed.

Solution: From the above table, we get

$$\frac{\text{Speed of sun}}{\text{Speed of arm}} = \frac{1+\frac{z_R}{z_S}}{1} = 1 + \frac{100}{60} = \frac{8}{3}$$

Therefore speed of the sun gear $= 9 \times \frac{8}{3} = 24$ rpm

Algebraic Method

The problem can be solved simply by using the following relation

$$e = \frac{n_L - n}{n_F - n_A} \tag{7.1}$$

where e = Train value relative to the train arm,

$$= \frac{\text{Product of number of teeth on driving gears in a train}}{\text{Product of number of teeth on driven gears in a train}}$$

n_F = Absolute angular speed in rpm of the first gear in the train

n_L = Absolute angular speed in rpm of the last gear in the train

n_A = Absolute angular speed in rpm of the train arm

This method is known as the algebraic method and is clarified in the following example.

Example 7.2 : Figure 7.6 gives certain basic data from which it is required to find the ultimate speed ratio between the driver and the follower. The annular or the ring gear is fixed.

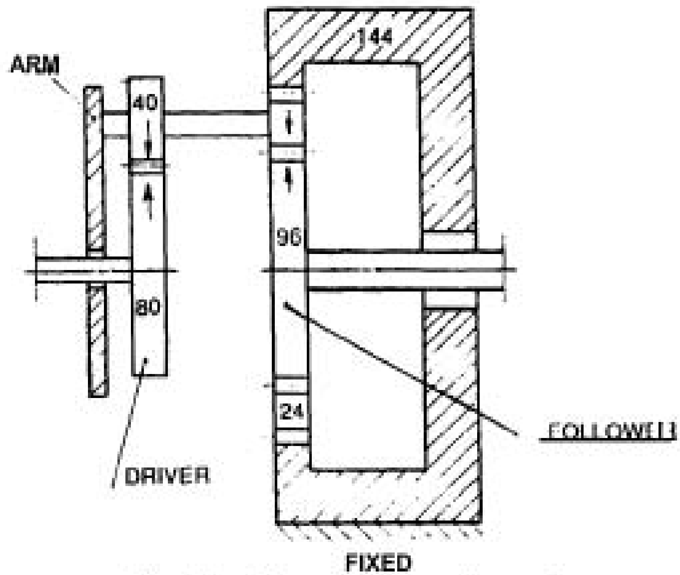


Fig. 7.6 Compound planetary system

Solution: For calculation, the gear train is split in two sequences.

(i) Gears having teeth *80–40–24–144.*

Let the first gear in this train make one revolution, i.e. $n_{F1} = n_{80} = 1$

$$e_1 = \frac{-80 \times (-24)}{(-40) \times (-144)} = -\frac{1}{3}$$

Here, the negative sign indicates a direction of rotation opposite to that of the driver which is assigned a positive sign. Applying Eq. 7.1, we have

$$e_1 = \frac{n_{L1} - n_A}{n_{F1} - n_A} = \frac{0 - n_A}{1 - n_A} \text{ or } -\frac{1}{3} = \frac{0 - n_A}{1 - n_A} \text{ Therefore } n_A = \frac{1}{4}$$

(ii) Gears having teeth 80–40–24–96.

In the second train, the last gear has *96* teeth. Therefore, $n_{L2} = n_{96}$.

The train value of this system is given by

$$e_2 = \frac{80 \times (-24)}{(-40) \times 96} = \frac{1}{2}. \text{ Therefore } \frac{1}{2} = \frac{n_{L2} - n_A}{n_{F2} - n_A} = \frac{n_{L2} - \frac{1}{4}}{1 - \frac{1}{4}}, n_{F2} = n_{F1} = 1$$

whence $n_{L2} = \frac{5}{8}$

Therefore, the ultimate speed ratio of the whole system is given by

$$\frac{\text{Speed of driver}}{\text{Speed of follower}} = \frac{n_{F1}}{n_{L2}} = \frac{n_{80}}{n_{96}} = \frac{1}{5/8} = \frac{8}{5}$$

Hence, the last gear or the follower having 96 teeth makes 5/8 revolutions for one revolution of the first gear or the driver having 80 teeth.

Several common applications of planetary gear trains will now be solved with the help of the algebraic method using Eq. 7.1.

1. Figure 7.7 (a). In this case sun S is fixed, arm A is the driver and planet P is the follower. Using the same notations as before, we have

$$e = \frac{z_P}{-z_S} = \frac{0-n_A}{n_P-n_A} = \frac{-n_A}{n_P-n_A}$$

or $$z_P n_P - z_P n_A = z_S n_A \text{ or } n_A (z_P + z_S) = z_P n_P \text{ or } \frac{n_A}{n_P} = \frac{z_P}{z_P+z_S}$$

Therefore, for one revolution of arm A, the follower makes

$$n_P = n_A\left(\frac{z_P+z_S}{z_P}\right) = 1 \times \left(1 + \frac{z_S}{z_P}\right) = 1 + \frac{z_S}{z_P} \text{ revolutions}$$

2. Figure 7.7 (b). Here, the train value is given by

$$e = \frac{z_P}{z_R} = \frac{0-n_A}{n_P-n_A} = \frac{-n_A}{n_P-n_A} \text{ whence } \frac{n_A}{n_P} = \frac{z_P}{z_P-z_R}$$

For $$n_A = 1, n_P = \left(\frac{z_P - z_R}{z_P}\right) = 1 - \frac{z_R}{z_P}$$

3. Figure 7.7 (c). In this case, we take the power flow in the reverse direction for case of calculation, and take the sun gear as the driver. Proceeding as before, we get

$$e = \frac{z_S-z_x}{-z_x-z_R} \text{ or } -\frac{z_S z_y}{z_x z_R} = \frac{0-n_A}{n_S-n_A}$$

By transposing, we get the transmission ratio

$$i = \frac{n_A}{n_S} = \frac{z_S z_y}{z_x z_R + z_S z_y}$$

For one revolution of the driver $(n_A = 1)$ the follower, i.e. the sun gear, makes

$$n_S = 1 + \frac{z_x z_R}{z_y z_S} \text{ revolutions}$$

4. Figure 7.7 (d). In this case

$$e = \frac{z_R}{z_P} \times \frac{z_P}{-z_S} = -\frac{z_R}{z_S} = \frac{0-n_A}{n_S-n_A} = \frac{-n_A}{n_R-n_A}$$

By simplifying, we get

$$i = \frac{n_R}{n_A} = \frac{z_R + z_S}{z_R} = 1 + \frac{z_S}{z_R}$$

For $n_A = 1$, the driver speed

$$n_R = 1 + \frac{z_S}{z_R}$$

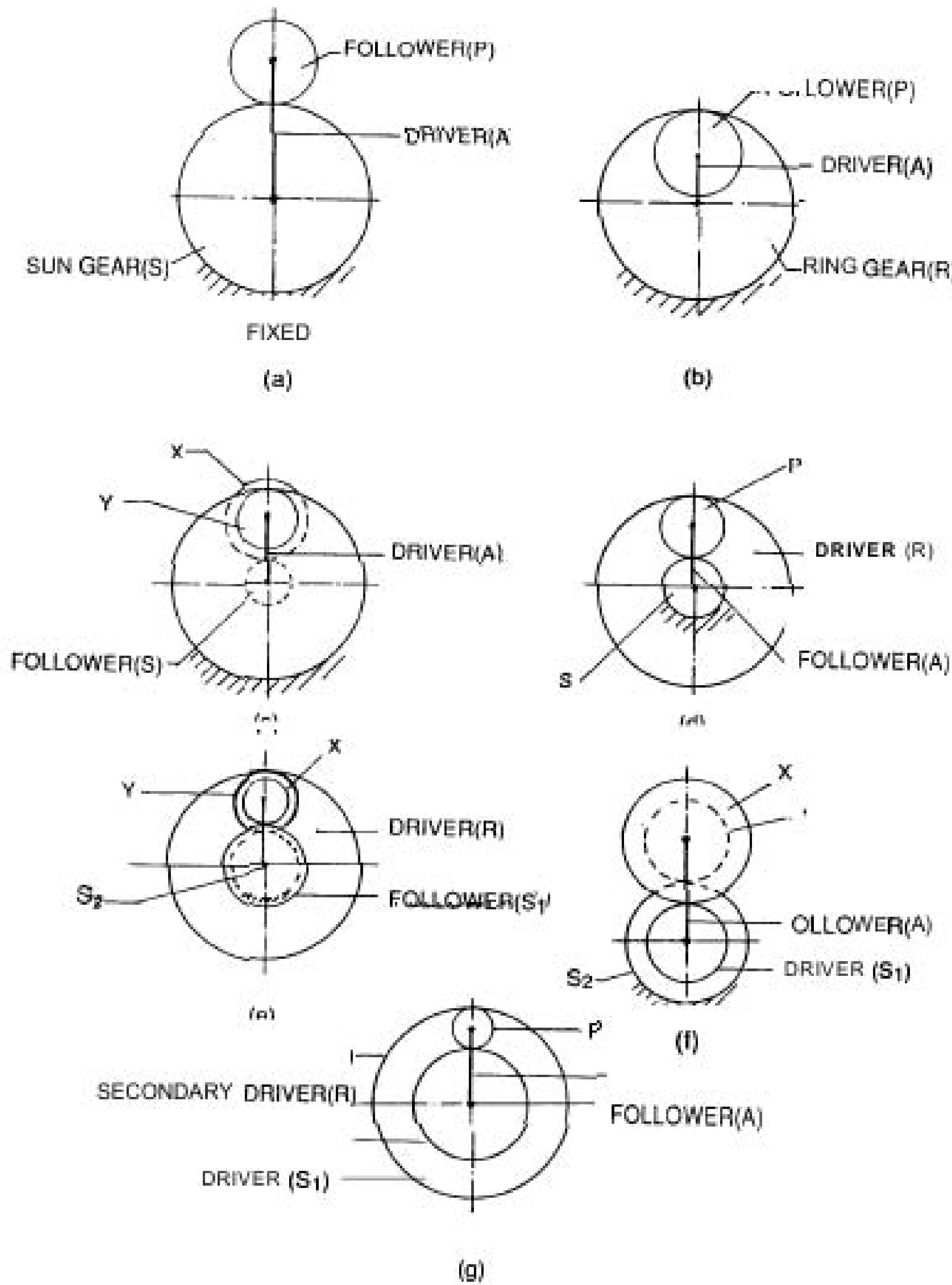


Fig. 7.7 Differentcombinations of planetary system

5. Figure 7.7 (e). Here, the sequence is split into two gear trains as shown in the beginning while explaining the algebraic method. We have

Train e, comprising: $R - y - S_2$

Train e, comprising: $R - Y - x - S_1$

$$e_1 = \frac{z_R}{z_y} \frac{z_y}{-z_{S_2}} = \frac{-z_R}{z_{S_2}}$$

Therefore, for one revolution of the driver R, we get

$$-\frac{z_R}{z_{S_2}} = \frac{n_{S_2} - n_A}{n_R - n_A} = \frac{0 - n_A}{1 - n_A} = \frac{-n_A}{1 - n_A} \text{ whence } n_A = \frac{z_R}{z_R + z_{S_2}}$$

$$e_2 = \frac{z_R}{z_y} \times \frac{z_x}{-z_{S_1}} = \frac{n_{S_1} - n_A}{1 - n_A} = \frac{n_{S_1} - \frac{z_R}{z_R + z_{S_2}}}{1 - \frac{z_R}{z_R + z_{S_2}}}$$

By transposition, we get

$$n_{S_1} = \frac{z_R z_{S_1} z_y - z_R z_{S_2} z_x}{z_R z_{S_1} z_y + z_{S_1} z_{S_2} z_y}$$

Therefore,
$$i = \frac{n_R}{n_{S_1}} = \frac{1}{n_{S_1}} = \frac{(z_R z_{S_1} z_y + z_{S_1} z_{S_2} z_y)}{z_R z_{S_1} z_y - z_R z_{S_2} z_x}$$

6. Figure 7.7 (f).
$$e = \frac{z_{S_1}}{-z_x} \times \frac{-z_y}{z_{S_2}} = \frac{z_{S_1} z_y}{z_x z_{S_2}} = \frac{0 - n_A}{1 - n_A} = \frac{-n_A}{1 - n_A}$$

Simplifying, we get

$$n_A = \frac{z_{S_1} z_y}{z_{S_1} z_y - z_{S_2} z_x}$$ = Number of rotation of follower (arm) for one revolution of the driver (S_1)

Therefore
$$i = \frac{1}{n_A} = 1 - \frac{z_{S_2} z_x}{z_{S_1} z_y}$$

7. Figure 7.7 (g). In this case there are two drivers — the main driver S and the secondary driver R. The rotational motion of both drivers are added to rotate the follower A. Denoting the number of revolutions of the secondary driver as n_R for one revolution of the driver S, we have

$$e = \frac{z_R}{z_P} \times \frac{z_P}{-z_S} = -\frac{z_R}{z_S} = \frac{1 - n_A}{n_R - n_A}$$

or
$$-z_R n_R + z_R n_A = z_S - z_S n_A$$

or
$$n_A = \frac{z_R n_R + z_S}{z_S + z_R} \quad \text{Therefore } i = \frac{1}{n_A} = \frac{z_S + z_R}{z_S + z_R n_R}$$

The planetary gearing system using bevel gears has many application; the most common being the differential of cars, cone pulley drives of certain machine tools and other equipments. Some common arrangements are discussed here.

8. Figure 7.8 (a). In this case the reduction ratio $i = z_s/z_p$.
9. Figure 7.9 (a). Here, $i = \cos \alpha + z_s/z_p$, where a = Shaft angle

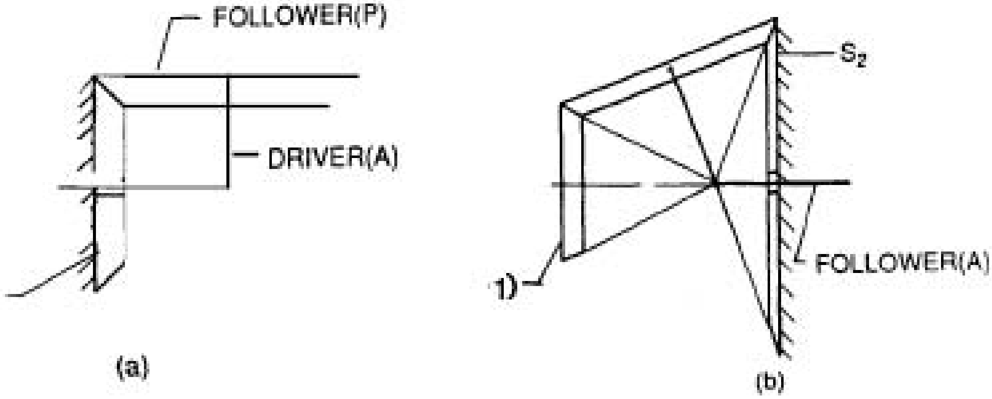


Fig. 7.8 **Bevel** planetary **systems**

The above expression for the reduction ratio can be established in the following manner. Referring to Fig. 7.9 (b), consider AB and CD to be axes of the sun and the planet respectively, GEF representing the arm, all referred to on the plane of the paper. We will now only consider the revolution of CD about its own centre F as the arm rotates about its centre E. Since, for the present, we are not considering the rotation of the planet which is caused by its tooth engagement with that of the sun, AB and CD have been shown separated.

Now, had AB been integral with the shaft GE, i.e. fixed by a key or similar device, the angular magnitude of rotation of GE would have been equal to that of AB. Hence, a rotation of AG by an amount $d\phi$ may be considered as if GE has been rotated by that amount. This rotation causes CD to assume the new position $C'D'$ on the surface of the imaginary cone as shown in the figure. It can be seen from Fig. 7.9(b) that due to this shifting, CD has rotated by an amount $d\theta$ about it own centre F.

From geometry, the following relation can be obtained

$$\text{arc } AA' = AG\, d\phi$$

$$AG = OA \cos \alpha$$

The arc AA' can also be taken as equal to OA $d\theta$ in the infinitesimal range.
Therefore AG $d\phi = OA\, d\theta$ or $OA \cos \alpha\, d\phi = OA\, d\theta$. Therefore $d\theta = \cos \alpha\, d\phi$
For a complete rotation of the arm, we can now write the following equation

$$\int d\theta = \cos \alpha \int_0^{2\pi} d\phi$$

By integrating, we get $\theta = \cos \alpha [\phi]_0^{2\pi} = \cos \alpha\, 2\pi$

Hence, in terms of rotation, when the arm **makes** one complete revolution (angle = 2π), the planet axis CD makes $\cos a \times 1 = \cos a$ revolution about its own centre .

If now the two gears are brought into mesh, the total number of revolutions **of the** planet **for** one revolution of the arm is given by

$$n_p = \cos\alpha + \frac{z_s}{z_p} \tag{7.2}$$

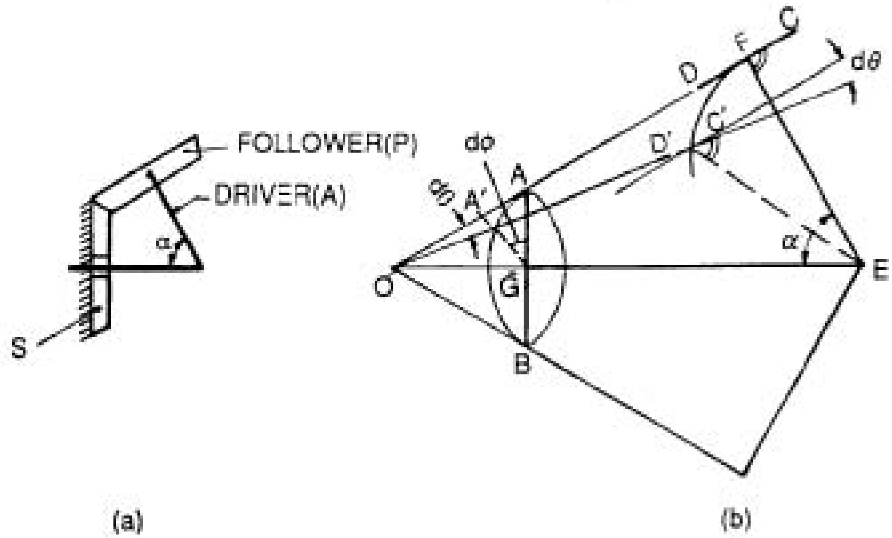


Fig. 7.9 Bevel planetary with shaft angle other than 90°

It is interesting to note that in case 1 the axes of the two gears are parallel. Hence $a = 0°$

Therefore $$n_p = \cos 0° + \frac{z_s}{z_p} = 1 + \frac{z_s}{z_p}$$

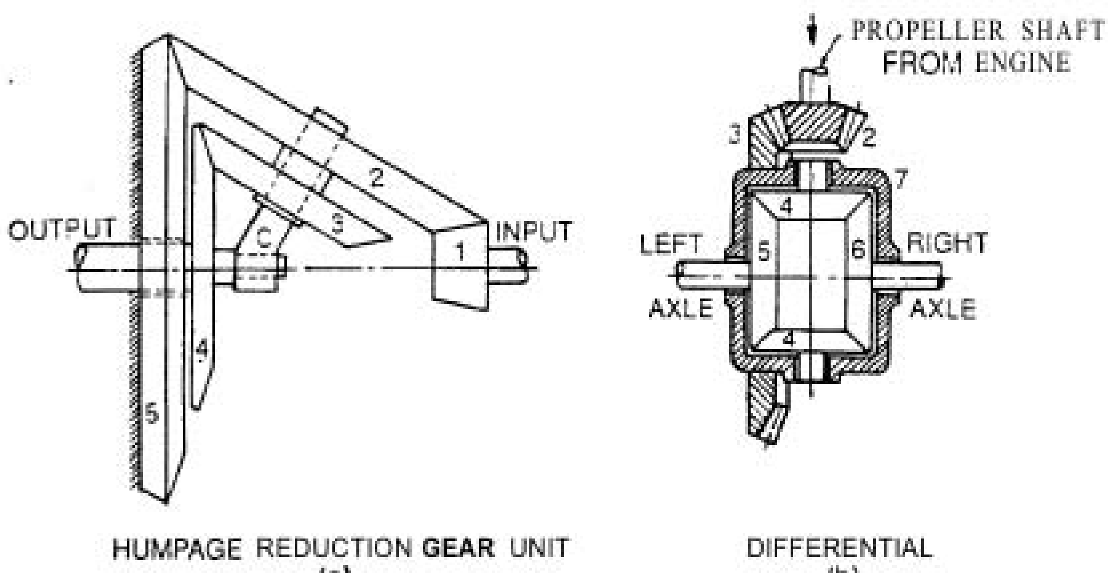


Fig. 7.10 Common applications of bevel planetary systems

which tallies with the equation previously given for case 1.
In case 8, $a = 90°$

Therefore $$n_p = \cos 90° + \frac{z_S}{z_P} = 0 + \frac{z_S}{z_P} = \frac{z_S}{z_P}$$

Equation 7.2, therefore, is a generalised relation for any two meshing bevel gears having shaft angle a. In cases 10 and 11 which follow, the effect of the shaft angle cancel out due to the ultimate co-axiality of the driver and the follower gears.

10. Figure 7.8(b). Here, for one revolution of follower A, the number of rotations of driver S_1 is given by

$$n_{S_1} = 1 + \frac{z_{S_2}}{z_{S_1}}$$

11. Figure 7.10. This type of arrangement is known as the Humpage reduction gear unit which is sometimes used in machine tools. For one revolution of follower 4, the number of rotations of driver 1 is given by

$$n_1 = 1 + \frac{z_5}{z_1} \Big/ 1 - \frac{z_5}{z_2} \times \frac{z_3}{z_4}$$

Geometrical Method

To visualise exactly what happens when motion takes place in an epicyclic train, consider the following geometrical treatment depicted in Fig. 7.11. The two gears, the sun and the planet, touch initially at the pitch point P, their centre being O_1 and O_2. The sun is fixed and the arm turns clockwise which makes the planet to roll over the periphery of the sun to the new position with O_2' as the centre. The initial and the new positions of the arm makes an angle θ at centre O_1 as shown. The new pitch point is P' after rolling of the planet. The initial pitch point P on the planet now occupies a new position Q. Since the rolling is without any slippage, it can be seen that

$$\text{arc } PP' = \text{arc } QP'$$

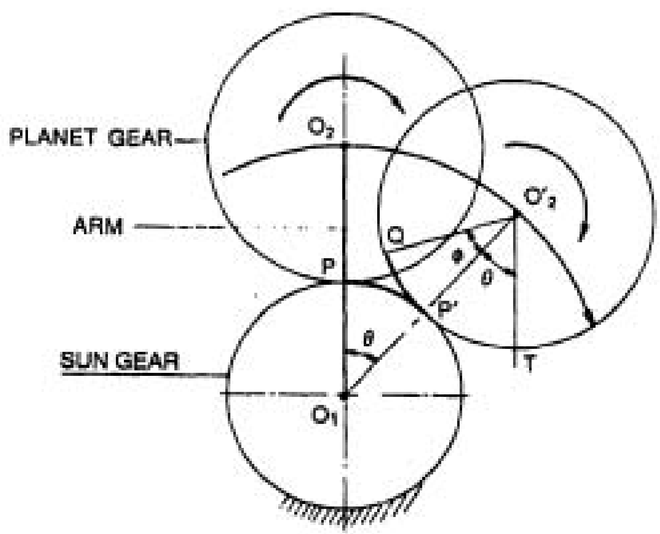


Fig. 7.11 Epicyclic movement of planet gear

It can also be seen that the initial line O_2P now assumes the new position $O_2'Q$ as a result of the rotation of the arm. Furthermore, the new vertical axis of the planet gear is $O_2'T$. Since the initial vertical axis O_2P now occupies the position $O_2'Q$ after rotation, it can be easily seen that the planet gear as a whole has rotated through an angle $TO_2'Q$. From geometry we have

$$\text{Angle } TO_2'Q = \theta + \phi$$

Now, since arc PP' = arc QP', we have

$$O_1P' \times \theta = O_2'P' \times \phi$$

or

$$\phi = \frac{O_1P'}{O_2'P'} \times \theta = \frac{r_s}{r_p} \times \theta = \frac{mz_s/2}{mz_p/2} \times \theta = \frac{z_s}{z_p} \times \theta$$

where r_s and r_p are the pitch radii and z_s and z_p are the number of teeth of the sun and the planet respectively, and m is the module.

Therefore angle $TO_2'Q = \theta + \phi = \theta + \frac{z_s}{z_p} \times \theta = \theta\left(1 + \frac{z_s}{z_p}\right)$

In other words, if the arm makes a complete revolution around the centre O_1, i.e. $\theta = 2\pi$, then the planet gear makes an angle of $2\pi(1 + z_s/z_p)$, that is, $(1 + z_s/z_p)$ revolutions. Therefore, the reduction ratio is given by

$$i = \theta\left(1 + \frac{z_s}{z_p}\right)\bigg/\theta = 2\pi\left(1 + \frac{z_s}{z_p}\right)\bigg/2\pi = 1 + \frac{z_s}{z_p}$$

It should be noted here that had there been no rotation of the arm and the sun gear were free to rotate, then the system would have been reduced to a simple train with $i = z_s/z_p$.

Differential

The differential is a mechanism consisting of bevel epicyclic gear system by means of which a rear wheel of an automobile is permitted to roll faster than the other while negotiating a curve. It is a simple but ingenious system of planetary gearing. There are many types of complex differential mechanisms also. A simple, common type of differential is depicted in Fig. 7.10(b) and described here.

The power to both the rear axles of an automobile comes from the engine through the gear box to the propeller shaft which carries a bevel pinion (usually hypoid) at its end numbered **2** in the figure. Pinion **2** meshes with the ring gear **3**. Gear **3** is fitted to the differential case 7. On 7, two bevel gears **4** are freely mounted by means of pins. Bevels **4** mesh with bevels **5** and 6, each of which is fitted to the inner ends of the two rear axles, the outer of which carry the two rear wheels. The whole mechanism is encased in a housing (not shown in the figure).

When the car is moving on a straight path, **2** drives **3**. This rotates the case 7, carrying the pins of **4**. Bevels **4** in turn rotate **5** and **6** by tooth engagement. Bevels **5** and 6 rotate the respective axles and the axles rotate the wheels. It can be seen that the components 3, 7, 4, 5, 6, and the axles rotate en **bloc**, and there is no relative movement between them.

Now, suppose the car takes a left turn. During the same time period, the right wheel has to cover a longer curvilinear path than the left wheel. Obviously, the right wheel must rotate faster than the left wheel to make this possible. As bevel 6 tends to rotate faster, bevels 4 begin to rotate about their individual pins on which they are freely mounted. As may be seen from Fig. 7.10(b), the teeth of bevels 4 push the teeth of the bevel 6 which helps it (bevel 6) to rotate faster. On the other hand, the teeth of bevel 4 try to push the teeth of bevel 5 in the opposite sense. Thus, a differential angular speed ensues between the bevels 5 and 6. Depending on the road friction, bevels 4 may rotate bevels 5 and 6 or they may simply roll or "walk" on bevels 5 or 6, while all the time the main rotational movement remains. That is, bevels 5 and 6 continue to being bodily rotated by bevels 4 through tooth engagement.

The whole sequence is reversed when the car takes a right turn.

7.3 Non-Circular Gears

When a continuously variable angular velocity ratio is required throughout a single revolution, the non-circular gears offer a simple method of attaining this condition. Often these gears are provided with the proper linkage systems to obtain the type of motion needed for the mechanism. In any non-circular gear system, the driver rotates at a constant angular velocity and the driven gear executes a non-uniform angular motion, resulting in a variable velocity ratio at each instant.

Although it is always possible to design a pair of mating gears having special shapes to attain virtually any desired mutual velocity and acceleration characteristics between the two gears, only a handful of such systems are used in practice.

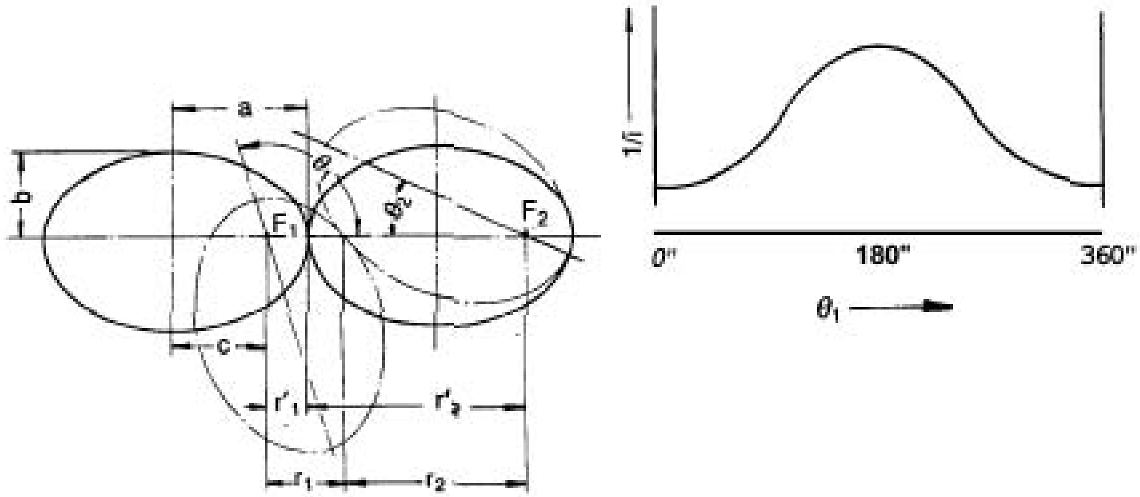
Non-circular gears are usually more expensive than cams, linkages or similar devices used to obtain variable velocity ratios, but by using modern production methods, the cost has decreased considerably. Among the many manufacturing methods employed, gear cutting by the tape-controlled gear shaper is the most accurate one.

From the application point of view, these gearing systems can be categorised into two groups—(i) cases where only an over-all change in the angular speed characteristics of the driven component is needed, and (ii) those cases where precise angular movements are required to produce certain specified non-linear functions. Examples of the first group include quick-return drives of machines and intermittent mechanisms of certain equipments. Examples of the second groups include mechanical calculators for extracting roots of numbers, raising numbers to any power and for treating problems involving trigonometrical and logarithmic functions. The pitch curves of the non-circular gears may be closed as in the case of elliptical gears or they may be open as in the case of logarithmic spiral gears used in computing devices. Unlike the closed types, the open type gears can be rotated only for a portion of a revolution.

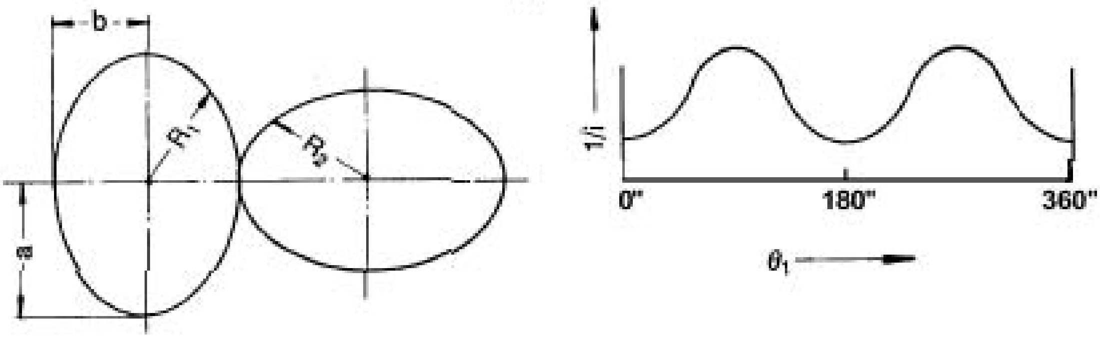
We shall discuss three most common types of non-circular gear systems in this section.

Type 1. This gearing system features two ellipses rotating about their foci. Such systems comprising elliptical gearings have been mainly used for obtaining quick-return motion for equipments like shapers, planers, slotters and similar machine tools in which the cutter, fixed on a reciprocating member, moves comparatively slowly during the forward stroke and returns quickly during the idle stroke. Some pumps, shears and punches also make use of this type of mechanism.

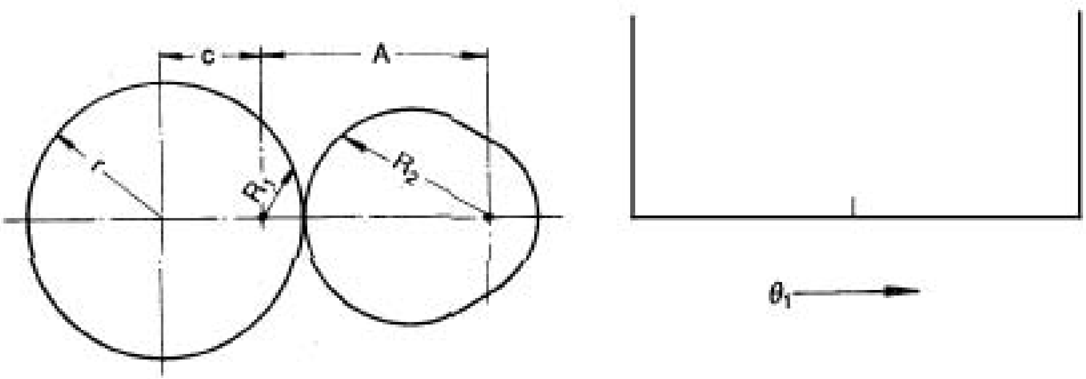
In Fig. 7.12 (a) the pitch curves of two identical elliptical gears which are mounted on their foci F_1 and F_2 are shown in mesh. The angular velocity ratio i or ω_1/ω_2 varies according to the respective radii of the driving and the driven members at the point of contact. When the gears are in the position shown in solid line in the figure, the angular velocity ω_2 of the driven gear 2 is minimum, the angular velocity ω_1 of the driven gear 1 being constant all the time. At this position, therefore, i is maximum, which means that $1/i$ or ω_2/ω_1 is at a minimum. This is shown in the accompanying curve at $\theta_1 = \theta_2 = 0°$. As the driver rotates, its radius r_1 at the point of contact



(a)



(b)



(c)

Fig. 7.12 Non-circular gears

gradually increases till θ_1 becomes 180° where ω_2 is maximum. Consequently, i is minimum at this point and $1/i$ is at its maximum. The situation is reversed during the remaining half of the rotation. The number of rotations made by each gear in a given time is, of course, the same.

In Fig. 7.12(a) a and b are the semi-major and semi-minor axes respectively, c is the distance between the geometric centre and one of the foci, r_1' and r_2' are the distances from F_1 and F_2 to the point of contact at the position shown in the figure, r_1 and r_2 are the distances from F_1 and F_2 to any instantaneous point of contact in question whereby the axes have rotated by the angles θ_1 and θ_2. It can be seen from the figure that

$$r_1' + r_2' = r_1 + r_2 = 2a \tag{7.3}$$

The angular velocity ω_1 of the driving gear is constant and the instantaneous angular velocity of the driven gear is given by

$$\omega_2 = \frac{d\theta_2}{dt}$$

so that the instantaneous velocity ratio is given by

$$\frac{\omega_1}{\omega_2} = \frac{\omega_1}{d\theta_2/dt} = \frac{r_2}{r_1} \tag{7.4}$$

Using polar equation of the ellipse, we can write

$$r_1 = \frac{a(1-e^2)}{1+e\cos\theta_1} = \frac{b^2}{a[1+\cos\theta_1\sqrt{1-(b/a)^2}]} \tag{7.5}$$

and

$$r_2 = \frac{b^2}{a[1-\cos\theta_2\sqrt{1-(b/a)^2}]} \tag{7.6}$$

Here, e is the numerical value of eccentricity of the ellipse and is given by

$$e = \frac{\sqrt{a^2-b^2}}{a} = \frac{c}{a} \tag{7.7}$$

The instantaneous velocity ratio is, therefore, given by

$$i = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{1+\cos\theta_1\sqrt{1-(b/a)^2}}{1-\cos\theta_2\sqrt{1-(b/a)^2}} \tag{7.8}$$

When $\theta_1 = \theta_2 = 0$, velocity ratio is maximum and using the above equation, we get

$$i_{max} = \frac{1+\sqrt{1-(b/a)^2}}{1-\sqrt{1-(b/a)^2}} = \frac{r_2'}{r_1'} \tag{7.9}$$

Minimum value of i is obtained when $\theta_1 = \theta_2 = 180°$

$$i_{min} = \frac{1-\sqrt{1-(b/a)^2}}{1+\sqrt{1-(b/a)^2}} = \frac{1}{i_{max}} \tag{7.10}$$

Often the centre distance of the shafts F_1F_2 is given, and the relation between a and b is to be calculated for a particular value of i_{max}. This can be found by using the relation

$$\frac{b}{a} = \frac{2}{i_{max}+1}\sqrt{i_{max}} \tag{7.11}$$

The above equation can also be written as

$$\frac{b}{a} = 2\sqrt[4]{K}/(1+\sqrt{K}) \tag{7.12}$$

where

$$K = \left[\frac{r_2'}{r_1'}\right]^2 = \left(\frac{a+c}{a-c}\right)^2$$

The variable i as a function of θ_1 is given by the relation

$$i = \frac{r_2}{r_1} = \frac{\omega_1}{\omega_2} = 2\left(\frac{a}{b}\right)^2 \left(1+\cos\theta_1\sqrt{1-(b/a)^2}\right) - 1 \tag{7.13}$$

This relation is represented by the curve in Fig. 7.12(a).

***Type* 2.** This variety, shown in Fig. 7.12(b), has two pitch curves of gears with axes off-set at an angle of 90°. These closed curves resemble ellipses and are called "higher order" ellipses. In this particular case, the ellipses belong to the second order and their contour differs slightly from that of a basic ellipse. The gears rotate about their geometric centres and execute two complete speed cycles per rotation as shown in the curve. At the position shown in the figure, $\theta_1 = 90°$ and $\theta_2 = 0°$. The relevant equations are

$$R = \frac{2ab}{(a+b) - (a-b)\cos 2\theta} \tag{7.14}$$

$$\frac{1}{i} = \frac{\omega_2}{\omega_1} = \frac{s+1+(s^2-1)\cos 2\theta_2}{2s} \tag{7.15}$$

where $s = a/b$.

***Type* 3.** In this variety, as shown in Fig. 7.12(c), an eccentrically mounted standard spur gear mates with its conjugate gear which has a special shape. The speed ratio *versus* angle characteristic curve is also shown in the figure. The gears are off-set at 180° so that in the position shown, $\theta_1 = 180°$. The relevant equations are

$$R_1 = c\cos\theta_1 + \sqrt{r^2 - c^2\sin^2\theta_1} \tag{7.16}$$

$$\frac{1}{i} = \frac{\omega_2}{\omega_1} = \frac{R_1}{R_2} = \frac{R}{A - R_1} \tag{7.17}$$

7.4 Intermittent Gears

When rotary motion of intermittent nature is required, spur gears having special shapes are **used.** In this type of gearing, the driver rotates at a constant angular velocity, when is the driven member also rotates at a constant angular velocity so long as it **is in** mesh with the driving gear. But when the teeth of the gears are not in mesh, the driven gear is locked against rotati . that it becomes and remains motionless for a prespecified period of rotation t . activated by the tooth of the driver (Fig. 7.13).

The driven gear rotates so long as the teeth of the mating driven gear are pushed by the teeth of the driver. The teeth of the pair come out of mesh after the specified period and the gears enter the toothless zone. The convex circular portion of the driver now fits snugly into the concave circular portion of the driven gear, thereby locking the rotational motion of the driven gear. It resumes its rotation when the tooth of the driver starts to mate with the tooth of the follower.

A variety of intermittent rotary motions can be obtained by properly designing the gear pair. Thus, the driving gear or the driven gear or both can have teeth so arranged that the interval or rest period may be made to vary, depending upon the number of teeth each gear is provided with at successive intervals. For example, the driver may have one, two, three or more teeth intercepted by smooth, convex, toothless circular portions in-between the sets of teeth. The driven member may have other numbers of teeth, intercepted similarly by concave arcs, so that in a single rotation itself variable durations of rotary motion of the follower can be attained, if required.

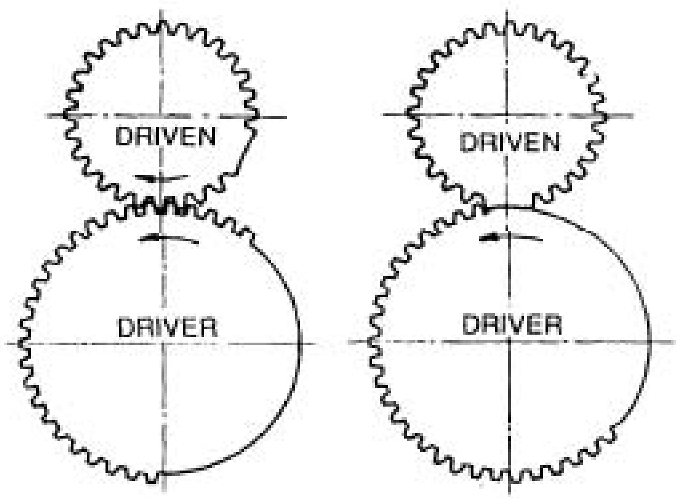


Fig. 7.13 Intermittent gears
Based on Gear Design and Application, Chironis, 1967 edition, Fig. No. 1, p. 361, McGraw-Hill, New York

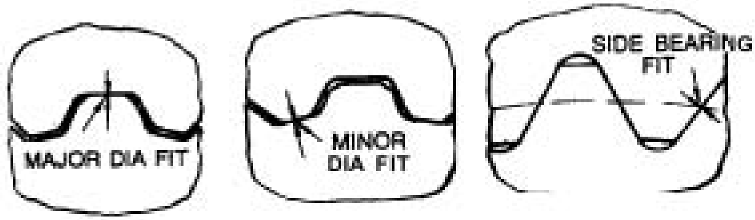
Intermittent gearing systems of different types are often employed in counting mechanisms, motion picture projectors and in many industrial appliances.

7.5 Involute Splines

For transmission of torque from the shaft to the gear or *vice versa*, three kinds of connections are normally resorted to

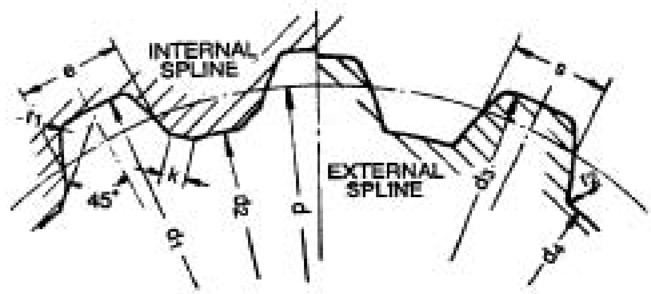
1. Connection through a shrink-fit. This is discussed in Sec. 8.13.
2. Connection through different types of keys. Details of these keys are given in the relevant Appendices at the end of this book.
3. Connection through splines. Splines can be straight-sided or with involute profile. Data on straight-sided splines are given in Appendix R. Only involute splines will be discussed in this section.

(a) TOOTH FORMS FOR INVOLUTE SPLINES

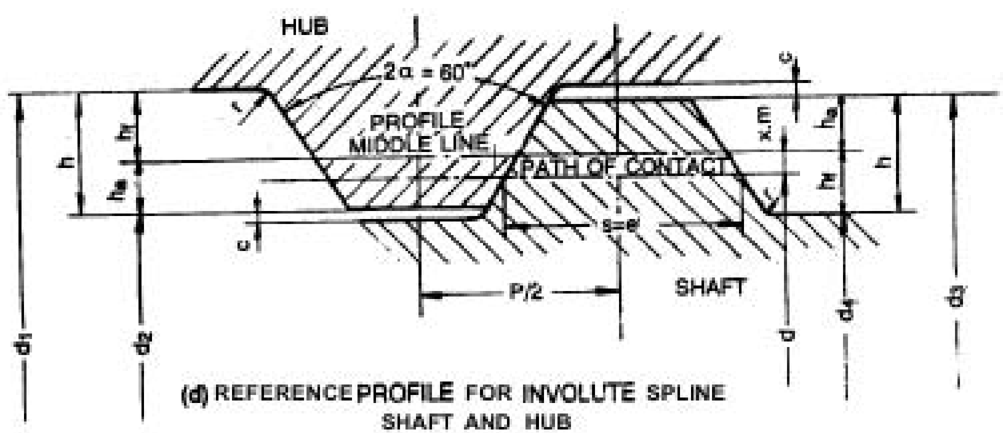


(b) TYPES OF SPLINE FITS

PRESSURE ANGLE 30'



(c) PARAMETERS OF INVOLUTE SPLINE



(d) REFERENCE PROFILE FOR INVOLUTE SPLINE SHAFT AND HUB

Fig. 7.14 Involute spline

Besides the above modes of connections, sometimes combinations are used. Thus, the shaft or the hub may be driven through keys or splines along with either a shrink-fit or a press-fit between the shaft and the hub.

Splines are multi-keys made integral with the shaft. Whenever torques of large magnitude are to be transmitted, the splined shafts are the suited. In a splined shaft, the parallel multi-keys mate with the corresponding grooves or serrations machined inside the hub of the gear or other power-transmitting machine elements. In shafts where torque is transmitted by conventional keys, slots are cut into the shaft and the hub, that is, the key-ways, inside which the keys are fitted. This considerably weakens the torque-transmitting components. Splines are superior to keys because of their greater strength, ability to take impact and reversible loads, and have bigger area of contact. Centering of the jointed components is advanta:eous and the shaft can be made to move accurately, guided along the hub length in case of sliding fit.

Types of Fits

Three types of fits described below are normally used bet   n the hub and the splined-shaft.

1. Major diameter fit: This is the easiest fit to obtain. ...s controlled by varying the major diameter of the external splines. For centering, the mating components contact at the major diameter.
2. Minor diameter fit: This type of fit is controlled by varying the minor diameter of the internal splines. Centering is done by contact at the minor diameter.
3. Side bearing fit: "his type of fit is attained by varying the tooth thickness.

All the above three types have been shown in Fig. 7.14 (b). The type of fit or centering to be adopted will depend upon the amount of accuracy required and the load involved. For high loads, side centering is usually recommended because this type of fit permits better load distribution between the splines. Side fit is not as accurate as the other two types. Hence, when accuracy is the main design criterion, then major or minor diameter fit is employed.

All the above types of fits can be sub-divided into two classes — sliding-fit where there is a relative movement between the mating parts, and press-fit where no relative motion is permitted.

Dimensions of Splines

Both the straight-sided and involute splines have been standardised and the dimensions can be obtained from relevant standards or can be computed by using relevant formulae. For selection of spline size, the undermentioned guidelines may be followed. Taking into account the non-uniform distribution of load on the spline keys, we assume that the effective load carrying capacity is around 75%. The torque is then given by

$$T(\text{N cm}) = 0.75\, p h L z r_m \tag{7.18}$$

where p = Allowable bearing pressure
= 8000 to 12,000 N/cm² for fixed joints without heat treatment. This can be increased to 12000 to 20000 N/cm² with special heat treatment
= 1000 to 2000 N/cm² for ordinary sliding joints

h = Height of spline key (cm)

L = Length of spline key (cm)

n = Number of keys

r_m = Middle radius = (major diameter + minor diameter)/4 (cm)

Involute splines are much superior to straight-sided ones. These splines have the following advantages.

1. Power transmitting capacity is higher than the other types.
2. Involute splines can be produced by the same manufacturing processes and machines as in the case of ordinary gears.
3. These splines have a self-centering action under load. Even a loose-fitted assembly will centre itself when torque is applied.
4. Since involute splines can be made by hob cutters and other generating processes, no special machine is required other than those with which a machine shop is generally equipped. Consequently, the cost of production is also reduced.

Normally, external splines are cut by hobbing, rolling or shaping and the internal ones are produced either by broaching or by shaping in a gear shaper. However, since it is almost the universal practice to broach the internal splines, it is the internal spline which is held to the basic dimensions and the external spline dimensions are varied to control the fit.

The involute spline-keys are thickened from top to bottom. There is no sudden change at the root of the keys. Endurance is greater, failure due to stress concentration is much less, centering is assured, service life is prolonged, and of course, load carrying capacity is much higher.

A hub with involute spline is self-aligning under load. Sometimes crowned splines are used to impart flexibility in the system and to take care of misalignments up to 5 degrees. The pressure angle of an involute spline is normally 30°. As per the American practice, other tooth forms employing different pressure angles are used [Fig. 7.14(a)].

Involute splines seldom fail due to wear or bending stress. Hence, these are not important factors in the design of spline teeth. Though splines are similar to gear teeth, the action of an involute spline joint is different from that of a mating pair of gears. In an involute spline, there is no rolling action. All the teeth of the two components are in mesh at once and since there is no relative motion, wear poses no problem in design. Usually, the splined shafts fail in torsion. Hence, it should be checked against torsion by the usual formulae of strength calculations. Splines also fail by fretting corrosion and by fatigue. The teeth of splines sometimes shear off on the pitch line. A splined system may also fail because the internally toothed member may burst because of applied torque. Besides, high speeds may create severe centrifugal force which aggravates the situation by being added to the bursting stresses. The shell of the internally toothed member becomes vulnerable to failure by rupture.

*Reference **Profile** of Involute Spline Shaft and Spline Hub*

The parameters of an involute spline and its reference profile are shown in Fig. 7.14(c and d). The tooth dimensions of the spline shaft and hub are determined as per the reference profile as well as the hub major diameter d, and the number of teeth z. The correction factor x lies between − 0.05 to + 0.45. This is done keeping in mind that the designer may select a spline with even number of teeth which is normally preferred. This arrangement also enables the designer to keep the mean pressure angle in a fixed and appropriate range which is required for self centering, precision in manufacturing and for restriction of normal pressures. However, in special cases where other design criteria are predominant factors, the dimensions need not be as per the relations given. These can be selected freely as per calculations. Obviously, in that case the restrictions as regards the profile correction factor x are also no longer valid. For normal applications, the following relations hold good.

Pitch $p = \pi m$

Pressure angle = 30°

Number of teeth $z = \frac{1}{m}(d_1 - 2xm - 1.1m)$

Profile correction $xm = \frac{1}{2}(d_1 - mz - 1.1m)$

Whole depth $h = 1.0\,m$

Addendum $h_a = 0.45\,m$

Dedendum $h_f = 0.55\,m$ = Addendum of cutting tool

Tip clearance $c = 0.1\,m$

Radius at root $r = 0.2\,m$

Pitch circle diameter $d = mz$

Base circle diameter $d_b = mz \cos \alpha$

Major diameter of hub d_1 = Reference diameter

Minor diameter of hub $d_2 = mz + 2xm - 0.9\,m = d_1 - 2\,m$

Major diameter of shaft $d_3 = mz + 2xm + 0.9\,m = d_1 - 0.2m$

Major diameter of shaft $d_4 = mz + 2xm - 1.1m = d_1 - 2.2\,m$

Width of tooth-gap of hub e = Width of tooth-thickness of shaft g

$$= \frac{\pi m}{2} + 2xm \tan \alpha$$

Table 7.1 shows the values of reference diameter *vis-a-vis* the number of teeth and module.

7.6 Gear Couplings

To connect shaft ends to transmit motion or torque, couplings are generally used. These machine elements can be broadly classified into two categories — rigid and flexible types.

When compensation for the misalignment between connecting shafts is imperative because of the prevailing operating conditions, the flexible couplings are used. They compensate for various types of misalignment, such as, lack of coaxiality between shafts, parallel as well as angular misalignments, and small axial movement of the shafts caused by temperature difference. Besides, they act as dampers against shock and impact-type of loads.

Various types of flexible couplings of widely different designs are marketed under various trade names. Gear couplings which are a type of flexible couplings, will be discussed in this section. These couplings are also designed in various ways, depending upon the type of service to which they are put. Thus, there are the common type, shear pin type, mill motor type, brake drum type, floating shaft type and a number of other types as well.

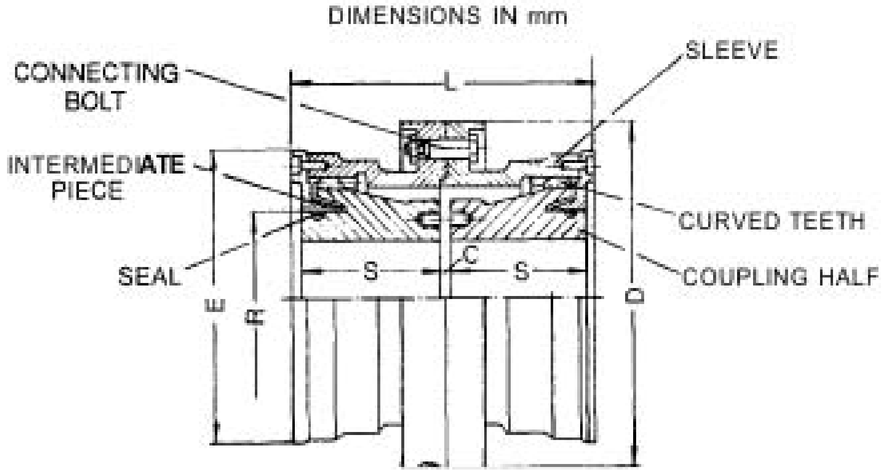
Gear couplings are also known as curved toothed gear couplings because the coupling halves are provided with built-in external gear teeth which are curved or crowned to compensate for the misalignment, thus imparting flexible properties. A typical gear coupling of common design has been illustrated in Table 7.2 along with the relevant data. It consists of the following essential parts: Two coupling halves having crowned external teeth, two sleeves provided with internal gear teeth which mate with the external teeth of the coupling halves, and intermediate pieces which act as a guard against the entry of extraneous materials into the interior of the coupling. Besides these, a complete coupling contains requisite number of bolts to connect the two sleeves which transfer the torque between the driving and the driven shafts, other fasteners, oil seals, gaskets, dowel pins, and an adequate amount of lubricating oil. In some designs, the intermediate pieces are dispensed with and the oil seals are mounted directly on the sleeves. Forged steels are normally used for mating gears. For other parts, 45 C 8 may be used.

Table 7.1

REFERENCE DIAMETER & MAJOR DIAMETER OF HUB d1 (mm)	NO OF TEETH z WITH MODULE m										
	1.0	1.25	1.5	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0
14	14										
16	16	13									
20	18	14	12								
22	20	16	13								
25	24	18	15								
28	25	21	17	12							
30	28	22	18	14							
32	30	24	20	14							
35	34	28	22	16	12						
38	36	29	24	18	14						
40	38	30	25	18	14	12					
42	40	32	26	20	15	13					
45	44	34	28	21	16	14					
48	46	37	30	22	18	14					
50	48	38	32	24	18	15					
52	50	40	33	24	19	16	12				
55		42	35	26	20	17	12				
58		45	37	28	22	18	13				
60		46	38	28	22	18	14				
62		48	40	30	23	19	14				
65		50	42	31	24	20	15	12			
68			44	32	26	21	16	12			
70			45	34	26	23	16	12			
72			46	34	27	22	16	13			
75				36	28	24	17	14			
78				38	30	24	18	14			
80				38	30	25	18	14			
82				40	31	26	19	15			
85				41	32	27	20	16			
88					34	28	20	16			
90					34	28	21	16			
92					35	29	22	17			
95					36	30	22	18			
98					38	31	23	18			
100					38	32	24	18			
105						34	25	20			
110						35	26	21	18		
120						38	28	22	20		
130						42	31	24	22		
150						45	34	26	24		
160						48	36	28	25		
170							38	30	27	19	
180							41	32	28	20	
190							44	34	30	21	
200							46	36	32	22	18
210							49	38	34	24	18
220							51	40	35	25	20
240							54	42	38	28	22
250								46	40	28	24
260								48	42	30	24
280								50	45	31	26
300								54	48	34	28
320								58	52	36	30
340								62	55	38	32
360									58	41	34
380									62	44	36
400									65	46	38
420									68	48	40
440									72	50	42
450									74	55	44
480									76	58	46
500									82	61	48

Values of no. of teeth z given in the space bounded by stepped lines are to be preferred.

Table 7.2 Curved toothed gear coupling



	Normal		Maximum			Finish Bore									Flange details		
Size	kW/ rpm	Torque kNm	kW/ rpm	Torque kNm	Max rpm	Min	Max	D	L	S	E	C	R	PCD	No. of Holes	Fit bolt size	Bolt length
1	0.06	0.55	0.09	0.85	9500	22	40	160	135	60	123	5	60	126	12	M10	35
2	0.12	1.2	0.18	1.8	8500	28	50	183	155	70	145	5	75	147	14	M10	40
3	0.20	2.0	0.30	3	7500	30	60	215	183	80	165	6	90	175	14	M10	48
4	0.34	3.4	0.52	5	6700	32	70	230	198	90	183	6	100	190	14	M10	50
5	0.48	4.6	0.68	6.5	6000	35	80	255	223	100	204	6	120	215	14	M10	55
6	0.68	6.8	1.04	10	5400	40	90	290	249	110	230	8	130	246	14	M12	60
7	0.97	9.5	1.50	13.5	5300	58	100	300	269	120	250	8	140	252	14	M14	70
8	1.26	12.5	1.95	18.5	4750	68	110	330	289	130	270	8	155	282	14	M16	75
9	1.94	18.5	2.87	28	4250	78	125	355	330	150	305	10	175	300	14	M16	80
10	2.76	25	4.07	39	3750	88	140	410	365	165	335	10	200	352	14	M18	85

(Contd)

Table 7.2 *(Contd.)*

Size	Normal kW/ rpm	Normal Torque kN m	Maximum kW/ rpm	Maximum Torque kN m	Max rpm	Finish Bore Min	Finish Bore Max	D	L	S	E	C	R	PCD	Flange details No. of Holes	Flange details Fit bolt size	Flange details Bolt length
11	4.10	39	5.97	58	3350	125	160	460	421	190	380	12	230	402	16	M18	90
12	5.67	52	8.21	84	3000	145	180	505	481	220	425	12	260	443	16	M20	95
13	7.84	75	11.57	110	2650	165	200	570	537	245	475	14	290	504	16	M22	105
14	10.60	100	15.67	150	2500	185	220	610	593	270	520	16	325	536	16	M24	105
15	22.00	210	32.09	320	2000	245	280	785	751	340	670	22	415	703	24	M30	115
16	41.04	395	61.19	585	950	260	350	960	830	400	840	30	590	075	18	M42	170
17	58.21	560	87.31	837	875	340	400	1070	890	430	915	30	650	980	24	M48	180

Note: Numerical values given in the table for kW/rpm, torque and max, rpm are rounded off values

Angular misalignment capacity normally amount to $\pm 1^\circ$ to $\pm 2^\circ$ or $\pm 3^\circ$, depending upon the design. Permissible axial movement $\pm$ 0.5 to $\pm$ 2 mm, maximum parallel misalignment $\pm$ 1.5 to $\pm$ 9 mm. The above values depend on the sizes of the coupling and are valid for sizes progressively from the smallest to the largest ones. Tolerances for hub and shaft will depend upon the sizes as well as on the design and type of the couplings. It is advisable to consult the manufacturers' catalogues or manuals regarding the recommended tolerances and other relevant aspects.

Transient peak loads occur during service leading to momentary increase in torque and this factor should be taken into account while selecting a coupling. Service factors (SF) which provide a basis of estimation and of allowance for combination of various driven equipments and types of loading are listed in Table 7.3.

Table 7.3 Service factors (SF) (Prime mover: electric motor or turbine)

Load,	**Driven** *Equipment*	*SF*
Uniform	Centrifugal pumps, Conveyors-uniform loading. Exciters. Fans and Blowers, Light duty generators, Uniform loading mixers	1.0
Light shock	Centrifugal pumps, Generators, Pulsating Grinders, Hydraulic pumps, Kilns, Line shafting machine tools, Oscillating pumps, Textile machinery, Wood working machinery	1.5
Medium shock	Air compressors-multi-cylinder, Ball and rod mills cranes elevators and hoists punch presses, Reciprocating pumps, Shears, Ship drives, Welding generators	2.0
Heavy shock	Air compressors-single cylinder, Dredgers, Drilling Rigs, Mining machinery, Rolling mill drivers, Rubber mixers	2.5
Extreme shock	Ore crushers, Barstock shears, Conveyors-vibrating	3.0

***Selection** of Size of Coupling*

For proper selection of the coupling, the following data are necessary — type of prime mover and driven machine, rated power of prime mover, speed in rpm, and service factor (SF) as given in Table 7.3. The following example illustrates the selection procedures.

Example 7.2. *Given:* Prime mover = 20 kW electric motor, speed = 1450 rpm, driven equipment = centrifugal pump, type of duty = light shock **load,** nominal power required for the reduction unit = 16 kW.

To select the required gear coupling.

Solution: The service factor is found to be 1.5 from Table 7.3.

Effective kW/rpm = 16 × 1.5/1450 = **0.017. From** Table 7.2 we find that size 1 is suitable for the purpose and is, therefore, selected.

The next step is to check whether the maximum torque required during operation **is** within the allowable limit specified in the table.

$$\text{Effective torque (Nm)} = 9550 \times \frac{16 \times 1.5}{1450} = 158$$

Since the values of torque given in the table are 0.55 kN m or 550 N m (normal) and 0.85 kN m or 850 Nm (max.) the selected coupling is good enough to sustain the required torque.

Finally, the diameters of the shafts to be connected are to be checked to ensure whether they are within the maximum permissible bores specified for the coupling selected. In case the shaft ends of the prime mover and the driven machine exceed these limits, then the coupling of the next higher size is to be selected.

Crowning

As stated before, gear coupling teeth are provided with top crowning to compensate for the misalignment. Besides top crowning, the teeth are also curved laterally so that the barrel shaped teeth can take misalignment in other planes as well.

Normally, the barrel-shaped contour of the tooth is a segment of a circular arc. An improved version of this contour consists of an arc at the central region of the tooth which gradually flattens off at the two ends. When angular misalignment occurs, the flattened curved area of the external tooth surface contacts with the straight surface of the internal tooth. This results in lower induced surface contact stresses per unit area, as we know from the Hertz equations that bodies with the smallest relative curvature have the largest area of contact under load. In other words, under a given load, bodies with the greater radii of curvature have smaller contact stresses.

Since a gear coupling is an item which is bought separately, the function of the designer is restricted to selecting the proper coupling as per the guidelines given in this section or according to the selection procedures given in the catalogues of standard manufacturers. Those readers who would like to delve deeper into the design aspects of these couplings, such as computation of the various stresses on individual components, determination of the correct amount of the crowning and barrel radii, and other parameters, should consult specialised books on the subject.

7.7 Pin Gearing

Pin gearing is an early form of gear mechanism used much before the present day gear technology came to be recognised. This type of gearing is still used in clock mechanisms and also in hoists, winches, turn-tables, as well as those cases where slow speed gear drive is required in the conveying systems.

In pin gearing, the two components of the pair are a wheel in which pins are inserted and a toothed wheel which resembles an ordinary gear (Fig. 7.15). The toothed wheel normally drives

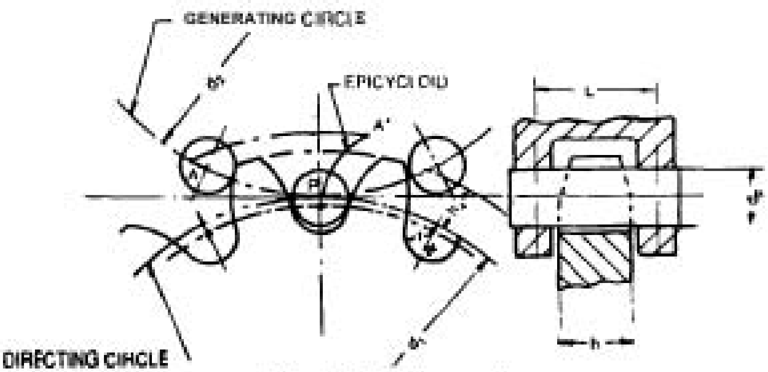


Fig. 7.15 Pin gearing

Based on Zahnraeder, Zapke, 11th edition. 1980, Fig No 36, p 36, VEB Fachbuchverlag, Leipzig the pin wheel. The pins in the pin wheel are sometimes fitted with rotatable rollers to minimise wear due to sliding. While in action, the pins slide (or roll) over the tooth surfaces inside the tooth spaces.

The profile of the tooth of the toothed wheel is an epicycloid. The generating circle contains the centres of the pins, i.e. the circle of diameter d_2 shown in the figure. The directing circle is the circle of diameter d_1 of the toothed wheel. (Reader may see Secs 1.6 and 1.7 on cycloidal curves for definitions of generating and directing circles.) A point on the generating circle, when rolled on the directing circle, generates the epicycloidal curve. Conjugate action takes place between the epicycloid PA' and points on the circle of diameter d_2. But since room has to be made to accommodate the pins which take the load, the epicycloids are shifted and tooth profiles are drawn parallel to the epicycloids as shown in the figure. If the pin wheel is of infinite diameter, analogous to a rack, then the profile of the other wheel is an involute.

Some practical values of the pin gearing system are given below.

Minimum number of teeth of the toothed wheel $z_{min} = 8$ to 12 for $v = 0.2$ to 1 m/s

$$
\begin{aligned}
d_1 &= z_1 m \\
h &= m(1 + 0.03z_1) \\
b &\approx 3.3\,m \\
d_p &= \text{Pin diameter} \approx 1.67\,m \\
L &= \text{Effective average length of pin} \approx b + m + 5\text{ mm} \\
r &= 0.5\,d_p = \text{Fillet radius} \\
a_r &\approx 0.15\,m \\
\text{Usual backlash} &\approx 0.04\,m
\end{aligned}
$$

The contacting surfaces of both the wheels should be adequately hardened to ensure longer life.

7.8 Novikov Gears

Though the involute form of gear toothing has replaced all other curves in power transmission gearing, researchers in the field are always on the look out for other shapes which may prove to be more efficient in power drive. One of these forms is the circular-arc type of toothing developed by Wildhaber in the USA and Novikov in the USSR. It is mainly due to the research work carried out by Novikov that this type of toothing has come to be recognised in gear technology.

The Novikov gears have circular tooth surfaces in the transverse section. The following three different types of curve combinations are generally used

(*i*) Tooth profile of pinion: Convex
Tooth profile of gear: Concave

(*ii*) Tooth profile of pinion: Concave
Tooth profile of gear: Convex

(*iii*) Face of pinion and gear tooth: Convex
Flank of pinion and gear tooth: Concave

Of the above three, the first form is most commonly used.

The teeth of Novikov gears in mesh have point contact. The height of a Novikov gear tooth is only about half that of the corresponding involute tooth having the same module. Different cutters are required for the manufacture of the toothing for the pinion and the gear.

Calculations and experiments have shown that the Novikov gearing can withstand a high amount of load. Compared to an involute gear drive of the same output, the weight requirement of a Novikov system is about half The efficiency can be as high as 99 to 99.5%.

The main demerits of the involute gearing *vis-a-vis* Novikov gearing are that the involute gearing has limited load capacity due to the small radii of curvature of the working surfaces of teeth. Resistance to wear is also poor and there is a heavy loss of power due to friction. Novikov gearing is practically free of these short-comings. In Novikov system, since the profile of teeth of one member is convex and that of the other concave, the surface stresses are considerably reduced. It can be seen from Sec. **2.23** on contact stress that the surface stress is inversely proportional to the square root of the equivalent radius of curvature, R_{eq}, given by $R_{eq} = R_1 R_2/(R_2 - R_1)$, where R_1 and R_2 are the radii of curvature of the two mating surfaces. Here, the negative sign is used because the two curves point to the same general direction of centres, as in the case of internal gears. If the difference $R_2 - R_1$ is very small, R_{eq} attains a very high value. As a consequence, the surface stresses are very much reduced.

Research carried out in different countries has yielded the following information.

1. Novikov system is better than the corresponding involute system in high-load and medium-load categories of applications where the speeds are not high.
2. Novikov tooth surfaces can take and withstand around 3 to 5 times the load which the involute gear tooth surface can take.
3. These high loads do not lead to the typical fatigue phenomena like pitting, wearing down of surfaces and other allied failures.
4. Novikov gears can be subjected to heat-treatment processes for increasing the load capacity just as in the case of involute gearing, e.g. hardening, nitriding, and other processes.
5. The lubricant retaining properties of Novikov gear teeth in mesh are excellent. Mating teeth tend to form a thick oil film, thus ensuring smoother transmission of motion.
6. The overall efficiency in power transmission in case of the Novikov system is higher than in case of involute gearing.

There are, however, certain disadvantages too. Compared to the involute system, the Novikov system is noisier and also it is unduly sensitive to variations in centre distance.

Figure **7.16**(a) shows a Novikov gear pair in mesh. It can be seen that the convex-concave tooth profiles conform to one another, thus enveloping the mating components. This is in contrast to the conventional (external) involute toothing in which case tooth surfaces in mutual engagement are both convex in form.

We have seen in Chaps 1 and **2** that to achieve a constant velocity ratio during rotation, the common normal to the profiles of the meshing pair of teeth at the point of contact must pass through a fixed point on the line of centres, termed the pitch point P. Such condition produces conjugate action. The common normal must obey this law in all positions of contact in order to attain conjugate action. In case of involute spur gears, we know that the mating teeth make contact along a straight line across the tooth width b. This contact line changes its position as the gears rotate, beginning near the root of the driving pinion tooth and ending near its tip. We have also seen that the two mating spur gears behave as if two pitch cylinders drive by friction without slippage, ensuring transmission of motion without (apparent) loss of power.

In Novikov system, a pair of spur gears contact only once during a complete rotation. In consequence, the common normal passes through the pitch point at only one position during a complete rotation. Hence, the conjugate action is not achieved. However, if we can visualise an infinite number of Novikov spur gears, each of infinitesimal thickness, joined together but each one slightly out of phase, then we can see that the contact point moves across the teeth from one side to another as the gears rotate. In other words, it amounts to a pair of helical gears in mesh,

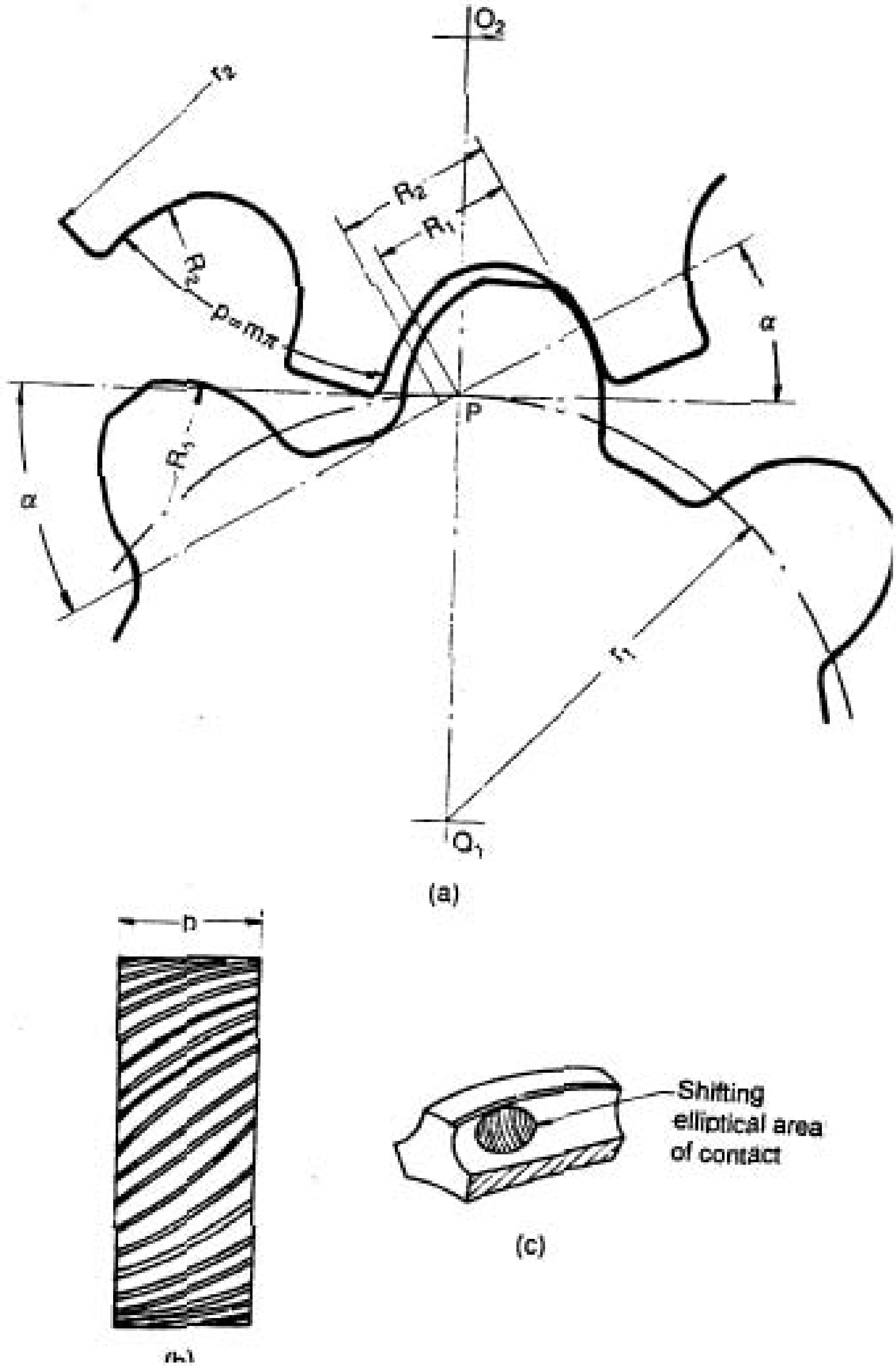


Fig. 7.16 Novikov gear

Based on Gear Design and Application, Chironis, 1967 edition, Fig. No. 2 & 5, p. 125 & 127, McGraw-Hill Book Co. Inc. New York each looking like the one shown in Fig. 7.16 (b), In such a case, conjugate action is ensured and the pressure angle a remains constant.

The system makes a point contact theoretically only. Actually, it is an elliptical area which travels across the width of the teeth as shown in Fig. 7.6 (c). Close conformity of the mating teeth, deformation of the point under load and gradual wearing result in such an area.

We have seen in Chap. 2 that this area of contact due to deformation by load and the surface stresses thereof can be calculated by the Hertz equations. We have also noticed that the convex-concave combination of gear tooth curves in mesh reduces the surface stresses. This, among others, is one of the contributary factors of the high load capacity of Novikov gearing.

8
Miscellaneous Topics

8.1 Reduction Gear Units

Broadly speaking, reduction gear units are contrivances used for providing high torques with comparatively low rotational speeds at the output end. They are normally placed in-between the prime-movers such as electric motors, and the machines to be driven for the **work** they are meant **for.** On both input and output sides of the reduction units, suitable type of couplings are provided. These units may also be used as step-up devices, that is, as speed increasing units.

In Sec. 1.1 a general clasification of various gear drives, their usual reduction ratios, stages, power, speed, torques and other relevant parameters have deen discussed. **A** reduction gear system is usually encased in **a** housing which is impermeable to, oil, dust and other outside agencies and is normally provided with an oil bath. The term "gear box" is used to indicate such a housing plus the internals it contains, viz., the gears, shafts, pinion shafts, seals and other components. The term may also be used sometimes to imply the housing or the casing only. The housing along with other connected accessories will be discussed in Sec. 8.2. In this section and elsewhere in this book, the term "gearbox" has been used to indicate the complete reduction unit as a whole.

A reduction gear unit may be of a built-in type as used in machine tools or it may be an integral part of a prime mover such as geared-motors, or it may be of the conventional independent unit type which is commonly known as a gear box.

Depending on the orientation of the driving and the driven shafts, reduction unit can be categorised into the following classes.

1. Spur and helical gear reduction units. These types have parallel shafts and may consist **of** one or more stages of reduction (Fig. 8.1).
2. Bevel gear reduction units. The axes of the shafts in these units are generally at right angles to each other (Fig. 8.2). The axes may be co-planer intersecting each other or they may be non-intersecting type as in the case of gear boxes using hypoid gears, viz., car differentials.
3. Worm and worm-wheel reduction units. The axes in these units are at right angles and are non-intersecting (Fig. 8.3).

Besides the above common types, gear boxes may consist of one or more combinations of the above classes, such as bevel spur or bevel helical gears where the input and the output shafts are

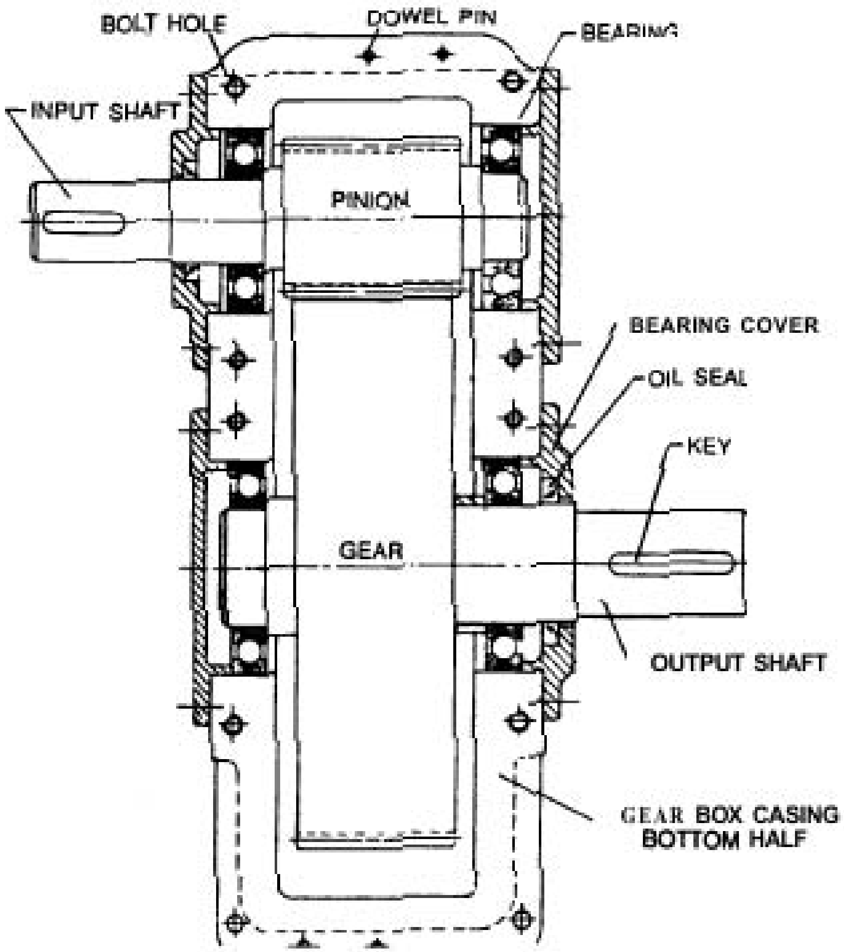


Fig. 8.1. Spur and helical gear reduction unit

at right angles to each other, spur or helical gears combined with worm drives and other assorted drives. In a combination drive with worm, the worm is used as a high-speed component because the efficiency of a worm drive is greater at high speeds. Planetary gear boxes are used where a high reduction ratio and a compact design are the main criteria. For the same power and gear ratios, a planetary gear box can be so designed that its weight is only half or one-third of the weight of a conventional gear box. In co-axial gear boxes the axes of the input and the output shafts are in one straight line. The disadvantage of this type lies in the fact that provision of an intermediate projection from the floor of the gear box casing is required to hold the bearings of the two shafts. The arrangement is also prone to misalignment.

Rigid housing in a gear box ensures correct alignment of the shaft bearings. Gears with small modules have greater accuracy of drive and create lesser noise during operation besides reducing the manufacturing cost. Copious lubrication must be ensured to decrease wear and increase overall efficiency.

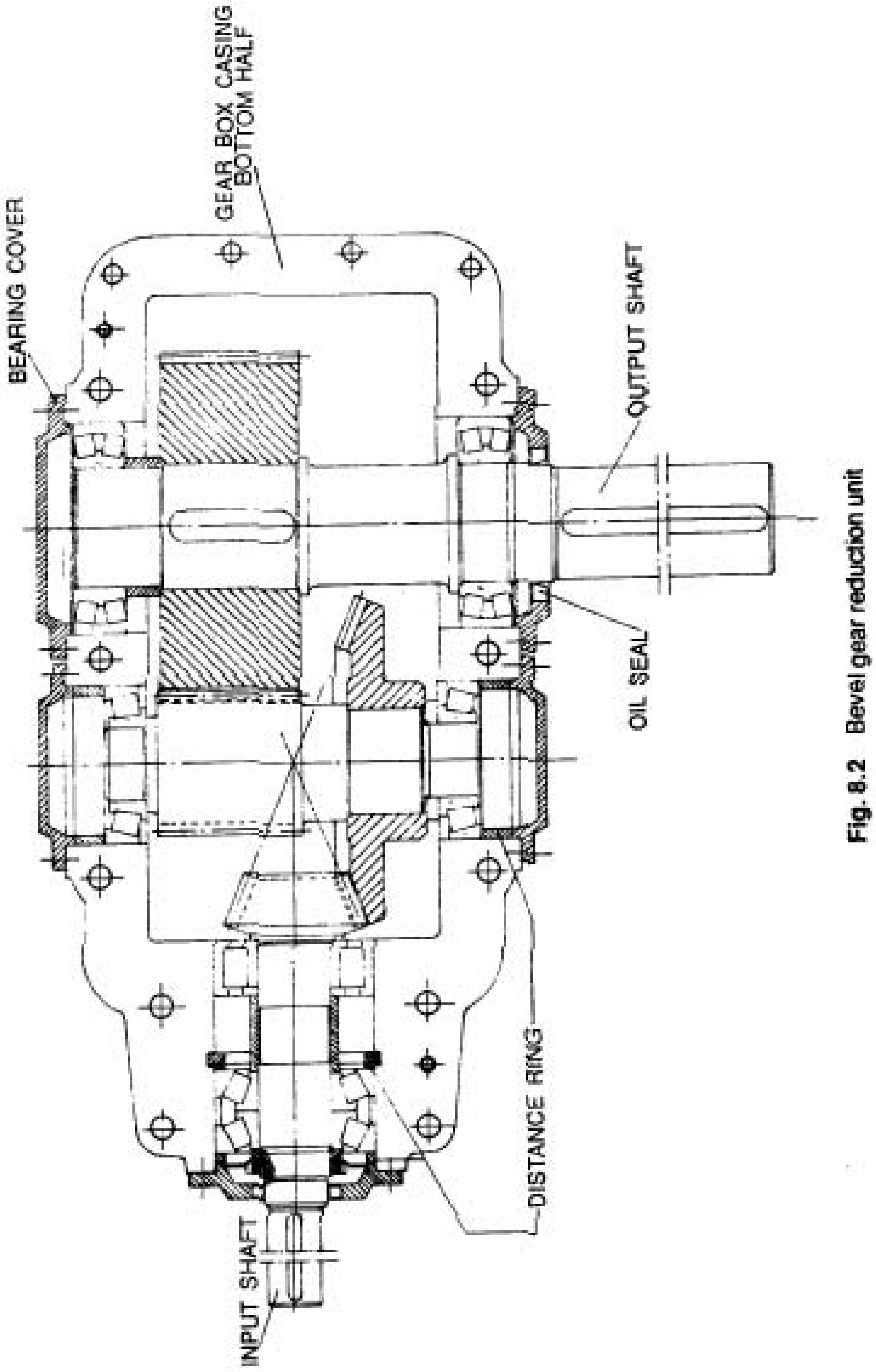


Fig. 8.2 Bevel gear reduction unit

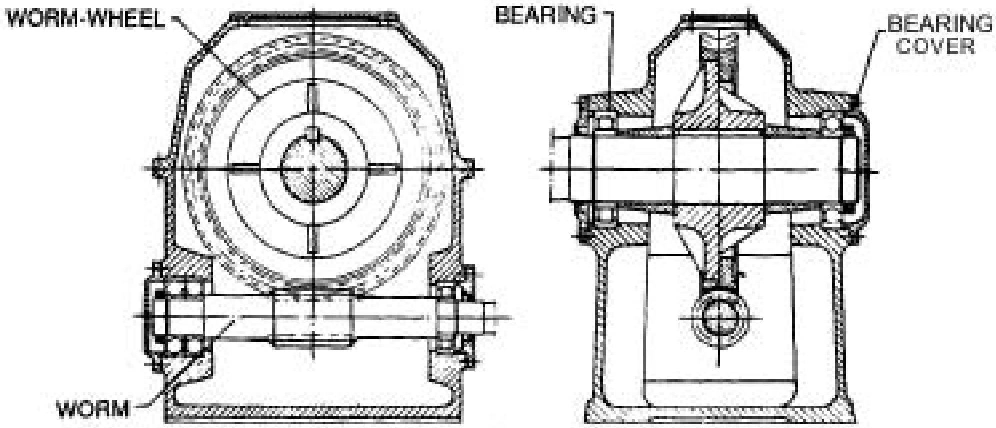


Fig. 8.3 Worm and worm-wheel reduction unit

Geared Motors

Geared motors are such units in which the prime mover (electric motor) and the speed reduction unit (gear box) are joined together, so that they make a single compact unit. Among many advantages, this arrangement saves a lot of space as it dispenses with the use of an intermediate coupling, and also as it allows the motor shaft to protrude inside the gear box. The motor and the gear box may be separate and then bolted together for compactness. Or, the electrical and the mechanical components may be enclosed in a common housing.

Depending upon the type of mounting and the internals used, geared motors are classified into flange-mounted or foot-mounted types. Both these types are used in industries, such as roller table drives for rolling mills, textile machines, and a host of appliances. Normally, they are meant for small power applications. As far as internals are concerned, these motorised speed reducers can be categorised according to the gearing system used, like the worm-gear drive, spur or helical gear train with parallel shafts, planetary gearing or any other suitable combination thereof. A flange-mounted geared motor is shown in Fig. 8.4.

For quietness of operation, high speed-reduction ratio and for shaft axes at right angles to each other, worm-type reduction unit is normally used. This can be of double reduction type if very low speeds are envisaged. For intermediate speed reduction range, parallel shaft type speed reducers are used. For large speed reductions where economy and compactness are the design criteria, the planetary gearings are employed.

Selection Procedures for Standard Gear Boxes

Standard worm, spur, helical and other type of gear boxes are manufactured by firm specialised in the field. These firms offer catalogues and other relevant technical literature giving detailed information concerning their product range as well as the data to help select the right types and sizes of gear boxes for the service conditions in question. Besides, IS: 7403 lays down selection procedures for worm and helical gear boxes. While selecting the standard gear box, the relevant calculations should be made as per IS: 7403, the salient points of which are discussed below.

For selecting the correct type and size of the gear box, the following data concerning both the prime mover and the driven machine are essential.

i. Type: For prime movers, e.g. electric motors, internal combustion engines
For driven machines, e.g. cranes, machine tools
ii. Power: For prime mover — rated power
For driven machine — actual power requirements
iii. Speed
iv. Operating conditions, e.g., ambient temperature, duty factors, any special working conditions.

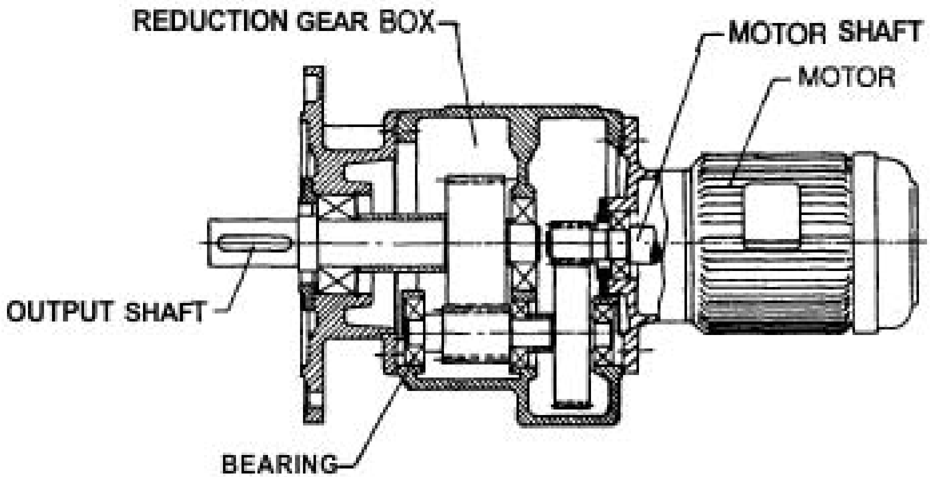


Fig. 8.4 Geared motor

Commercial gear boxes are assigned power ratings by the manufacturers of the gear boxes. These power values, called the "rated power", are given in the manufacturers catalogues along with the relevant data. One sample data sheet is shown in Table 8.1.

A gear box is classified according to its size, rated power, maximum torque capacity, maximum speed, reduction ratio, input and output speeds, and dimensional constraints, if any. To facilitate systematic selection of the correct gear box cotnmensurate with the operating conditions, the following guidelines in the form of steps are given.

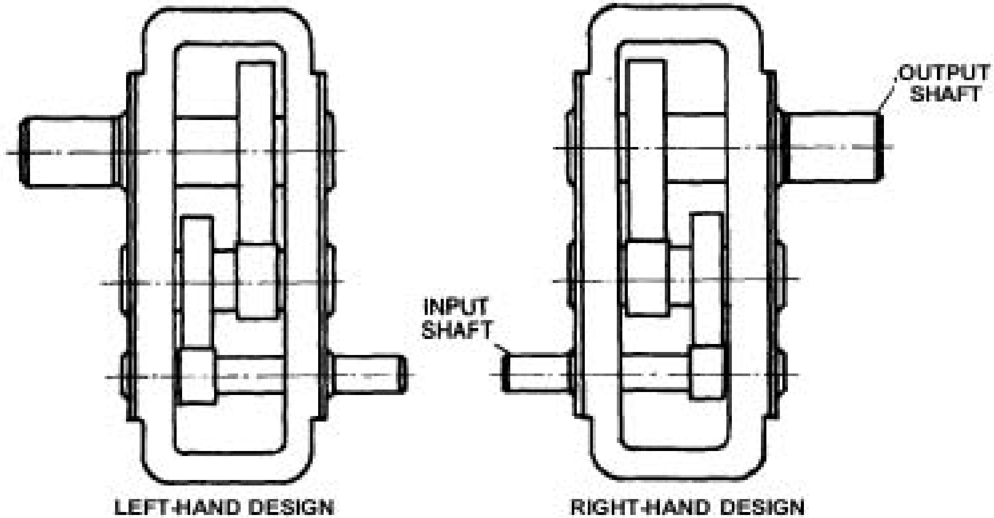
1. Calculate the actual power required for driving the machine at the desired speed.
2. Determine the type of load (U, M, H or HI) which is likely to be encountered by the driven machine during service from Table 8.2.
3. Depending upon the prime mover, duration of service and type of load as given in Table 8.2, select the service factor (SF) from Table 8.3.
4. Calculate the "equivalent power" by using the relation

$$\text{Equivalent power} = \text{Actual power} \times \text{Service factor}$$

5. Select the right size of the gear box from the manufacturers' catalogues, so that the rated power of the gear box selected is equal to or greater than the calculated equivalent power for the given speed ratio of the gear box.

Besides the above criteria, other aspects listed below should be checked as well.

Table 8.1 Power **ratings of a** double **reduction** gear box



Nominal trans-mission ratio	*Nominal speeds [1/min]*		*Size of gear unit*																	
			110	125	140	160	180	200	225	250	280	315	355	400	450	500	560	630	710	800
i_N	n_1	n_2	*Nominal gear box rating* P_N [kW]																	
	1500	240	36	50	70	105	145	205	285	370	530	790	1060	1450	2020	3740	5060	7020		
6,3	1000	160	24	34	47	71	100	145	215	280	400	560	800	1100	1520	2650	3650	4780	7120	
	750	120	18	25	36	54	74	110	170	230	310	425	600	900	1200	1990	2790	3600	5420	7700
	1500	210	36	48	66	100	140	195	280	380	490	730	990	1350	1900	3400	4760	6200		
7.1	1000	140	24	32	44	66	93	135	200	255	365	490	720	1000	1400	2330	3270	4210	6270	
	750	105	18	24	33	50	71	100	150	210	275	370	550	790	1050	1760	2470	3170	4730	6700
	1500	188	32	44	62	91	125	180	255	350	450	660	920	1300	1750	3070	4300	5600		
8	1000	125	22	30	41	60	85	125	180	245	335	450	680	950	1270	2120	2970	3820	5700	
	750	94	16	22	31	46	65	92	135	190	250	340	520	710	950	1590	2230	2870	4270	6070
9	1500	167	29	40	56	83	130	185	225	320	450	580	820	1100	1500	2740	3840	5000		
	1000	111	19	27	38	56	86	125	160	215	300	430	620	800	1120	1890	2640	3400	5070	

(Contd.)

Table 8.1 *(Contd)*

Nominal transmission ratio i_N	Nominal speeds [1/min] n_1	n_2	110	125	140	160	180	200	225	250	280	315	355	400	450	500	560	630	710	800
			Nominal gear box rating P_N [kW]																	
	750	83	15	20	28	43	67	95	125	170	235	340	500	650	900	1470	2080	2750	4020	5760
	1500	150	25	35	50	74	100	150	210	280	390	540	760	1050	1420	2540	3560	4590		
10	1000	100	17	24	33	49	68	95	145	195	265	360	540	750	1000	1700	2380	3060	4560	
	750	75	13	18	25	37	50	80	110	155	210	280	420	600	800	1320	1860	2460	3600	5160
	1500	134	22	32	45	66	95	140	180	250	330	480	680	900	1250	2270	3180	4090		
11,2	1000	89	15	21	30	45	65	95	130	175	245	360	500	680	940	1530	2140	2750	4320	
	750	67	11	16	22	35	49	72	95	130	185	270	400	500	720	1180	1660	2200	3220	4610
	1500	120	21	29	40	55	80	110	170	225	320	430	640	850	1200	2020	2830	3630	5420	
12.5	1000	80	14	19	27	37	52	77	115	165	220	300	450	600	850	1390	1970	2600	3800	
	750	60	11	15	20	28	42	58	88	125	165	225	330	450	640	1050	1480	1950	2860	4090
	1500	107	18	26	35	48	68	100	150	205	280	380	550	710	950	1790	2510	3230	4820	
14	1000	71	12	17	24	32	46	70	105	145	195	265	400	520	710	1240	1750	2310	3380	
	750	53	9	13	18	24	35	52	78	110	145	200	290	420	560	930	1310	1730	2530	3630
	1500	94	15	22	30	43	60	90	135	185	250	340	490	650	860	1590	2230	2870	4270	
16	1000	62	10	15	20	29	40	62	92	130	175	235	350	490	650	1100	1550	2050	3000	
	750	47	8	11	15	22	32	47	69	97	130	175	270	370	500	820	1170	1540	2250	3220
	1500	83	13	19	27	37	55	73	120	140	220	310	430	550	740	1470	1760	2570	4020	
18	1000	56	8.5	14	19	25	39	51	80	98	145	230	320	410	540	970	1230	1820	2730	
	750	41	6.5	10	15	19	30	40	62	77	110	180	250	340	440	770	950	1440	2140	3110
	1500	75		17	24	35	49	73	110	140	210	280	410	520	700	1320	1860	2460	3600	
20	1000	50		12	17	23	33	49	74	98	140	190	280	380	500	880	1240	1640	2400	
	750	38		9	12	18	25	38	58	77	110	145	230	310	400	700	990	1290	1920	2780
	1500	67			21	30	41	65	99	135	185	250	390	490	660	1180	1550	2020	3110	
22,4	1000	45			14	20	27	44	66	92	125	170	260	350	460	790	1050	1360	2100	
	750	33			11	16	21	34	52	70	98	130	200	280	370	620	790	1040	1600	2370

Table 8.2 Load characteristics values for different applications

U = Uniform Load
M = Moderate Shock Load
H = Heavy Shock Load
HI = High Inertia Load, Manufacturer to be consulted

Driven machine	*Type of load*	*Driven machine*	*Type of load*
Agitators:		Bucket	M
Pure liquids	U	Chain	M
Liquids and solids	M	Flight	M
Liquids—variable density	M	Live roll	HI
Blowers:		Oven	M
Centrifugal	U	Reciprocating	H
Lobe	M	Screw	M
Vane	U	Shaker	H
Brewing and distilling:		***Cranes:***	
Bottling machinery	U	Main hoists	U
Brew kettles—continuous duty	U	Bridge travel	HI
Cookers—continuous duty	U	Trolley travel	HI
Mash tubes—continuous duty	U	***Crusher:***	
Scale hopper—frequent starts	M	Ore	H
Can filling machines	U	Stone	H
Cane knives	M	Sugar	M
Car dumpers	H	***Dredges:***	
Car pullers	M	Cable reels	M
Clarifiers	U	Conveyors	M
Classifiers	M	Cutter head drives	H
Clay working machinery:		Jig drives	M
Brick press	H	Manoeuvering winches	M
Briquette machine	H	Pumps	M
Clay working machinery	M	Screen drive	H
Pug mill	M	Stackers	M
Compressors:		Utility winches	M
Centrifugal	U	***Dry dock cranes:***	
Lobe	M	Main hoist	M
Reciprocating multi-cylinder	M	Auxiliary hoist	M
Single-cylinder	H	Boom, luffing	M
Conveyors—uniformly** Loaded **or fed:		Rotating swing or slew	M
Apron	U	Tracking, drive wheels	H
Assembly	U	***Elevators:***	
Belt	U	Bucket—uniform load	U
Bucket	U	Bucket—heavy load	M
Chain	U	Centrifugal discharge	U
Flight	U	Bucket continuous	U
Oven	U	Escalators	U
Screw	U	Freight	M
Conveyors—heavy** duty **not uniformly fed:		Gravity discharge	U
Apron	M	Man lifts	HI
Assembly	M	Passenger lifts	HI
Belt	M	***Fans:***	
		Centrifugal	U
		Induced draft	M

Table 8.2 (Contd)

Driven machine	Type of load	Driven machine	Type of load
Large—mine, etc	U	Planer tilting hoist	M
Large—industrial	M	Re-saw merry-go-round conveyor	M
Light—smaller diameter	U	Roll cases	H
		Slab conveyor	H
Feeders:		Small waste conveyor-belt	U
Apron	M	Small waste conveyor-chain	M
Belt	M	Sorting table	M
Disc	U	Tipple hoist conveyor	M
Reciprocating	H	Tipple hoist drive	M
Screw	M	Transfer conveyors	M
Food industry:		Transfer rolls	M
Beef slicer	M	Tray drive	M
Ceral cooker	U	Trimmer feed	M
Dough mixer	M	Waste conveyor	M
Meat grinders	M	Machine tools	
Generators—hot welding	U	Bending roll	M
Hammer mills	H	Punch press—gear driven	H
		Notching press—belt driven	HI
Hoists:		Plate planers	H
Heavy duty	H	Tapping machine	H
Medium duty	M	Other machine tools and main drive	M
Skip	M	Auxiliary drives	U
Laundry washers reversing	M	Metal mills	
Laundry tumblers	M	Draw bench carriage and main drive	M
Line shafts:		Pinch dryer and slitters	M
Driving processing equipment	M	Table conveyors non-reversing group drives	H
Light	U	Wire drawing and flattening machine	M
Other line shafts	U	Wire winding machine	M
		Reversing	HI
Lumber industry			
Barkers—hydraulic-mechanical	M	Mills—rotary type	
Burner conveyor	M	Ball	M
Chain saw and drag saw	H	Cement kilns	M
Chain transfer	H	Dryers and coolers	M
Craneway transfer	H	Kilns—other than cement	M
De-barking drum	H	Pebble	M
Edger feed	M	Rod	
Gang feed	M	Plain	M
Green chain	M	Wedge bar	M
Live rolls	H	Tumbling barrels	H
Log deck	H		
Log haul—incline	H	Mixers:	
Log haul—well type	H	Concrete mixers—continuous	M
Log turning device	H	Intermittent	M
Main log conveyor	H	Constant density	U
Off bearing rolls	M	Variable density	M
Planer feed chains	M	Oil industry	
Planer floor chains	M	Chillers	M
Paraffin filter press	M	Rubber calenders	M

(Contd)

Table 8.2 *(Contd)*

Driven machine	Type of load	Driven machine	Type of load
Rotary kilns	M	Rubber mill—2 on line	M
Paper mills:		Rubber mill—3 on line	U
Agitators (mixers)	M	Sheeter	M
Barker—auxiliaries-hydraulic	M	Tubers and strainers	M
Barker—mechanical	M	Warming mills	M
Barking drum	H	Sand muller	M
Beater and pulper	M	Sewage disposal equipment:	
Bleacher	U	Bar screens	U
Calendars	M	Chemical feeders	U
Calendars, super	H	Collectors	U
Converting machine, except cutters-platers	M	Dewatering screws	M
Conveyors	U	Scum breakers	M
Couch	H	Slow or rapid mixers	M
Cutters-platers	H	Thickeners	M
Cylinders	M	Vaccum fillers	M
Dryers	M	Screens:	
Felt stretcher	M	Air washing	U
Felt whipper	H	Rotary—stone or gravel	M
Jordans	H	Travelling water intake	U
Log haul	H	Slab pushers	M
Presses	U	Stockers	U
Pulp machine reel	M	Sugar industry:	
Stock chest	M	Cane knives	M
Suction role	U	Crushers	M
Washers and thickeners	M	Mills	M
Winders	U	Textile industry:	
Printing presses	HI	Batchers	M
Pullers:		Calendars	M
Barge haul	H	Cards	M
Pumps:		Dry cans	M
Centrifugal	U	Dryers	M
Proportioning	M	Dyeing machinery	M
Reciprocating:		Looms	M
Single acting, 3 or more cylinders	M	Mangles	M
Double acting, 2 or more cylinders	M	Nappers	M
Single acting, 1 or 2 cylinders	HI	Pads	M
Rotary		Slashers	M
Gear type	U	Soaps	M
Lobe, vane	U	Spinners	M
Rubber and plastic industries:		Tenter Frames	M
Crackers	H	Washers	M
Laboratory equipment	M	Winders	M
Mixing mills	H		
Refiners	M		
		Windlass	HI

Table 8.3 Service *factors*

Prime mover	*Duration of service* (hours/day)	*Service factors*			
		Nature of load on gear unit from driven machine			
		Uniform	Moderate shock	Heavy shock worm	Heavy shock Helical and spur
Electric motor or steam turbine	2	0.75	0.9	1.25	1.40
	4	0.8	1.0	1.3	1.5
	8	0.9	1.1	1.45	1.65
	12	1.00	1.25	1.55	1.76
	24	1.25	1.5	1.75	2.00
Multi-cylinder internal combustion engine	2	0.9	1.1	1.25	1.65
	4	1.0	1.25	1.4	1.75
	8	1.1	1.35	1.6	1.90
	12	1.25	1.5	1.75	2.00
	24	1.5	1.75	2.00	2.25
Single-cylinder internal combustion engine	2	1.1	1.35	1.75	1.90
	4	1.25	1.5	1.85	2.00
	8	1.35	1.65	1.95	2.15
	12	1.5	1.75	2.05	2.25
	24	1.75	2.0	2.25	2.50

Momentary or peak load: This means a load which acts for a duration of not more than 15 seconds. Normally, the conventional gear boxes are so designed that they can take momentary loads of twice the rated capacity, which means that they can take 100% overload. In case the peak power requirements of the driven machine exceed even this amount) then it is to be checked that the selected size should have such power rating that its rated power is equal to or greater than half the peak power.

Brake torque: When the prime mover is equipped with braking arrangements, it is to be ensured that the torque rating of the selected gear box is greater than that of the brake.

Overhung load: Sometimes, the output shaft of the gear box carries a sprocket, pulley or a pinion. This imposes overhung load on the shaft, The equivalent overhung load can be calculated by using the formula

$$\text{Overhung load (N)} = 9555 \times 10^{3} \times \frac{\text{Shaft power (kW)} \times \text{Load factor}}{\text{Shaft speed (rpm)} \times \text{Pitch of sprocket, etc. (mm)}}$$

The load factor is to be taken from the following table

Overhung member	*Load factor*
Sprocket	1.00
Spur pinion	1.25
Vee belt pulley	1.50
Flat belt pulley	3.00

The overhung load thus calculated should be less than the maximum permissible overhung load on the gear box.

Determination of Speed

The angular speeds obtainable in a conventional gear box are normally calculated on the basis of geometrical progression. This method is particularly applicable in case of machine tool gear boxes for varying spindle speeds or feed movements.

While designing geared transmission systems, the problem of properly proportioning a train of gears to attain a given velocity ratio or a given series of speeds is often encountered. In such cases, a uniform reduction between the different stages is conducive to a high efficiency and a logical sequence of events. When geometrical progression is used, successive speeds are obtained by multiplying each preceding term by a ratio or constant multiplier. This is elaborated below.

When the maximum and the minimum speeds as well as the number of speeds are known, then the following formula is applicable

$$f = \sqrt[S-1]{\frac{n_{max}}{n_{min}}} \tag{8.1}$$

Using logarithm, the above equation can be rewritten as,

$$\log f = \frac{1}{S-1}(\log n_{max} - \log n_{min})$$

where n_{max} and n_{min} are the maximum and minimum speeds in rpm, S is the number of speeds of the gear reduction unit, and f is the ratio or the common factor with which any speed is to be multiplied in order to get the next higher speed. Example 8.1 illustrates the procedure.

Example 8.1: *Given:*

$$n_{max} = 1000 \text{ rpm}$$
$$n_{min} = 50 \text{ rpm}$$

To find the speeds of a double reduction unit.

Solution: Since it is a double reduction unit, the number of stages of reduction is **2**, and the number of speeds is **3.** The number of shafts is **3** as shown in the figure accompanying Table 8.1. The speed of the input shaft is 1000 rpm, and the speed of the output shaft is 50 rpm. Therefore, the reduction ratio i is 1000/50 or 20.

Using **Eq. 8.1,** we have

$$f = \sqrt[3-1]{\frac{1000}{50}} = \sqrt[2]{20} = 4.472136$$

The derived speeds of the shafts are as follows

3rd shaft (output):	50 rpm
2nd shaft (intermediate):	**50 x 4.472136 = 223.6068** rpm
1st shaft (input):	**223.6068 x 4.472136** = 1000 rpm

The job of a gear designer in general is not so much as to design a gear box than to select one. For proper selection, catalogues and manuals of standard gear box manufacturers contain exhaustive guidelines.

As an example, one such data sheet is shown in Table 8.1 for a double reduction gear box. Ultimate selection of the proper size of the gear unit is made after considering various service factors, thermal considerations, cooling systems, lubrication and other relevant parameters.

For numbering the size of the gear box, different manufacturers adopt different yardsticks. Often the size indicates the centre distance between the input and the output shafts in mm. Still other companies use some other parameters or may assign the size number arbitrarily or in numerically ascending order. In any case, once a size is selected, the gear box housing becomes fixed in dimensional parameters. Different nominal power ratings for a particular gear box are obtained by varying the input and the output speeds, keeping the transmission ratio as constant. This can also be achieved by varying the transmission ratio by changing the internal components of the gear box as in a change-gear system, keeping the centre distance the same.

It can be seen from Table 8.1 that the power rating does not necessarily vary on pro rata basis. For example, in case of size 200 and transmission ratio 18, the power ratings for speeds 750 and 1500 rpm are 40 and 73 kW respectively, and not 40 and 80 kW. The difference is attributable to various practical operational factors and manufacturing constraints.

For the sake of rationalisation of parts, and also to simplify manufacturing procedures as well as to minimise production costs, gear box components are not always made strictly as per the designed or calculated values. Thus, for example, a shaft of a particular diameter may be meant for catering to different torques within allowable limits.

8.2 Gear Box Housing and Accessories

Gear box housings or casings are containers in which the internals, namely the gears, shafts, pinion shafts, bearings, oil seals, bearing covers and other components are mounted.

The prerequisites for a reasonably free, long lasting, non-jamming, vibration free and efficient load transmitting gear-drive are proper mounting and alignment of the bearings, maintenance of the correct centre distance and provision of lubricating arrangements ensuring proper and regular supply of lubricants, besides other factors. Closed gear box housings, if properly designed, can achieve such objectives.

As material for gear box casings, good quality cast iron is used in most of the cases. Steel castings or light metal castings are also popular, but they are used in special cases. Fabricated housings are also not uncommon.

Cast iron housings have good damping properties and freedom from noise. In spite of the fact that patterns are required for such housings, it often pays to use CI castings, even when small numbers are required, because the initial costs for patterns are nullified in the long run. Steel castings, which are obviously much costlier than CI housings, are used only in those cases where CI casings are not strong enough to withstand the operational stresses involved. Because of their lighter weight, light metal housings are usually used in automotive applications. Gear housings are also made of fabricated, welded steel plates and sections. Housings of this type are recommended for a single piece or for very small number of pieces. For very big housings, the cost of pattern is saved if fabricated design is used. The welded construction also affords the designer to reduce weight of the housing considerably. Noise damping property, however, is not as good as that of the CI castings. Fabricated casings are often provided with ribs for extra strength. They must be heat-treated to relieve thermal stresses.

It is relevant to mention here that, in broad terms, the approximate cost break-up for a spur or helical gear box is as follows:

Housing:	36%
Gears:	22%
Shafts:	21%
Bearings:	9%
Others:	12%

A gear box housing in general consists of two halves — the upper half and the lower half. The plane of separation of the two halves also normally contains the axes of the shafts and bearings. Such arrangement facilitates easy mounting and dismantling of shafts and bearings. "hemating surfaces of the two halves are properly machined and suitable gaskets are provided between them to secure tightness against entry of dust and leakage of oil. The upper and the lower casings are then bolted together and are also provided with dowel pins for proper alignment. Oil seals are fitted inside the grooves on the bearing covers through which the shafts project out. These serve the dual purpose of preventing the gear oil from leaking out and extraneous contaminants from entering the gear box. Felt sealing rings are also used for the purpose. The radial oil seals, which are usually fitted to the gear box bearing covers, are of specifications as per IS: 5129-1969. Bolt holes are bored on the bottom flange of the lower casing for securing the gear box to its support or to the civil fouhdations.

Figure 8.5 gives the dimensional parameters of a gear box. The following guidelines may be used for determining the main parameters of a cast iron gear box.

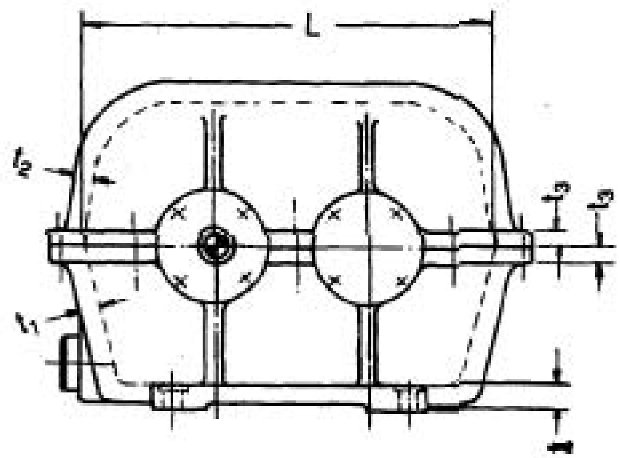


Fig. 8.5 Dimensional parameters of a gear box

If the overall length of the housing is L mm, then the wall thickness of lower half will be $t_1 = 0.012L + 5$ mm, wall thickness of upper half $t_2 = 0.8$ to $1\,t_1$, thickness of mating flanges of casing $t_3 = 1.5t_1$, and thickness of bottom flange $t_4 = 2t_1$.

Gear Box Bearings

Depending on the type of design, size and operational parameters of the gear box, both anti-friction bearings and journal bearings are used. Anti-friction bearings are mainly suitable for gear drives with small and medium loads and speed. These bearings require little maintenance and their starting resistance is negligible. They are lubricated by grease or by gear oil, depending on the overall design. Deep groove ball bearings are mostly used because they can take both radial and axial loads. Cylindrical bearings are suitable when only radial loads are encountered. For high radial and axial loads, self aligning spherical ball or roller bearings as well as tapered roller bearings are generally used. Besides, suitable combinations of the above types of bearings are also employed. To alleviate space problems, needle bearings are sometimes used. Calculation of anti-friction bearings are generally based on a service life of at least 15,000 operational hours. The bearings are generally secured on the housing by means of bearings covers.

To compensate for the possible thermal expansion of the shaft during service, and also to take care of the assembly tolerances, one end of the shaft and mounting is usually provided with a floating bearing to allow axial movement. Such bearings do not transmit any axial or thrust loads. The floating bearing effect can be achieved by using separable type of cylindrical roller bearings, by using ball bearings but at the same time providing a little gap between the leg of the bearing cover which enters the gear box housing and the outer ring of the bearing, and by other methods, the details of which are given in the catalogues of standard bearing manufacturers. 'She other end of the shaft and mounting is normally arranged to have a fixed bearing. Only fixed bearings are used in a design when it is imperative that axial loads are to be sustained in both directions.

Anti-friction bearings are, however, not suitable for high speeds as they create problems of noise. Journal bearings are preferred for such cases. These bearings are generally used for big and high speed gear drives, and they are usually hydro-dynamically lubricated with pressurised oil. Lubrication of gear drives in general has been discussed in Sec. 8.6.

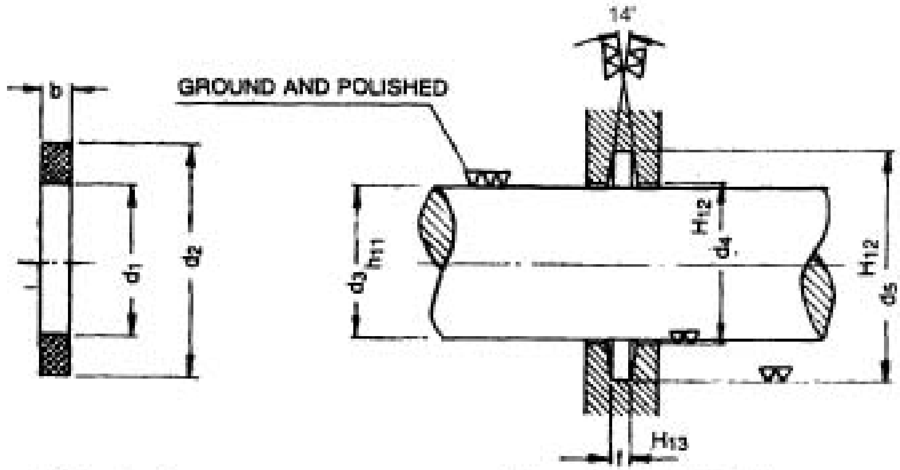
In any type of housing, the recesses or grooves in the housing halves for mounting the bearings must be bored to a high degree of precision. Both the halves are assembled, secured by bolts and pins and then bored in assembled condition. To prevent misalignment, it must be strictly ensured that the opposite recesses are perfectly coaxial. For proper seating of anti-friction bearings, taking into account the effect of stress concentration, the bearing manufacturers have standardised the values of the fillet radii of the stepped shafts and of the housings on which these bearings are mounted.

Bearing Covers

The bearing covers are bolted on to the two halves of the gear box casings. They serve the purpose of closing the holes of the casings made for mounting the bearings, helping to retain the bearings in their proper places and also for mounting the sealing devices. Normally, the sealing devices are in the form of felt sealing rings or radial oil seals. They are fitted inside the grooves or recesses inside the bearing covers which are made for the purpose. To facilitate preparation of manufacturing drawings, details of grooves and recesses are given in Table 8.4 for felt sealing rings and in Table 8.5 for radial or rotary shaft oil seals. A general arrangement of bearing mounting components is shown in Fig. 8.6.

Table 84 Groove for felt sealing rings

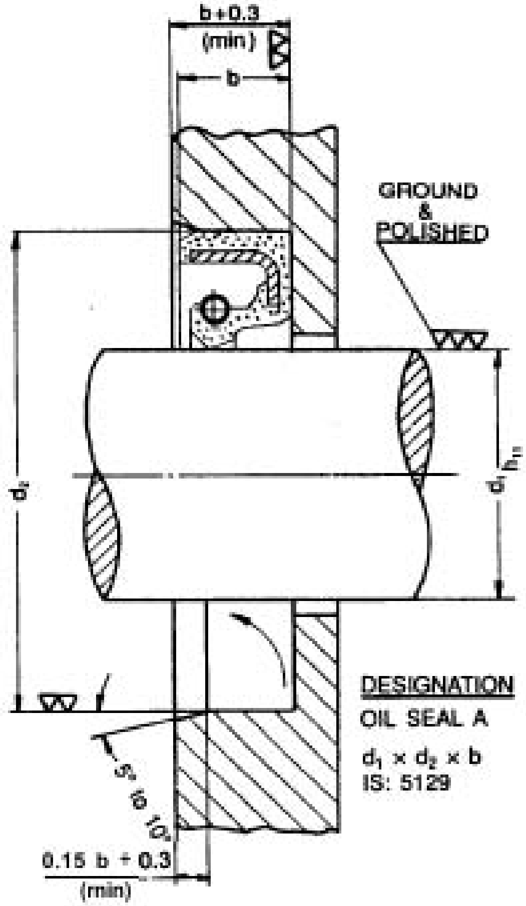
Dimensions in mm



Felt Sealing Ring Groove for **Felt Sealing** Ring

Sealing ring No.	$d_1 - d_3$	b	d_2	d_4	d_5	f
Fi 5	20	4	30	21	31	3
Fi 6	25	5	37	26	38	4
Fi 7	30	5	42	31	43	4
Fi 8	35	5	47	36	48	4
Fi 9	[illegible]	5	52	41	53	4
Fi 10	45	5	57	46	58	4
Fi 11	50	6.5	66	51	67	5
Fi 12	55	6.5	71	56	72	5
Fi 13	60	6.5	76	61.5	77	5
Fi 15	65	6.5	81	66.5	82	5
Fi 16	70	7.5	88	71.5	89	6
Fi 17	75	7.5	93	76.5	94	6
Fi 18	80	7.5	98	81.5	99	6
Fi 19	85	7.5	103	86.5	104	6
Fi 20	90	8.5	110	92	111	7
Fi 21	95	8.5	115	97	116	7
Fi 22	100	10	124	102	125	8
Fi [illegible]	110	10	134	112	135	8
Fi [illegible]	115	10	139	117	140	8
Fi 28	125	11	153	127	154	9
Fi 30	135	11	163	137	164	9
Fi 32	140	12	172	142	173	10
Fi 34	150	12	182	152	183	10
Fi 36	160	12	192	162	193	10
Fi 38	170	12	202	172	203	10
Fi 40	180	12	212	182	213	10

Table 8.5 Groove tor radial oil seals (based on IS: 5129-1969)



b Width of seal	t_1 (0.15*b* + 0.3) *min*	*t* (*b* + 0.3) *min*
7	1.35	7.3
8	1.50	8.3
9	1.65	9.3
10	1.80	10.3
12	2.10	12.3
15	2.55	15,3
20	3.30	20.3

While mounting, the lip of the oil seal should always be turned towards the side to be sealed. For representation of a seal in a drawing, an arrow directed towards the side to be sealed shall be used as shown.

Press fit allowances and tolerances on housing bore

(Dimensions in mm)

Type A Seals

Nominal bore diameter of housing d_2	Housing bore		Outside diameter of seal		Possible press fit variation	
	High limit	Low limit	High limit	Low limit	Maximum interference	Minimum interference
Up to 25	+0.03	−0.03	+0.20	+0.10	0.23	0.07
25-55	+0.03	−0.03	+0.25	+0.15	0.28	0.12
55-125	+0.03	−0.03	+0.30	+0.20	0.33	0.17
125-200	+0.04	−0.04	+0.38	+0.22	0.42	0.18
200 and above	+0.05	−0.05	+0.48	+0.32	0.53	0.27

Type B and C seals

Nominal bore diameter of housing d_2	Housing bore		Outside diameter of seal		Possible press fit variation	
	High limit	Low limit	High limit	Low limit	Maximum interference	Minimum interference
Up to 50	Nominal	−0.03	+0.12	+0.04	0.15	0.04
50-90	Nominal	−0.03	+0.14	+0.06	0.17	0.06
90-115	+0.03	−0.03	+0.18	+0.08	0.21	0.05
115-170	+0.03	−0.03	+0.20	+0.10	0.23	0.07
170-215	+0.04	−0.04	+0.23	+0.13	0.27	0.09
215-230	+0.04	−0.04	+0.25	+0.15	0.29	0.11
230 and above	+0.04	−0.04	+0.30	+0.20	0.34	0.16

Type A—Rubber-cased seal

Type B—Metal-cased seal.

Type C—Built up seals for shaft diameter from 22 mm onwards

8.3 Gear Cutting Processes

For producing gears, a number of methods are followed, namely, sand casting, die casting centrifugal casting, powder metallurgical processes, punching, broaching, extrusion and similar methods. But by and large steel gears carrying large loads compared to their size are cut with form cutters or are produced by one of the gear generation processes.

Such processes are broadly classified into three categories mentioned below.

Processes using form cutters This method is normally used in a milling machine and a cutting tool having the shape of the space between the teeth is utilised. Circular milling cutters having profiles matching the shape of the tooth space are widely used for short production schedules and for gears where absolute accuracy is not of prime importance. A gear milling cutter has been shown in Fig.8.7. For each pitch and gear diameter ideally a different cutter is required because gear teeth having specific pitch, base circle diameter as well as pressure angle have a unique tooth shape, and as such necessitate a particular cutter. In practice, however, standard cutters are available and a set of 8 cutters is normally sufficient to cover the range from a 12-teeth gear

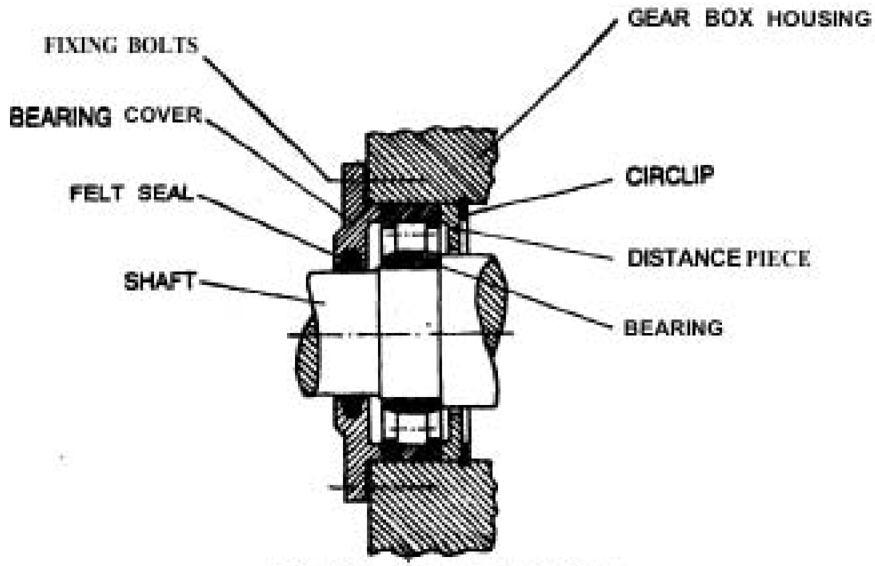


Fig. 8.6 Bearing mounting

to a rack with reasonably accurate profile. A separate set of cutters is, of course, required for each module or pitch. Details about milling cutters and other types of cutters and generating tools are discussed in Sec. 8.5.

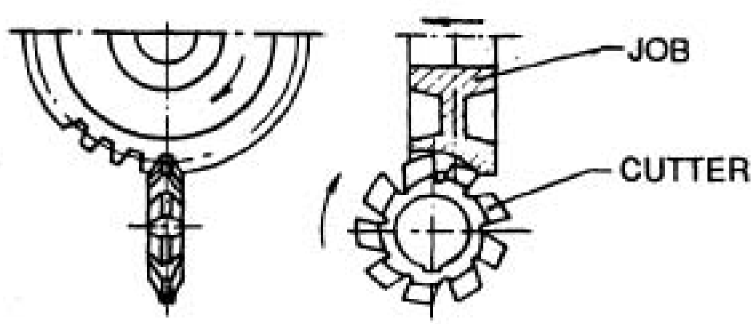


Fig. 8.7 Milling cutter

A form cutter may also be used in reciprocating type of machines, such as, a planer or a shaper. In such cases, the cutter is not circular but resembles a planing or a shaping tool. But the process being slow, it is of no practical use.

Processes using templates or master formers Here the tool is guided by a tracer arrangement. The tracer point moves over a template having the desired shape of the gear tooth profile and the tool duplicates this motion on the gear blank, and consequently the curvature of the gear tooth is cut and produced on the blank. This method is mainly used for cutting very large gears.

Processes using generation principles These are by far the most frequently used methods employed for accurate and high degree of production. The generation processes are broadly

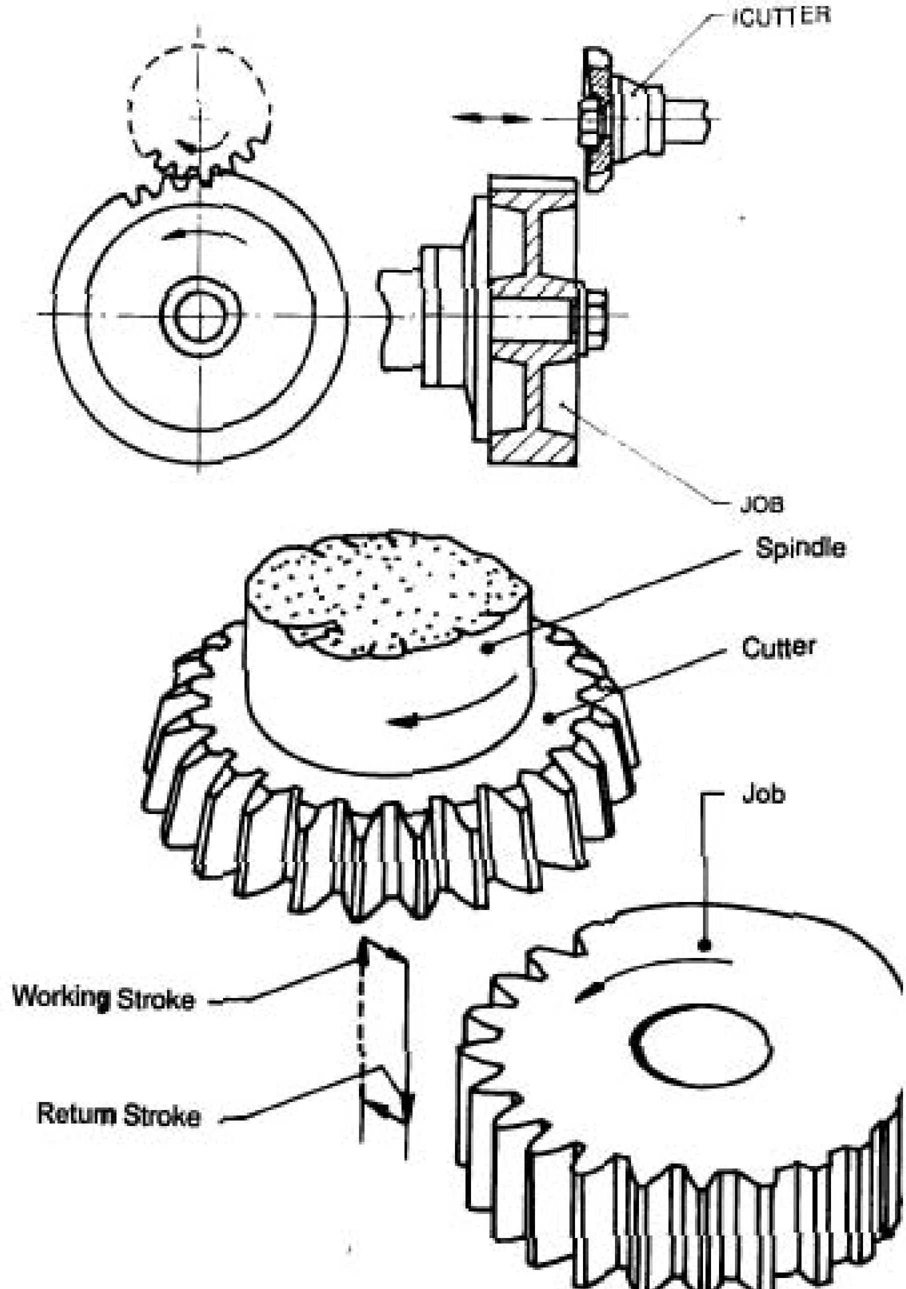


Fig. 8.8 Gear shaping by pinion-type cutter

classified into gear shaping and gear hobbing. In all types of generation processes, a cutter makes its mating component; its action being, analogous to that of a wood screw which makes its own counterpart or female thread as it spirals inside the wood. Similarly, if a hard gear is revolved in contact with a soft blank, the resulting rolling action would be to generate matching teeth on the soft blank. If the hard gear is in the form of a cutter, having cutting angles, edges, reliefs and other features, its teeth will roll into contact with the blank and cut teeth having the curvature required for proper meshing of the mating pair.

In gear shaping type of generation, the cutter may be in the form of a pinion as in the case of a Fellows gear shaper, or in the form a rack having the same pitch as the gear to be generated. These methods are shown in Figs 8.8. a id 8.9.

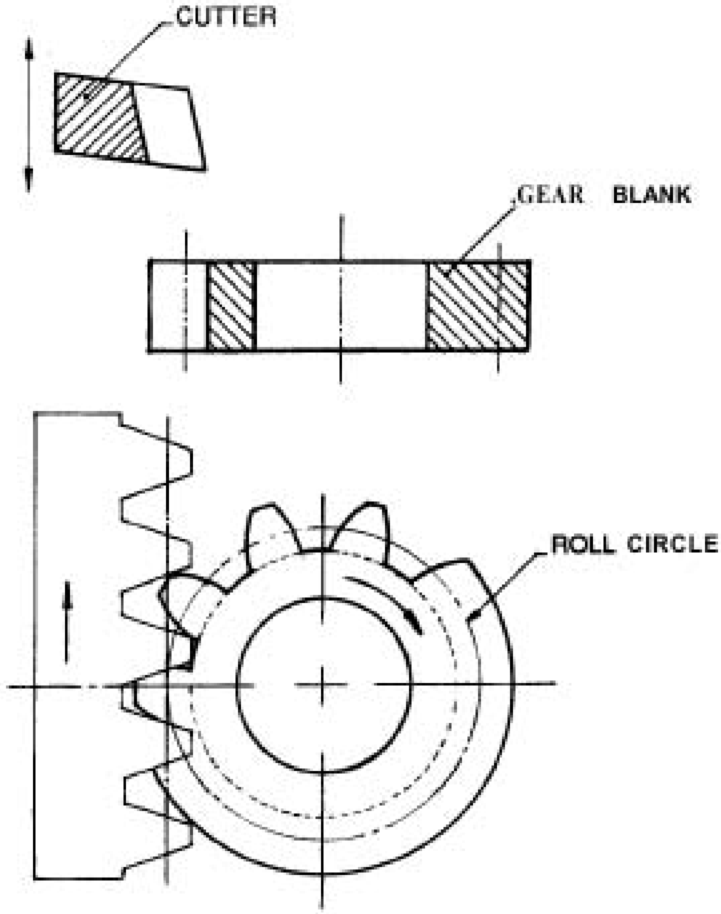


Fig. 8.9 Rack-type cutter

The pinion cutter simultaneously reciprocates and rolls on the face of the gear blank which is also imparted a rolling motion. Thus, both the cutter and the blank slowly revolve together, i.e. the same movement as in the case of two gears in mesh in conjugate motion. In a pinion type cutter, however, the possibilities of application are limited as compared to the rack type cutter.

In a rack type cutter, the blank is given a rolling movement relative to the cutter, and the tooth is produced on the blank by the reciprocating motion of the cutter. In all the cases of gear generation, the "cutting" and "meshing" motions act simultaneously in a synchronised manner. In general, parts with narrow face widths are particularly suitable for generation of gear teeth with gear shaping methods.

In Fig. 8.10 *MM* represents the pitch line of the tool which is a rack type cutter, the basic rack being used as the generating tool. We have seen in Chap. 1 that for involute-shaped teeth, the basic rack has straight-sided faces and flanks. This fact enables the tool manufacturer to make rack teeth to almost any degree of accuracy. For generation, the essential requirement is that ***MM*** rolls on the pitch circle of blank without slip. In Fig. 8.10 the pitch circle has been kept stationary and *MM* has been shown to roll on it on either side of the pitch point *P*. In this way it is easier to represent the generation principle on paper. Thus, the successive positions of the rack are indicated by *a,b,c, ..., r* as it rolls on the pitch circle. Positions *a* to *k* represent rolling

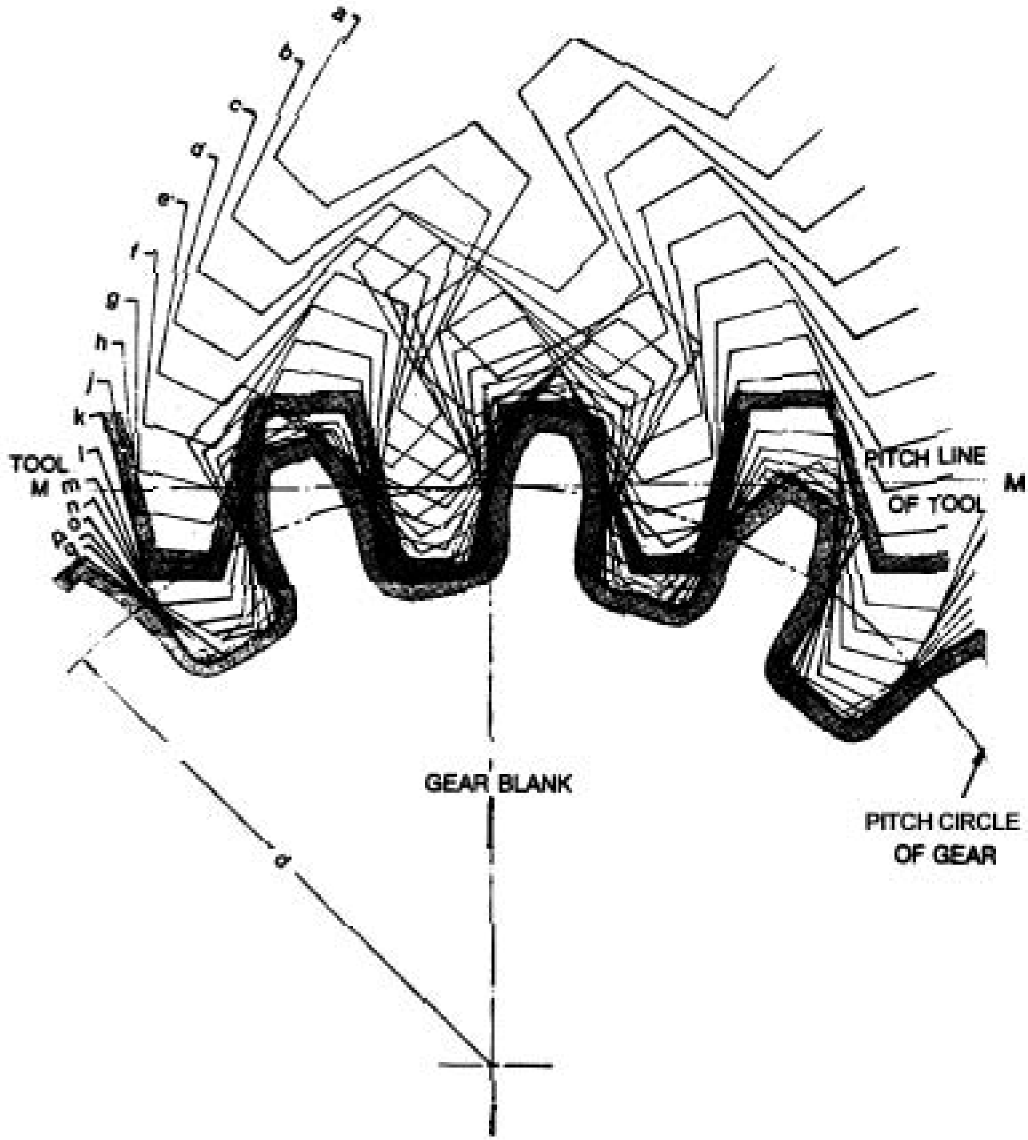


Fig. 8.10 Generation of Gear teeth

of the pitch line on the right hand side of P, while those from k to r show rolling on the left side of P. (See also Sec. 8.5 and Appendix A).

The hobbing process employs a rotating cutter, called hob, which is in effect a worm having gashed teeth to form cutting edges with appropriate relief (Fig. 8.11). The blank revolves in a horizontal plane while the rotating hob moves downward across the face of the gear to generate the teeth. Indexing system maintains the relative turning ratio between the pair, and change gear combinations permit the desired number of teeth to be generated.

In any of the gear generation processes, since conjugate forms are produced on the blank by the cutter, conjugate action is ensured which is one of the fundamental criteria of gear drive, as we have seen in earlier chapters.

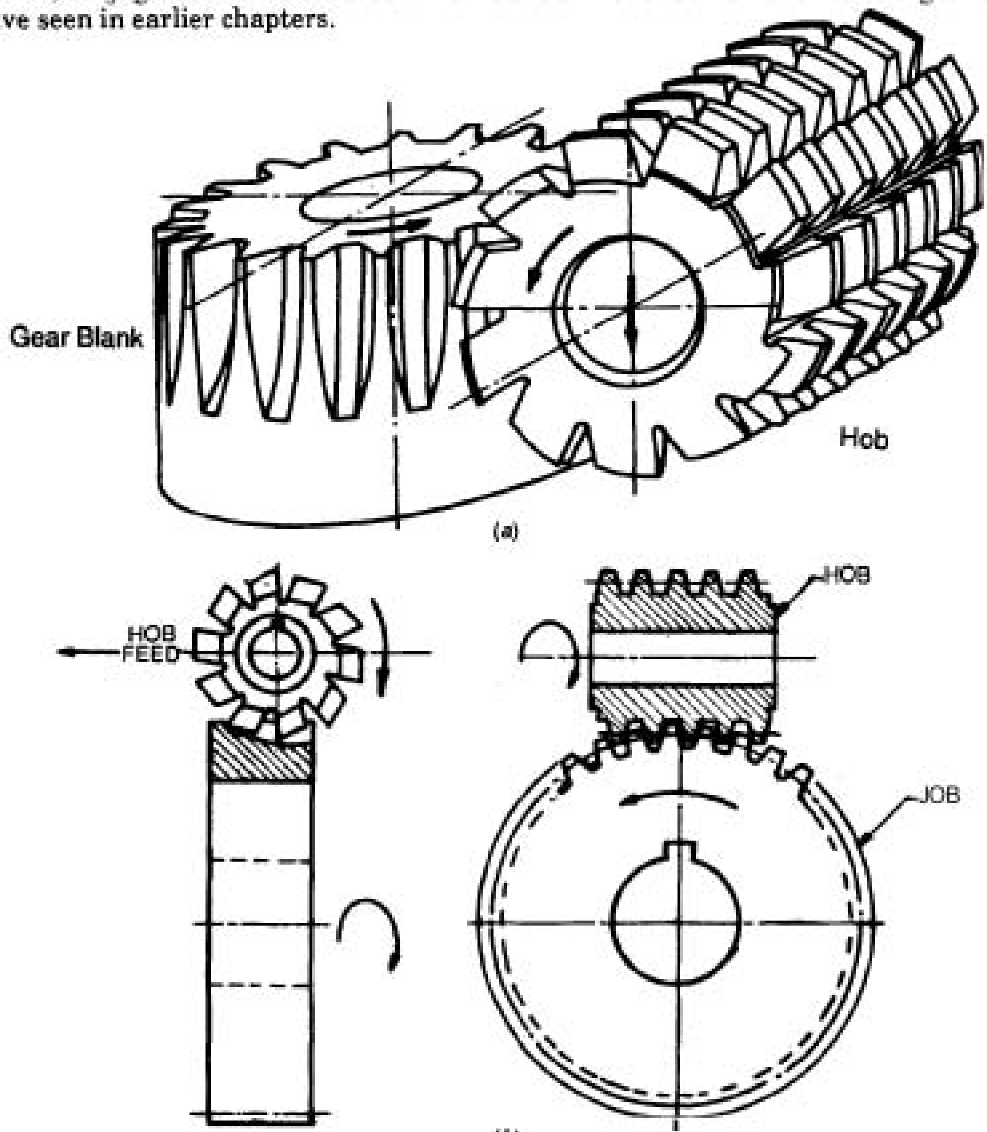


Fig. 8.11 Generating a spur gear with a hob

The straight type of bevel gears are produced by a generating machine which reciprocates a cutter in a motion more or less like a shaper, but the cutter does not resemble a pinion or 'a rack'. The special geometry of the bevel gear necessitates the use of a special tool to cut each side of the bevel gear tooth.

A number of generation methods have been developed for spiral bevel gears. The principal ones among them are: the Gleason system, Klingelnberg Palloid system in which a conical hob is used, and Oerlikon gears which are generated by annular cutters.

8.4 Gear Finishing Processes

After cutting teeth on the gears by any of the processes described in Sec. 8.3, finishing operations are required to improve surface finish, to correct errors left by the gear cutting machine, to achieve a desired accuracy in tooth profile, to attain a certain prespecified precision grade, and to rectify the distortion of the teeth caused by heat treatment. The undesirable effects caused by tooth inaccuracy become particularly pronounced in case of gears running at high speeds and transmitting large forces, because these gears then become vulnerable to additional dynamic forces caused by the errors in tooth profiles. The finishing processes are necessary to reduce these errors as much as possible. Several methods or combinations thereof are employed to effect finishing processes. The most common of these processes — shaving, grinding, lapping and burnishing — are briefly discussed below.

Shaving

In this finishing process, a cutting tool in the form of a hardened pinion or rack is used. The teeth of the cutter are serrated with many small and fine notches or cutting edges. This cutter meshes with and rotates the gear to be finished. The rotation is synchronised with a longitudinal motion, so that the entire face of each gear tooth comes in contact with the cutter. Both the cutter and the gear are run together in mesh at high speed. As a prerequisite, the gear should be of machineable hardness, usually up to HRC 38, if shaving is to be effectively used as a finishing process, though gears may be shaved having hardness up to HRC 47 in certain case. Generally, shaving is a much faster process than grinding.

Grinding

The process is usually used after the gears have been hardened. Finishing by grinding involves similar methods as adopted in the case of the forming process or the generation principle, only the cutter is in the form of an abrasive wheel. When the gear teeth are hardened to a high degree, then grinding is the only process to finish gears.

When the hardness of parts exceeds HRC 38, it becomes very difficult to cut them by conventional means. When they are in the range of HRC 60 in hardness, it is practically impossible to cut them. The hardness of a fully hardened martensitic steel is generally around HRC 58 to HRC 63. Hence grinding is the only solution in such cases. Broadly speaking, of all the different finishing processes, grinding produces the most accurate tooth profiles.

Lapping

In this process the unfinisliedgear and a lapping gear are rotated in mesh in the presence of an abrasive compound. Lapping is an inexpensive process for correcting slight errors which may be caused during hardening, Sometimes a gear is lapped after it has been shaved as this finishing process produces high degree of precision. Sometimes a pair of production gears may also be lapped with each other.

Burnishing

Burnishing is a finishing process in which the machine operates by rolling the gear to be finished in contact with a master burnishing gear or gears. Pressure is applied to the burnishing gears. Hardened mating gears can be burnished by running them in mesh with one another. This process is suitable for gears which have been cut but not heat treated.

8.5 Profiles of Gear Cutters and their Actions

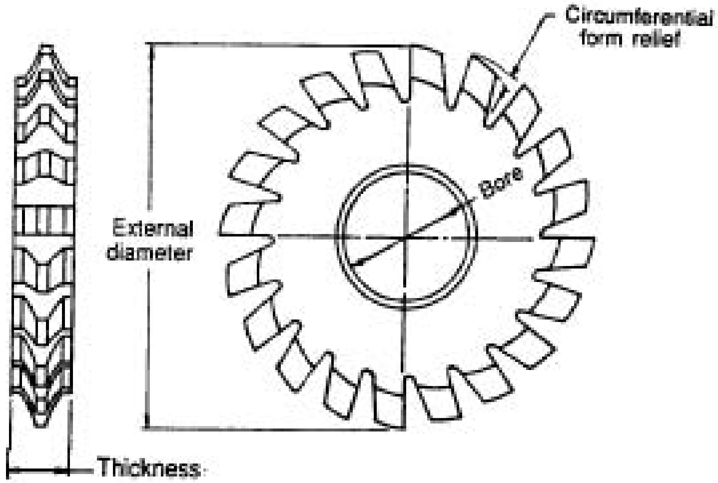
It has been stated before in Sec. **8.3** that all gear cutting methods can be categorised into two general classes: the forming method and the generation method. The cutting tool belonging to the first category is made to have a shape which corresponds to the desired tooth space, as in the case of milling cutters. The profile of a typical circular type milling cutter is shown in Fig. 8.12. The tool in the generation method is given a shape which is conjugate with the form of the tooth to be cut. The cutter and the gear blank roll in contact in timed relation to each other and simultaneously, they are also given a feed relative to each other to remove metal from the blank.

Hobbing is a generation process which is used for the manufacture of spur, helical or worm gears as well as herringbone-gears, crossed-helical gears, single and multi-start worms, sprocket-wheels, splines, serrations, besides some special applications, such as the hour-glass and cone type worm and worm-wheels.

The cutter, called the hob, is like a worm. For making **a** hob, a cylindrical blank is first turned and then a helical thread analogous to a worm thread is milled. At this stage, the hob is actually a worm. To convert it into a cutter, flutes are milled across the thread, so that the thread is interrupted at a number of places. These flutes may be either parallel to the hob axis or at right angles to the course of thread. Afterwards, relief is provided at the sides and the tops of the individual hob tooth thus produced, the hob is hardened, ground and sharpened. The profile of a typical hob cutter is illustrated in Fig. 8.12.

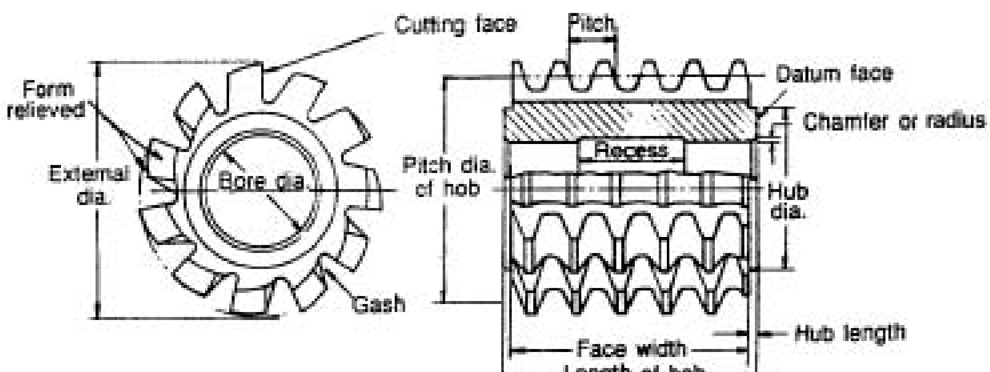
The hob cutter and the gear blank being cut can be compared to a worm and worm-wheel in mesh. The cutting action continues till the blank is finished. Precision gears can be generated with a high degree of accuracy! and at a high rate of production by hobbing method. Another advantage of this process lies in the fact that the profile of the hob thread is nearly the same as the shape of a straight rack-tooth, the difference being practically negligible. This aspect makes it possible to produce hobs readily and with a very high degree of accuracy, unlike curved cutter-teeth used in some other generation processes.

In the involute system, since a straight-sided rack can mesh with a gear having any number of teeth, a single hob can be employed to generate gears of any number of teeth, and all of theni will mate properly with each other and also with a rack. While setting a hob on the machine, the



GEAR MILLING CUTTER

(a)



GEAR HOB

(B)

Fig. 8.12 Profiles of gear cutter

axis of the hob for cutting gears is tilted according to the hob-thread angle ω in case of spur gears, and in addition, also to the helix angle p in case of helical gears, taking into account the hand of the helix, i.e. $\omega \pm \beta$, the sign depending upon the hands of the hob flute and the gear-tooth helix.

The same hob may be used for cutting spur gears as well as helical gears of either hand or of any helix angle.

Four classes of hobs in general use are mentioned below.

Class A: Precision ground, used for finishing gears of highest accuracy.
Class B: Commercial ground, satisfactory for finishing general purpose types of jobs.
Class C: Accurate but unground, suitable for finishing gears of finer pitches.
Class D: Unground, used commercially for roughing cuts.

In any class of hobs, the cutters must be properly and accurately sharpened to produce good quality results.

The speeds and the amount of feed for cutting spur and helical gears are functions of many factors. Some general recommendations are given below.

For cast iron: 12 to 30 m/min. (usual value 25 m/min)

For steel: 18 to 45 m/min. (usual value 30 m/min)

On specially designed hobbing machines, the speed for hobbing steel with ordinary HSS hobs can go up to 150 m/min. Non-metallic materials can be cut at still higher speeds using carbide tipped hobs.

For a single-cut job, the feed per revolution of blank is 0.8 to 1.3 mm. For roughing, it can vary from 1.5 to 5 mm. In selecting the amount of feed, care should be taken in considering the safe metal-removing capability of the particular cutter. After rough cut, finishing cuts are given, the feed being 1 to 2.5 mm depending on the finish required.

For hobbing helical gears, the speeds are the same as in spur gears. However, to maintain the same quality of finish, the feed per revolution should be reduced as the helix angle increases. Other factors remaining same, the required feed for a helical gear is found by multiplying the corresponding appropriate spur gear feed with the cosine of the helix angle.

While generating gears with hobbing method, proper coolant should be used. The recommended coolants are: sulphurised mineral oil combined with lard oil for hobbing steel, and soluble oil in water for brass and bronze. Cast iron generally does not require any coolant expect malleable CI which needs coolant.

Gear Cutter Profiles

For generating gear teeth which will conform to the parameters of the basic rack discussed in Sec. 2.1, the reference profile of a gear cutting tool should be properly laid down. This has been done in Table 8.6. For standard tooth profiles, the cutter profiles have been divided into the following four categories.

1. Reference profile of cutter with cutter addendum given by

$$h_{a0} = 1.167\,m$$

The above cutter conforms to the DIN basic rack as per DIN 867. Here, the dedendum of tooth of the gear is given by $h_f = 1.167m$. This profile does not conform to the basic rack as per IS: 2535 described in Sec. 2.1 and is included here for reference only.

II. Reference profile of cutter with cutter addendum given by

$$h_{ca} = 1.25\,m$$

This cutter produces a dedendum of teeth $h_f = 1.25\,m$, and conforms to the IS basic rack. Reference profiles III and IV also conform to the above basic rack.

Both the reference profiles I and II are meant for processes in which the gears are finish-cut, namely, hobbing, gear shaping with rack-type or pinion-type cutters. In each case, however, a very small amount of machining allowance may be provided for shaving or other finishing processes.

III. Reference profile of cutter with cutter addendum given by

$$h_{ca} = 1.25\,m + 0.25\,\sqrt[3]{m}$$

A larger amount of machining allowance is provided in this case so that the gear teeth are first rough-machined followed by finishing operations, such as grinding and shaving.

IV. Reference profile of cutter with cutter addendum given by

$$h_{ca} = 1.25\,m + 0.6\,\sqrt[3]{m}$$

This profile provides for machining allowances which are still larger than those provided by profile III.

In Table 8.6, the symbols used are defined as follows

h_c = Height of the cutter tooth

h' = Whole depth of the gear tooth. This is the ultimate height of the gear tooth, and is equal to the depth-setting of the gear-cutting tool. The value includes the effect of machining allowance and the tooth thickness tolerance, if any. Otherwise $h' = h$ = Whole depth of a standard tooth

$h_{ca}I$ = Cutter addendum of reference profile I

Similarly, $h_{ca}II$, etc. refer to the corresponding profiles

h_f = Dedendum of gear tooth

p_c = Pitch of cutter = πm

S_c = Tooth thickness of cutter = $p_c/2$

r_1, r_2 = Radii of cutter

p = Machining allowance per flank of gear tooth, measured perpendicular to the flank

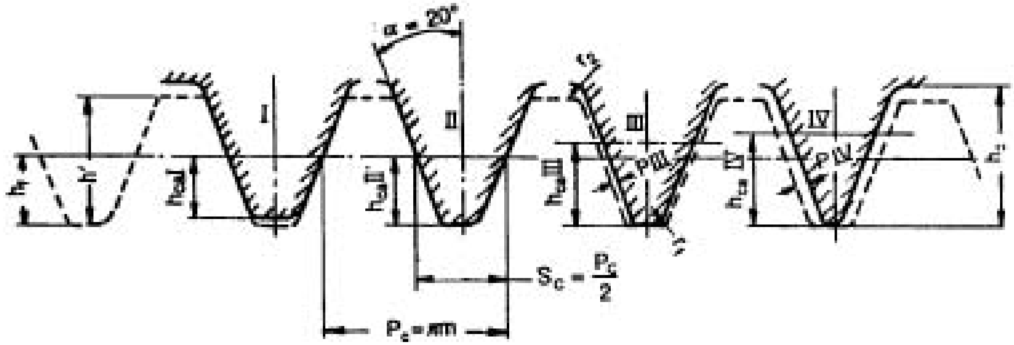
A_{SU} = Upper tooth thickness tolerance of gear in mm (see Sec. 2.8)

The following relations hold good for the different reference profiles.

I. $h_f = h_{ca}\,I = 1.167\,m$ $\quad h_c \geq 2.367\,m$ $\quad h' = 2.167\,m - p/\sin\alpha + A_{SU}/2\tan\alpha$

II. $h_f = h_{ca}\,II = 1.25\,m$ $\quad h_c \geq 2.45\,m$ $\quad h' = 2.25\,m - p/\sin\alpha + A_{SU}/2\tan\alpha$

Table 8.6 Reference profiles of gear cutting tools



m	p_c	s_c	h_{ca}I	h_{ca}II	h_{ca}III	ρIII	h_{ca}IV	ρIV	$r_2 = 0.2m$	r_1
1	3.1416	1.57	1.167	1.25	1.50	0.09	1.85	0.21	0.08	The radius r_1 on the tip of the tool is made equal to r_2 except for pinion type cutters. In the case of the latter only the sharp edge is slightly chamfered. If radius differing from the values listed is required, this shall be specified at the time of the ordering
1.25	3.9270	1.96	1.46	1.56	1.83	0.09	2.21	0.22	0.12	
1.5	4.7124	2.36	1.75	1.88	2.16	0.10	2.56	0.24	0.20	
1.75	5.4978	2.75	2.04	2.19	2.49	0.10	2.91	0.25	0.25	
2	6.2832	3.14	2.33	2.50	2.82	0.11	3.26	0.26	0.30	
2.25	7.0686	3.53	2.63	2.81	3.14	0.11	3.60	0.27	0.40	
2.5	7.8540	3.93	2.92	3.13	3.46	0.12	3.94	0.28	0.50	
2.75	8.6394	4.32	3.21	3.44	3.79	0.12	4.28	0.29	0.50	
3	9.4248	4.71	3.50	3.75	4.11	0.12	4.62	0.30	0.60	
3.25	10.2102	5.11	3.79	4.06	4.43	0.13	4.95	0.30	0.60	
3.5	10.9956	5.50	4.08	4.38	4.75	0.13	5.28	0.31	0.70	
3.75	11.7810	5.89	4.38	4.69	5.08	0.13	5.63	0.32	0.75	
4	12.5664	6.28	4.67	5.00	5.40	0.14	5.95	0.33	0.80	
4.5	14.1372	7.07	5.25	5.63	6.04	0.14	6.60	0.34	0.90	
5	15.7080	7.85	5.84	6.25	6.68	0.15	7.28	0.35	1.00	

(Contd)

Table 8.6 *(Contd)*

m	p_e	s_e	h_{ca} I	h_{ca} II	h_{ca} III	ρ III	h_{ca} IV	ρ IV	r_a=0.2m	r_f
5.5	17.2788	8.64	6.42	6.88	7.32	0.15	7.92	0.36	1.10	
6	18.8496	9.42	7.00	7.50	7.95	0.16	8.59	0.37	1.20	
6.5	20.4204	10.21	7.59	8.13	8.59	0.16	9.24	0.38	1.30	
7	21.9911	11.00	8.17	8.75	9.23	0.16	9.90	0.39	1.40	
8	25.1327	12.57	9.34	10.00	10.5	0.17	11.20	0.41	1.60	
9	28.2743	14.14	10.50	11.25	From and incl	—	12.50	0.43	1.80	
10.	31.4159	15.71	11.67	12.50	module 9 refe-	—	13.79	0.44	2.00	
11	34.5575	17.28	12.84	13.75	rence profiles I	—	15.08	0.46	2.20	
12	37.6991	18.85	14.00	15.00	and II or IV are	—	16.37	0.47	2.40	
13	40.8407	20.42	15.2	16.25	used, depending	—	17.66	0.48	2.60	
14	43.9823	21.99	16.3	17.50	on the size of	—	18.95	0.49	2.80	
15	47.1239	23.56	17.5	18.75	thegrindingor	—	20.23	0.51	3.00	
16	50.2655	25.13	18.7	20.00	shaving alow-	—	21.51	0.52	3.20	
					ance provided					

III. $h_f = 1.25\,m$ $h_c \geq 2.45\,m$ $h' = 2.25\,m + 0.25\sqrt[3]{m} - p/\sin a + A_{SU}/2 \tan a$

$p = (h_{ca}\,\text{III} - h_f) \sin a = 0.25\sqrt[3]{m}\sin a$

IV. $h_f = 1.25\,m$ $h_c \geq 2.45\,m$ $h' = 2.25\,m + 0.6\sqrt[3]{m} - p/\sin a + A_{SU}/2 \tan a$

$p = (h_{ca}\,\text{IV} - h_f) \sin a = 0.6\sqrt[3]{m}\sin a$

Gear teeth can be cut by form milling cutters, using a set of eight cutters, valid for each pitch or module. The cutter series are tabulated below

Cuner No.	Number of teeth of gear to be cur	Cuner No.	Number of teeth of gear to be cut
1	135-a rack	5	21-25
2	55-134	6	17-20
3	35-54	7	14-16
4	26-34	8	12-13

In gear milling, the tooth surface shapes produced are approximate within reasonable limits. Those gears, however, meet the ordinary operational requirements. In contrast to the milling method, generation processes produce accurate tooth surfaces. Moreover, as stated earlier, any generating type of cutter of a given module can be used for any number of teeth of the gears to be manufactured.

Besides the disc-type or circular-type milling cutters discussed so far, end-mill type of cutters are also used. These are generally used for cutting spur or helical gears of large modules (10 mm and above) where it is not possible to cut the teeth by hobbing method.

In case of worm and worm-wheels, the worm threads can be made on a lathe with proper tool and fixture. Worms are also produced by milling. Production of worms of different profiles has been discussed in Sec. 4.2.

For hobbing a spur or helical gear, the diameter and the helix angle of the hob are not functions of the gear. With a worm-wheel hob, however, the matter is different. Such a hob must be a copy of the worm. As a result, parameters, such as the hob diameter, angle of the thread, lead of thread, must be the same as these elements of the worm. Generally, two types of hobbing are done to produce worm-wheel teeth — the in-feed bobbing where the hob is fed radially into the gear blank by gradually reducing the distance between the tool and the blank, and the tangential-feed hobbing where, keeping the centre-to-centre distance between the axes of hob and blank constant, the tool is fed along the axis. In other words, the tool-feeding directions of the two methods are at right-angles to each other. The second method produces more accurate results. Besides above, single-tooth fly-cutting hobs are also used to produce worm-wheel teeth. These have low production, capacity.

For ordinary purposes, the hobs which are most widely used for worm-sheels are those based on a worm having straight-sided linear section which corresponds to the Archimedean worm. This gives high accuracy.

For generation of straight-toothed bevel gears, two reciprocating tools or cutters are used in a special type of machine. For curved-toothed bevel gears, face-mill type cutters are used. Spiral bevel gears can also be generated in special machines by means of conical hobs.

Finally, in choosing the most appropriate method from the many processes available for manufacture of gear teeth, the gear designer should bear in mind the following considerations: The capacity of the machine to be commensurate with the size and shape of the gear, proper material selection, the magnitude of production range, the length of time involved, the technical competence of the machinist, and most important — the economic viability of production.

8.6 Gear Lubrication and Cooling

Lubrication of gears is required to ensure smooth operation of the drive. Hence, a thorough knowledge of the different aspects of lubricating methods and nature of lubricants is imperative for a gear designer. In recent years, the technology of lubrication and lubricants has become extremely complicated and sophisticated. This section deals with the broad features of the subject.

Lubrication and Lubricants

In any gear drive, the fundamental types of motion are rolling and sliding. These occur simultaneously, but their magnitudes are functions of the type of gears and speed of operation. For example, in case of hypoid or worm-drives, it is the sliding component which is of greatest relevance.

To effectively meet the detrimental effects caused by these motions in an inter-meshing gear system, lubrication is essential if the tooth surfaces are to have a reasonable length of service life. A quick failure by seizure of the tooth surface may be the result of the absence of a lubricating medium. Sliding friction leads to the generation of heat which may be sufficient enough to raise the local temperature to the melting point of the metal. Welding at spots may occur with the resulting dislocation of pieces of tooth surface. Inadequate lubrication may lead to gradual wearing away of the tooth surface. With proper selection of gear material and lubricant, metallic contact can be prevented when the lubricant has sufficient film strength. The lubricating medium should have enough viscosity to develop a suitable oil film between the tooth surfaces and to sustain this film under load. The main considerations, therefore, are that the lubricant must withstand the load, and the gear material must be strong enough to resist the contact pressures transmitted through the lubricant without succumbing to fatigue failure.

Basically, the gear lubricant is intended to serve the following main purposes.

1. To reduce the wearing off of mating surfaces in general.
2. To reduce friction and power loss.
3. To act as a coolant by dissipating heat.
4. To prevent pitting, welding and breakage.
5. To carry additives to the tooth surfaces.
6. To carry away undesirable contaminants in the effluents.
7. To minimise noise, vibration and shock.
8. To prevent corrosion.

Besides these some other minor functions are also carried out by the lubricant.

Proper selection of gear lubricants involves considerable knowledge and experience. The designer must know the basic parameters like type of gearing, operating conditions and the desirable characteristics of the requisite lubricant. Operating conditions involve temperature to which the gear set is subjected, tooth pressure due to load, service speed, exposure to contamination by non-lubricating, outside agencies in the form of solids, liquids or gases. Oil companies furnish extensive charts which incorporate these factors along with such essential data as viscosity, load-carrying capacity and other information to facilitate the selection of proper lubricants.

Besides causing heavy functional deterioration, faulty lubrication leads to many types of the surface defects discussed in Sec. 8.7. It is, therefore, imperative that correct lubricant must be selected and the protecting film of lubricant maintained uniformly and continuously on the meshing teeth of the gears. This film may be very thin in case of boundary lubrication. In that case it should be ensured that the lubricant has ample adhesive properties and resistance to rupture of the film. Some wear will still take place under boundary lubrication conditions, but the extent of serious damage can be avoided by regularly renewing the film. Even with splash or circulating type of lubrication, boundary lubricating conditions may sometimes appear with possible rupture of lubricating film resulting in the usual harmful consequences such as welding of metals, surface tear and galling. The development of sophisticated extreme pressure (EP) oils has reduced these damaging effects to a large extent as discussed later in this section.

The development of gear lubricant technology for the last 3 or 4 decades has been mainly due to an overall advancement in the engineering field. It is also due to the wide use of automobiles and automotive vehicles that the improvement of gear oils has been achieved to a large extent. When hypoid gear differential drive came to be used, the service requirements demanded high performance from the gear oils which resulted in the development of the EP oils.

Apart from a few open type gear systems where grease is used, gears are normally lubricated by oils. Use of straight mineral oils is sufficient in most of the cases of gear-drives. In certain special cases where maximum oxidation stability is essential, turbine-quality oils of viscosity grades 50 to 90 centistokes at around 40°C are required. In general, wear reduces with increasing viscosity of oil.

As regards EP oils, although they were developed primarily for the lubrication of hypoid gears, other types of gears are also serviced by this category of oils because of the many advantages they offer. Another type of oils which operate at high pressures is the multi-purpose (MP) gear lubricant. Many industries prefer these oils to straight mineral or EP oils, and quite a few consumers use this type only in all gear sets in their plant units. This leads to rationalisation of stock and simplification of inventory control due to reduction in the various grades of oils which have to be kept in stock.

In short, for normal gear-drives, straight mineral oils without additives should meet the requirements in most of the cases. For higher duty, gear oils containing mild additives can be used. When this is not considered sufficient, the EP oils containing high percentage of additives are employed.

As pointed out earlier, selection of a proper lubricant will depend upon the gear parameters and service conditions, Table 8.7 is intended to give an idea of such selection criteria which are commensurate with the peculiarities of the drive. Broadly speaking, smaller the velocity and greater the tooth pressure and surface roughness, greater must be the viscosity of the oil. Besides, a higher viscosity is conducive to a higher hydrodynamic strength.

Table 8.7 Relation between viscosity, velocity and tooth pressure(for selection of lubricant)

Circumferential velocity v (m/s)	*Type of duty*		
	Light $p < 400$ N/cm²	*Medium* $p = 400$-1000 N/cm²	*Heavy* $p > 1000$ N/cm²
	Kinematic viscosity of oil in centistoke (cSt) at 50 °C		
Below 0.5	150	250	455
0.5-2	90	150	225
2-6	60	90	150
6-12	45	60	90

Here p (N/cm²) = Tooth pressure $= \dfrac{F_t}{\pi b m}$, and the parameters, m, F_t, and b have the usual meanings.

Table 8.8 gives guidelines in which the viscosity has been shown as a function of the velocity only. It is valid for closed gear-drives with oil temperatures ranging between 45 and 90°C.

Table 8.8 Relation between viscosity of oil and velocity of gear

Based on Mashinenelemente. Niemann. vol 11, 1965 edition, table no 122/1, p. 122 Springer Verlag, Hiidelberg

Velocity v (m/s)	0.25	0.4	1.0	1.6	2.5	4.0	6.3	10	16	25
Viscosity in cSt at 50° C	175-350	145-290	100-200	83-166	69-138	57-114.	47-94	39-78	32-64	27-54

To arrive at the above relation, the following formula may be applied

$$\text{Viscosity (cSt)} = 100/v^{0.4} \text{ to } 200/v^{0.4} \quad (v \text{ in m/s})$$

Thus, for $v = 4$ m/sec, the viscosity $= 100/4^{0.4}$ to $200/4^{0.4} = 57$ to 114 cSt, which tallies with the values given in Table 8.8.

Surface finish of teeth has a direct bearing on the efficacy of maintaining the lubricating film. In general, when loads are moderate or low and speeds are medium or high, the more accurate the gears are, the lower is the viscosity of the oil required. However, with highly polished tooth surfaces and extremely low sliding velocity, it is difficult for a hydrodynamic film to be established or maintained.

Characteristics of Lubricants and their Nature of Action

In a pair of rotating gears with lubricated teeth, a hydrodynamic wedge is formed which tends to keep away the meshing teeth of the pair from one another by forming a thick fluid film in between the teeth. This, however, is true only when the load is low. With increasing load, the tooth pressure in the contact zone increases which in turn decreases the thickness of the separating film. Eventually, when the load becomes very high, the fluid film fails to prevent metal to metal contact at raised spots, resulting in deterioration of tooth surface. Nature of wear depends on the speed of the system. So far as gear lubrication is concerned, speed can be divided into three zones—slow speed which extends up to around 1000 rpm, medium speed from 1000 to 8000 rpm and high speed from 8000 to 30,000 rpm. These three regions are characterised by absence of wear, abrasive wear and scoring respectively. With high loads, EP oils are recommended, as stated earlier.

To serve its functions satisfactorily, a gear lubricant should have the following characteristics.

1. It should stick on the tooth surface, resisting the action of centrifugal force.
2. It must be compatible when mixed with other lubricants or additives.
3. Its component ingredients must not precipitate or settle down.
4. It must not react chemically or otherwise with gear box parts and fittings such as oil seals or gaskets.
5. It must possess "oiliness", that is, it must have such additives so as to prevent film rupture as the type of lubrication changes from thick-film to boundary lubrication during course of action.
6. It should be resistant to oxidation.
7. It must be amenable to demulsification.
8. It must have good detergent qualities, film strength and foam inhibition characteristics.
9. It must have satisfactory viscosity-temperature characteristics and shear stability.
10. It must be reasonably priced, i.e. available at an economic price.

Gear lubricants, whether oils or greases, are blended products. Products from mineral oils are the main constituents, but a number of additives are used to lend the desired properties to the lubricants. Besides oil and grease, solid powdery materials like MoS_2 or synthetic materials like PTFE is also sometimes used as lubricants in mechanisms where fluid lubricants are not admissible. In some cases, the lubricant may be a gas.

Methods of Application and the Respective Fields of Lubricants

Lubricants can be applied in several ways to the gears, namely: by hand; by feeding by drips; either by gravity or by circulation system; by bath or splash; by force-feed system using oil under pressure created by pumps; by spray or jet; and by creating a mist of oil in the system.

Selection of the type of lubrication for a gearing system will depend upon the nature of its duty, circumferential velocity, availability of facilities, cost and other factors.

Grease lubrication is resorted to when hydrodynamic lubrication is not possible, e.g. in case of low circumferential velocities. The coefficient of friction is higher compared to that in case of oil lubrication. The heat transfer through lubricant is practically nil.

Splash lubrication affords simplicity of operation and does not need elaborate and complicated system. Here the portions of the running gears dip into the oil sump and the teeth surfaces are thus lubricated. Sometimes spray discs are provided which scoop oil from the bottom and throw it into the desired places of the gear. The depth to which the gears are supposed to be dipped should not exceed the value of six times the module. On the other hand, it should not drop below the value of one module. In any case, when splash lubrication is used, it is to be ensured that a high oil level is avoided, because in that case the gears have to dip deeper and a high amount of power loss, generation of heat and a high level of noise will ensue.

Normally, a velocity of **13-15** m/s should be the upper limit in case of splash-type lubrication. With higher velocities, pressurised spray-type lubrication is used. In planetary gear drive, oil is also sometimes delivered at the tooth surfaces through holes in the shaft and thence to the holes at the roots of the teeth of the planet pinions.

As for the quantity of oil required, the following guidelines may be followed.

For splash lubrication: 4-8 litres of oil fillings are needed for each kW lost
For spray-type lubrication : 1.5 litres per minute per 100 kW power output.

It has been found that at high velocities, best results are obtained when lubrication is done by thin oils sprayed directly on the tooth flanks. Such objective can be achieved by centralised circulatory pressure-lubricating system. For such cases, the typical values of velocity vs. viscosity are as follows

$$v = 12\text{-}20 \text{ m/s and viscosity} = 45\text{-}53 \text{ cSt}$$

$$v > 20 \text{ m/s and viscosity} = 30\text{-}45 \text{ cSt}$$

Higher the velocity, finer should be the oil-spray. The mist thus formed is sufficient to lubricate the gear teeth in the proper manner.

In spray-type lubrication, the pressurised oil is normally provided by a gear pump. The oil pressure in the pipe-lines is usually around 100-300 kPa. The spraying is done throughout the length of the teeth like showers. Quantity of oil required for such lubrication is about 0.5 litre per minute per centimeter of tooth width.

Types of lubricating system as a function of the circumferential velocity are given in Table 8.9.

Table 8.9 Lubrication system *vis-a-vis* circumferential velocity

Velocity, v (m/s)	*Type of lubrication system*
Up to 0.8	Application of grease is sufficient
0.8-4	Splash lubrication in case of high rotational speed: otherwise, grease lubrication is enough
4-12	Splash lubrication
Over 12	Spray or jet lubrication

In case of worm-drive, grease lubrication should be adequate in the velocity range $v \leq$ 0.8 m/s, where v = the circumferential velocity of the worm. Splash lubrication is recommended up to v = 10 m/s.

For oil lubrication of worm gears, the general recommendations that can be given are: For low speeds an oil of viscosity of about 43 cSt; for average conditions of about 32 cSt and for high sliding velocities of about 17 cSt to be used; the reference temperature being 100°C in each case. Straight mineral oil is good enough for ordinary worm-drives.

For lubrication of bevel gears, the methods recommended are the splash method and the pressure or jet method. The splash type of lubrication is suitable for gear velocities up to 10 m/s. When the speed goes higher, churning of oil takes place resulting in overheating. For velocities greater than 10 m/s, pressurised jet of oil directed towards the tooth surfaces should be resorted to. The lubricants normally used for spur and helical gears are also used for normal running of straight, zerol or spiral bevel gears.

Heat generation and cooling: It has been mentioned before that one of the functions of the lubricating medium is to act as a coolant to carry away heat generated during gear operation. Heat is developed when there is relative sliding contact and motion between any two metallic surfaces, including gear teeth faces and flanks. It has been found that inspite of the presence of lubricating films, surface temperatures may reach several hundred degrees Celsius even at small loads and sliding velocities. Gear oils help in dissipating heat generated by friction. Generally, oils are not the ideal coolants because of their low specific heat. Besides, effective cooling is a function of many factors, namely, the quantity of oil coming in contact with the gear surfaces, ambient temperature, viscosity of oils and the method of application of oil over the gear teeth. As a general rule, low viscosity oils are more effective in heat transfer.

High capacity gear units are usually provided with pressure lubrication. The pressure oil system generally consists of a sump, a pump, a heat exchanger and nozzles to direct oil streams on to the gear teeth. The heated oil which drains back to the reservoir is cooled by the heat exchanger and is then pumped to the jets which direct it to the gears. The spray should be so oriented that it strikes the leaving sides of the teeth for better effect.

A portion of the heat generated will be dissipated through radiation and convection, but the bulk of it is carried away by the cooling oil. In case of light duty drives, no external cooling system is provided for. But with increasing loads, improved cooling is effected by built-in fans or impellers on the shafts. Fins or shrouds on the housing are also sometimes provided for ensuring better cooling effects.

Although a gear drive is the most efficient mechanical system as yet devised for the transmission of power between shafts, the heat developed due to frictional losses may sometimes become quite considerable, specially when the gears operate at high speed or when the tooth pressure is high. Oil meant for cooling should be directed towards the gear body and not towards the area of meshing for better results.

Though the temperature of the gear casing indicates the state of the gears as far as heating is concerned, the actual temperature of the gears themselves is considerably higher. High-speed gears mostly are provided with sufficient backlash so that the teeth do not seize when they expand due to heat.

Heat dissipation will depend, among other factors, on the operating conditions and the state of the surrounding air. The following guiding values can be given for ordinary purposes.

In still air the rise in temperature of a gear casing can be approximately given by

$$t\,(^\circ\mathrm{C}) = 9 \times 10^{-4} \times \frac{F_t v}{\mathrm{A}} \tag{8.2}$$

where **A** is the area of exposed surface of gear case in m^2 and other symbols have the usual meanings. In case there is a natural circulation of air around the gear casing, then

$$t\,(^\circ\mathrm{C}) = 6.7 \times 10^{-4} \times \frac{F_t v}{\mathrm{A}} \tag{8.3}$$

Tests on gearing mounted on plain bearings have shown that in still air, the rate of heat dissipation will be around 380 J/min/m^2 of exposed area. This can be increased to 1080 J/min/m^2 with a flow of air at a rate of about 150 m/min.

In all the above cases, the expression $F_t v$ represents the total power, that is, the total sum of the entire power transmitted inside the gear box in question. Additional cooling system should be provided if the rise in temperature is more than around 50°C.

In a gear box, the total power loss P_L (J/sec) is transformed into heat energy H (kcal/hr). Taking 1 J = 1/4187 kcal, we have,

$$\text{Rate of heat production } H \text{ (kcal/hr)} = P_L \times 3600\ 14187 = 0.86\, P_L$$

The value of P_L is usually taken to be around 3-5% of the useful power delivered by the gear box. Heat dissipation is carried out through radiation, fan and lubrication, first to the gear box walls and thence to the atmosphere. Ribs on the walls help dissipation. Experience shows that with a supply of unhindered current of air, an amount of heat up to 22 kcal/m^2·h·°C can be dissipated. This figure comes down to around 7 kcal/m^2·h·°C in case of closed gear boxes without any arrangement of fan. Heat to be dissipated is proportional to the temperature difference between the temperature of the gear box (which approximately correspond to the oil temperature and which should not exceed 70° C) and the outside temperature which should be around 35°C.

8.7 Types of Gear Failures and their Causes

For preparing an efficient design, the gear designer should have a thorough knowledge of the possible causes of gear failures and the remedial measures thereof. He must be in a position to predict the service life of the gear set with reasonable accuracy after assessing all the relevant factors involved. For this, a good knowledge of failure criteria is imperative. Correct analysis of failure should be made and in this, the designer can draw from past experience to arrive at the real cause of trouble. Various causes of gear failures will be discussed and evaluated in this section.

According to gear experts, there are eighteen recognised ways in which the gear tooth surface may fail and two ways in which tooth breakage may take place. Some of these causes of failures have already been discussed in Secs 2.23 and 2.25. These will be briefly reviewed here along with other causes of failures.

We have seen in earlier sections that the meshing action in a pair of gear teeth involves a combination of rolling and sliding motions. The line of contact continuously changes and shifts its position during action. The line is actually a band having a certain width under tooth pressure, so that the area of contact describes it more aptly. It has been mentioned in Sec. 2.23 that the entire contact area is subjected to continuously fluctuating stresses and this makes the teeth surfaces vulnerable to fatigue failures. In fact, it has been emphasised before that the failure of gear teeth is largely due to fatigue of one kind or another. The whole gamut of the causes of failure need not be discussed here, only the common ones have been described.

Wear Wear has been defined as a surface fatigue failure phenomenon in Sec. 2.23. Surface deterioration of this kind results in pitting which has been elaborated in the above mentioned section along with the possible remedial measures, such as hardening of the gear teeth.

Normal wear and abrasive wear When metal slides against metal, an inevitable consequence is the gradual loss of material from the surfaces of the teeth in mesh. This can be termed as normal wear. Proper lubrication is one of the effective ways of minimising this loss. Abrasive wear takes place due to surface injury or damage caused by particles trapped in between the tooth surfaces. These particles may be present in the lubricant as impurities or they may be flakes of material detached from the tooth surfaces.

Scoring The kind of gear failure known as scoring has been covered in Sec. 2.25. As pointed out before, scoring is essentially attributable to the lubrication failure. Course ridges and radial scratch lines are formed from the tip of the teeth down to the pitch circle. Lack of adequate lubricant may cause metal to metal contact, resulting in momentary welding between contacting surfaces due to molecular adhesion. High localised temperatures may induce plastic flow of metals which in turn may result in spot-welding. The contacting surfaces may tear apart when they separate out. Extreme pressure (EP) lubricants are used to prevent or minimise scoring as they contain welding-inhibiting agents, such as sulphur or chlorine. Besides improper lubrication, aspects such as misalignment, interference, involute profile error, tooth spacing error and poor surface finish are also contributory factors to scoring failures. Scoring is also termed as scuffing, galling, seizing and roping.

Tooth breakage This kind of failure occurs due to fatigue, and sudden overload or shock. Fatigue breakage is the result of a large number of repetitions of the load. This kind of failure starts with a crack which progressively widens till a portion or a whole tooth breaks away.

Tooth may break due to transient overload caused by momentary fluctuations of torque which may considerably exceed the normally transmitted torque for which the gear set is designed. Overload may also occur due to the inherent tooth errors which prevent the rotating masses to attain uniform angular speed. This in turn creates dynamic loads which add extra burden on the toothed system. This aspect has been fully treated in Sec. 2.22.

Corrosion This is caused by chemical action by the wrong kinds of lubricants or it may be due to agencies prevailing in the surrounding atmosphere which may be of corrosive nature.

Lubrication failures Besides scoring, there are other failures which may be attributable to lubrication. The desirable properties of lubricants and their proper functions have been discussed in Sec. 8.6. The failure of the lubricants to attain the requirements expected of them will result in tooth failure. However, it must be emphasised here that there are several factors which cause failure due to extraneous causes other than lubrication, but which are apparently thought of as failures due to faulty lubrication. For example, tooth surface finish is directly related to the possibility, or otherwise, of the failure of tooth surfaces. Better the finish, greater is the load carrying capacity and the ability for proper maintenance by the lubricant. There might be some inherent design defect which could be the real cause of failure, yet this failure would be blamed on the lubricant. Similarly, manufacturing defects may also give rise to such misconceptions.

Spalling This is also a surface fatigue failure similar to pitting. The damage to tooth surface may be extensive. Chunks of tooth break away as small or large flakes. Case hardened gears are more prone to spalling. Unlike pitting, the damage is not confined to the pitch line area, but may occur at the tip area.

Interference We have seen in Sec. 2.9 how interference and undercutting weaken the teeth which might eventually result in tooth breakage. By adopting a proper design and the right type of manufacturing process, this type of failure can be avoided.

8.8 Gear Noise

In spite of the monumental research work done on the subject, gear noise (as well as measures for its reduction) is one of the least understood areas of gear technology. Reduction of gear noise is being tackled with renewed interest these days as it is believed that loud discordant sounds or predominently high-pitched whine of a high speed gear-set has a particularly detrimental physical and psychological effect on the machine operater or others who have to work for a long time near such a system. Moreover, vibrations created by sound waves can be very disturbing for the normal functioning of machine tools, automobiles and marine engine drives.

Even if all the relevant machining instructions, tolerances, and other factors are observed faithfully, there is no guarantee that a noiseless gear-set will result. However, to develop a reasonably noise-free gear system, certain guidelines can be given. But the designer has to depend mainly on the results of practical experience to attain such goal.

Impulse forces or pulsations in a gear drive are created due to various causes. Impulse is generated during meshing of teeth. It follows, therefore, that noise in a gear system is a function of the accuracy of the component gears. Depending on the quality of manufacture, the various errors and other parameters of a gear will vary from the theoretical values. These include tooth-form error, tooth-pitch error, radial and axial run-out. During operation, the mating teeth bend under load. Pitch error leads to unequal transmission of motion. The resulting angular acceleration and deceleration generates additional dynamic forces and fluctuating torques on the rotating masses. These dynamic forces, along with other factors like the mutual sliding of meshing teeth and the behaviour of the anti-friction bearings, are the causes of the gear noise.

The following measures may prove to be effective for the reduction of noise level.

1. Gear noise is proportional to the square of the pitch-line velocity of the meshing gears, and therefore the reduced velocities produce less noise. This is one of the advantages of the planetary gears.

2. Due to the change in pitch and bending of teeth, the pair of teeth which come in mesh next are subject to an impact at the beginning of the contact. The ensuing noise is proportional to the magnitude of the pitch error. This effect is most pronounced in case of spur gears and increases with the increasing speed. This can be somewhat alleviated by providing tip relief of teeth and lapping the teeth to have a crowning. Experience shows that small modules produce less noise. Tooth errors are mainly responsible for noise especially in case of low loads. Therefore better quality of toothing, smoother tooth surfaces and improved alignment remove noise producing possibilities. Ground or scraped teeth surfaces give best results.

3. Longer duration of meshing period generally reduces noise level. Helical gears are better than spur gears in this respect, because the engagement of teeth is gradual and more teeth are in mesh simultaneously which helps to cancel out the bad effects of tooth error resulting in smoother operation. High helix angle ensures noiselessness. Also high contact ratio (about 2) in case of spur gears results in minimum noise.

4. Gears may become more prone to vibration due to resonance. Therefore, the natural frequency of the system must lie away from the critical zone or the inclination towards covibration should be corrected by providing appropriate shape and vibration-damping methods. The damping can be effected by shrink-fitting cast iron rings in the gear tooth rim, fitting the rim with lead rings, filling the cavity of the rim of big cast gears with noise-damping materials, mounting an elastic bodv (or bush) between the hub and the tooth-rim, and similar measures.

5. Gear box housing should be so designed that the resonance effect is avoided by damping. Webs and ribs should be appropriately placed to attain this objective. During assembly it is to be seen whether the housing could be isolated by a vibration-damping medium such as rubber.

6. The material should be so selected that it has daniping properties. In this respect cast iron gears give better results as against steel gears. Considering the susceptibility of gears towards accuracy as regards production of noise, non-metallic gears give better results as they can stand three to four times as much error as steel gears and can still operate without trouble arising out of inaccuracy. Hence, if silence during service is the main design criterion, non-metallic materials are primarily selected for gear. They are obviously not suitable for heavy duty gears

7. Higher viscosity and high-additive lubricants decrease the gear noise only marginally. Research in the field of gear noise gives the following relation

$$p = c v^{a} F_t^{b} \tag{8.4}$$

where p = Sound pressure in micro-bar (μb) v = Circumferential velocity (m/s), F_t = Tooth load (N). Indices a and b are found to be around 0.6-1.2 and 0.5-1.1 respectively. Factor c is a constant for the particular gear box. In terms of power output, the relation is

$$p \approx 0.11\, P (\mu b) \tag{8.5}$$

$$\text{Intensity of sound} = 55 + 20 \log P \text{ (phon)} \tag{8.6}$$

where P is the power in kW.

Generally speaking, the noise level at the work place should be as low as possible. In case of various gear drives, certain guiding values for intensity of sound can be given which have been measured at a distance of 500 mm and which are usually encountered in industry.

Very well machined worm-drive 70 to 75 phon.

Small to medium spur and helical gear drives in industrial gear boxes with low speed 75 to 85 phon

Turbine gear boxes 85 to 100 phon

Big marine gear boxes 100 to 105 phon

Rigid standards for permissible gear noise have not been made, but the American Gear Manufacturers Association (AGMA) has given the following provisional noise limits for high speed helical and herringbone gear-sets:

107 dB between 20 and 75 **Hz,** 99 dB between 75 and 150 Hz, **94** dB between 150 and **300** Hz, **92** dB up to 10,000 Hz (here dB (decibel) is the unit of sound level and Hz (hertz) for the frequency.)

8.9 Spare Part Drawing of a Gear from Sample

In the practical industrial field, an engineer working in the design office of a factory is often confronted with the problem of making a manufacturing drawing of a gear which is used as a spare part in an equipment or in a gear box drive. In such cases, the problem of the designer is the reverse of the usual one. That is, instead of designing a gear from the given drive-data, arriving at the proper dimensions of the gear from the relevant parameters, working conditions, strength and other requirements, the engineer in this case is required to make a shop drawing from the sample given, so that a spare part can be made accordingly to run the concerned machine.

Often these gears are in worn-out condition. The designer has to take several suitable decisions, namely, choice of materials, allocation of appropriate tolerances to the extent it is possible, specifications for machining, surface quality, heat-treatment, mounting instructions (if any), hardness, besides arriving at the relevant gear tooth data and gear body dimensions using the measurements taken from the sample. If the material is of prime importance, then drillings should be taken from the sample and analysed chemically to ascertain the composition and heat-treatment, if possible. Otherwise, suitable common materials will serve the purpose. Hardness can be similarly tested by suitable means or its value can be decided after considering operational constraints, service conditions and other allied factors.

It has to be assumed in such a case that the actual measured values may or may not tally with the original designed values or with the theoretical calculations which the designer makes for this purpose. In that case, slight adjustments here and there have to be made by trial and error method so that all conditions are more or less satisfied. Besides the actual measurements of the gears, of prime importance is the accurate measurement of the centre distance. The designer's gear calculations are vindicated if the gear or gears, which are made as per the spare part drawing, run smoothly when mounted on the shafts at the actual centre distance, meshing condition do not pose any problem and the gears have a reasonable operational life.

Though the designer may be asked to make the spare part drawing of one gear only, it is always considered advisable to make the drawing of the other gear of the pair too, so that both the members of the gear-set are changed if required. It may not be so important for spur gears, but for helical gears it is virtually imperative for smooth running of the system as the helix angle of the sample gear found by crude measurement could never be the same as the original one. If both the helical gears, having the new calculated value of helix angle, are changed, this problem will not arise. The Example 8.2 illustrates the procedures which should be adopted when a pair of mating gears are given for drawing.

Example 8.2: The following data are determined by actual measurement of the two gears in a medium to fast running drive: $z_1 = 18$, $z_2 = 73$, $d_{a1} = 102.42$, $d_{a2} = 380.52$, centre distance $(a) = 231.5$ mm.

To make manufacturing drawings of the gear pair.

Solution: In this case, since the gear has an odd number of teeth, viz. 73, its outside diameter is measured by suitable gadgets by mounting it on a mandrel and using dial indicators in the shop or in the inspection department. This way the measurement will be accurate. The tip diameter of the pinion is measured by vernier calipers.

The whole depth of tooth is measured and is found to be around 11 mm. Comparing with the table given in Appendix D, the corresponding module is found to be 5. It, is assumed that topping has been done since the gears appear to have been corrected.

Next, an impression is taken of the top land of the teeth to determine tentatively the helix angle at the tip circle (β_a). For this purpose, the top lands may be first smeared with industrial blue or a similar thing and then the gear in question is rolled on a white sheet of paper. The resulting developed impression will look more or less like Fig. 3.1 (d). The helix angle from the impression is then measured with the help of a protractor. This angle, however, is not the helix angle at the pitch circle β. To find β, we use

$$\tan \beta_a = \tan \beta \times \frac{d_a}{d} \qquad \text{(see Chap. 3)}$$

$\beta_a = 9°$ (by measurement). Hence

$$\tan 9° = \tan \beta \times \frac{102.42}{m z_1 \sec \beta} = \tan \beta \times \frac{102.42}{5 \times 18 \times \sec \beta}$$

or

$$\sin \beta \times \frac{102.42}{90} = \tan 9 \quad \text{whence } \beta = 8°$$

Therefore

$$d_1 = \frac{mz_1}{\cos\beta_1} = \frac{5 \times 18}{\cos 8^\circ} = 90.88 \qquad \text{and } d_2 = \frac{5 \times 73}{\cos 8^\circ} = 368.58$$

In this connection it is to be especially noted that though the magnitude of p is same for both the gears at the pitch circles, the helix angles at the tip circles of pinion and gear, β_{a1} and β_{a2}, are different. Therefore data relating to any one of the two mating components should be adhered to for calculation. In our example, we will use those of the pinion.

The transverse pressure angle α_t is found from

$$\tan 20^\circ = \tan\alpha_t \times \cos 8^\circ \qquad \text{whence } \alpha_t = 20^\circ\ 11'$$

From Table 3.3 the working pressure angle α_{tw} is found thus

$$231.50 = \frac{5}{\cos 8^\circ}\ \frac{18 + 73}{2}\ \frac{\cos 20^\circ\ 11'}{\cos\alpha_{tw}}$$

By solving, we get

$$\alpha_{tw} = 21^\circ\ 20,$$

From Table 3.3 we get the total correction factor

$$\text{inv } 21^\circ\ 20'' = \frac{2\tan 20^\circ\ (x_1 + x_2)}{18 + 73}\ \text{inv } 20^\circ\ 11'$$

We get the values of the involute functions from Appendix H. By substituting these values, we have

$$0.018217 = 2\ \frac{0.36397}{18 + 73}\ (x_1 + x_2) + 0.015333$$

By solving

$$x_1 + x_2 = 0.36$$

The individual correction factors x_1 and x_2 can be found by using Table 3.3.

$$102.42 = 2\ (231.50 + 5 - x_2\ 5) - 368.58$$

$$380.52 = 2\ (231.50 + 5 - x_1\ 5) - 90.88$$

By solving, we get

$$x_1 = 0.16 \text{ and } x_2 = 0.2$$

Therefore

$$x_1 + x_2 = 0.16 + 0.2 = 0.36$$

This tallies with the value of total correction as found before. In case the calculated values and the measured values do not tally, then slight adjustments are to be made in different values, e.g. tip diameters, helix angles, and correction factors, with a view to keeping the centre distance the same so that there is no difficulty in mounting the gears.

To check top clearance, we find first the value of topping by using the equation

$$y\,m = a_r + (x_1 + x_2)m - a = \left(\cdot \quad \frac{+\ 368.58}{2}\right) + (0.36 \times 5) - 213.5 = 0.03 \text{ mm}$$

The top clearance

$$c = a - \left(\frac{d_{a1} + d_{a2}}{2} + ym - h\right)$$

$$= 2315 - \left(\frac{102.42 + 380.52}{2} + 0.03 - 11.25\right) = 1.25 \text{ mm}$$

As per the basic rack (IS: 2535), $c = 0.25$ m. Therefore $c = 0.25 \times 5 = 125$ mm

The top clearance, therefore, is in conformity with the standard value.

Some more data are required to be given in a shop drawing for manufacturing and inspection purposes. With the help of Secs 2.27 and 2.28, the following values are calculated.

For block measurement, the number of teeth to be measured z', and the base tangent length W, are found from Sec. 2.28.

$$z' = 3 \text{ for the pinion as per its data}$$
$$W = 5\,(7.38033 + 0.014402 \times 18 + 2 \times 0.16 \times 0.34202)$$
$$= 38.745 \text{ mm}$$

For the type of application in question, a quality of 7 will suffice as per Table 2.27. Zone of tolerance selected is d for base tangent length W and J for centre distance. For inspection purposes, the double flank roll-gear test for total composite error S" is chosen.

From Appendix I, the tolerance on the tip diameter d_a is h8, and the permissible radial run-out of tip cylinder is found to be equal to

$$0.025 d_a + 15 = 0.025 \times 102.42 + 15 = 17.56 \mu m \approx 0.018 \text{mm}$$

The tolerances on W are found from Appendix K

$$-53\,\mu m = -0.053 \text{ mm}$$

$$-79\,\mu m = -0.079 \text{ mm}$$

These values are in the transverse section. To get the values in the normal section for actual measurement, the values given in the table are to be multiplied by $\cos\beta$ as explained in Sec. 2.28. The final values are

$$-0.053 \text{ mm}$$

$$-0.078 \text{ mm}$$

Appendix L gives the tolerance on the centre distance to be

$$\pm 0.036 \text{mm}$$

As explained in Sec. 2.27, these tolerances are to be multiplied by a factor

$$\frac{\tan 20°}{\tan \alpha_{t}} = \frac{\tan 20°}{\tan 21° 20'} = 0.932$$

The final values are

$$\pm 0.034 \text{ mm}$$

For the double flank total composite error test, the reader should refer to Sec. 2.27. The value is given by

$$F_i'' = 56 + 4.5\phi_p = 56 + 4.5\,(m + 0.25\sqrt{d}) = 56 + 4.5\,(5 + 0.25\sqrt{90.88})$$
$$= 89\ \mu m = 0.089 \text{ mm}$$

As before, this value is multiplied by **0.932**, giving

$$F_i'' = 0.083 \text{ mm}$$

After calculating all the data as described above, they are to be entered in table along side the relevant drawing as shown in Drawings 2.1 and 2.2 given in Sec. **2.30.**

In Drawing **2.2**, the values for z', W, F_i'' and the tolerances on W have been omitted. This has been deliberately done and is meant to be an exercise for the reader. These values as well as the run-out values along with other instructions meant for the note are to be determined and inserted in the drawing by the reader.

In selecting the quality and zone of tolerance and other relevant parameters, it is reiterated that there is no well defined thumb rule for the selection as this is entirely dependent on the experience and discretion of the designer and his assessment of the prevailing operational conditions. This has been already pointed out in Sec. 2.28. If for example, backlash warrants it, the tolerances can deviate from the standards and the tables given. These should be considered as broad guidelines only and not as inflexible ones.

8.10 Gear Pumps

These positive displacement pumps are quite frequently used in industrial equipments, hydraulic lines, and lubrication systems. The most common type is the external gear pumps shown in Fig. **8.13.** Spur gears of equal size are normally used which operate inside a casing. One of the gears is positively driven by means of a key and the other member of the meshing pair is normally an idler gear which runs free on the shaft. Various types of gears may be used, but the spur gears are most common. When the teeth of the rotating gears unmesh, a partial vacuum is created which draws fluid into the pump. The fluid is then carried to the discharge side by the rotating gear teeth as the fluid, which is confined in the space between the tooth-gap and the inside surface of the fixed casing, is forced to move continuously along the periphery till it is discharged on the delivery side. The fluid is forced out of the tooth space on the pressure side, and as the gears rotate, the space vacated by the moving teeth is filled with the fluid again, and the process thus continues. Gear pumps are normally provided with relief valves.

Efficiency of a gear pump will depend on the accuracy of components, their fittings, as well as on the magnitude of the leakage oil. Leaking causes a loss and brings down the efficiency. By suitably selecting the design criteria, gear pumps can produce pressures up to **1400** N/cm² or sometimes even higher, though these pumps are ordinarily limited to medium to low pressure service only. These pumps are small in relation to their capacity and are ideally suited for pumping light lubricating oil, though they can be designed to pump different types of fluids.

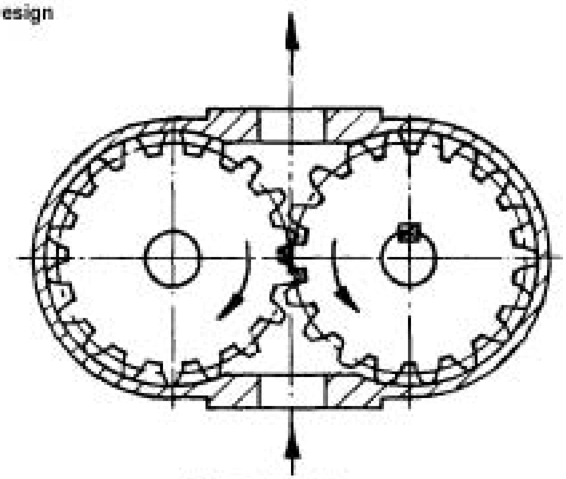


Fig. 8.13' Gear pump

These pumps are usually self-priming up to moderate suction heads. Helical or herringbone gears are also used in these pumps to reduce the high-frequency pulsation which causes noise.

The gears are closely fitted in the housing and the teeth form a seal against the housing. The material, tolerance, and other parameters for the components will vary according to design. Mentioned below are some common practical design data.

Whole depth of tooth = 2.2 to 2.25m Tolerance: quality and zone = **5** cb
Centre distance tolerance $= a$ $_{-}^{0.01}$ (generally)
Tolerance on tip circle diameter of gear $d_a = h6$ Tolerance on the inside diameter of housing = ***E* 6**

All surfaces including teeth should be ground. Tooth surfaces should be case-hardened to **HRC 63 ± 2.** Material of gear should preferably be 17 Mn **1 Cr** 95, IS: 1570.

Neglecting leakage oil, the theoretical value of flow in case of pump with equal gears of 20° pressure angle, is given by

$$V = \frac{1}{2}\pi b n \left(d_a^2 - a^2 - \frac{1}{s}\pi^2 d_b^2 / z^2\right) \text{ [Approximate formula } V = 6.9 \times d \times m \times b \times n\text{]}$$

The symbols have the usual meanings used in the gear technology. The unit of V **will** depend on the units used for the gear and other parameters.

8.11 Hunting Tooth System

It has been discussed that due to the sliding action between the tooth surfaces of mating gears and due to other reasons as well, wear on the tooth surfaces takes place. Although, theoretically the surfaces of all the teeth in a gear should have identical properties as regards hardness, surface finish, magnitude of different errors, material and metallurgical properties, tolerances and other tooth parameters, in practice this is seldom so. It is quite likely that after a reasonable running in time, each tooth develops individual characteristics. Cast gear teeth may have individual characteristics from the very beginning itself due to imperfections in castings, lack of homogeneity in the material, slag inclusions, blow holes, surface irregularities such as raised portions, cracks and uneven hardness.

If the reduction ratio of a gear pair is an integral number, the numbers of teeth of the two members comprising the drive have a common divisor. Consider, for example, a pair of gears having 24 and 72 teeth. The reduction ratio is 72/24 or 3. Now, for one complete revolution of the gear, one particular tooth of the pinion mates with 3 particular teeth of the gear. And this takes place for any number of revolutions the gear pair makes. Similarly, each of the other teeth of the pinion mates with a particular set of 3 teeth in the gear in each case. This results in uneven distribution of wear. One pinion tooth may be harder than its neighbouring tooth. When the first tooth mates with its own set of 3 gear teeth continually, it makes the 3 gear teeth in question to wear out faster than 3 gear teeth corresponding to the pinion tooth with lesser hardness. The case may also be reverse, that is, a harder gear tooth may make the pinion tooth vulnerable to wear and other undesirable effects.

To control this situation properly and to help in equalising wear on all the teeth of the pinion and gear, and also to improve accuracy of tooth-spacing, the "hunting tooth" system has been universally adopted. In this system, which is particularly desirable in case of teeth surfaces with low hardness, one of the two mating gears is provided with an extra tooth than the number exactly required for a particular ratio. Considering our previous example of a gear pair having 24 and 72 teeth, if the gear is made to have 73 teeth, instead of 72, keeping the number of teeth in the pinion unchanged, then the reduction ratio becomes 3.0416667, which is very near the required value of 3.

Since the numbers 24 and 73 do not have a common divisor, it is ensured that any tooth of one component of the gear pair will, in time, contact all the teeth of the mating coniponent as the two members rotate during service, and thus the continual meshing of the same pair of teeth at regular intervals is avoided. This progressive nieshing of teeth results in an even distribution of wear because all the teeth develop more or less the same wear pattern. The teeth are then eventually worn to a comparatively true and identical shape.

It is desirable that the number of teeth of one member should be a prime number, but so long as the numbers of teeth of the two gears have no common factors, the hunting action will take place. For example, consider the gears with 28 and 195 teeth. The factors are

$$2 \times 2 \times 7 = 28 \quad \text{and} \quad 3 \times 5 \times 13 = 195$$

Since no common factor is present, hunting action is assured. Besides, if a cutting tool operating on the basis of meshing action like a gear is used for machining, a tool having a common factor between number of teeth of the tool and number of teeth of gear to be machined should be avoided. Thus, a shaving cutter having 91 teeth should not be used to machine either gear as 91 can be factorised as

$$91 = 13 \times 7$$

Factors 7 and 13 occur in numbers 28 and 195 respectively.

8.12 The 05-System of Toothing

In this system, each gear of a mating pair is positively corrected by an amount

$$xm = +0.5m$$

That is, $x = +0.5$, irrespective of the number of tooth of the gear. Hence the name of the system.

This system is valid for all gears, having number of teeth 8 and above. The teeth have a relatively high load carrying capacity. This system can be used very easily with simple calculations by the use of tables given in DIN 3995 for the purpose. The system has, however, its own limitations, namely, it can be used mainly for transmission in slow speed or step down systems, and it cannot be used if a pre-specified centre distance has to be maintained.

In 05-toothing, the contact ratio is greater for number of teeth less than 12 and up to 15 than in case of toothing without profile correction, i.e. ordinary, uncorrected gears, because due to a high value of **x**, the undercutting is avoided. By greater number of teeth, however, the contact ratio is lower than that of the corresponding uncorrected gears.

Because the correction factor x is constant, all gears with 05-toothing can be paired off, if only the module is the same. This characteristic is advantageous for inventory control and sparepart availability because gears can be had "off the shelf" in the store or ex-stock from the market.

Recalling Eq. 2.35, the centre distance of a corrected gearing is given by

$$a = m\,\frac{z_1 + z_2}{2}\,\frac{\cos \alpha}{\cos \alpha_w}$$

The working pressure angle α_w, can be calculated from

$$\text{inv}\,\alpha_w = \frac{2\tan\alpha\,(x_1 + x_2)}{z_1 + z_2} + \text{inv}\,\alpha$$

Since $x_1 = x_2 = +0.5$ and $\alpha = 20°$, we get

$$a = m\,\frac{z_1 + z_2}{2}\,\frac{0.93969}{\cos \alpha_w} \tag{8.7}$$

$$\text{and inv}\,\alpha_w = \frac{2\tan 20°\,(0.5 + 0.5)}{z_1 + z_2} + \text{inv}\,\alpha = \frac{0.72794}{z_1 + z_2} + 0.014904 \tag{8.8}$$

From the above equations it is clear that for a particular module the centre distance depends only on the sum of the number of teeth of the mating gears, i.e. $z_1 + z_2$. This characteristic of 05-system is important for change-gear arrangements because the centre distance remaining the same, different transmission ratios are obtainable by changing the z_2/z_1 combinations, provided that the value of z_1+z_2 is kept constant.

The modern trend in gear design is to adopt the 05-system in as many cases as possible for the advantage enumerated above. In short, this system combines the advantages of positive correction as discussed in Chap. 2 with the added facilities for interchangeability if required.

Relevant equations for the calculation of the usual parameters of a 05-gear are given below. These can be arrived at by inserting the value $x = +0.5$ in the equations in the sections dealing with corrected gears, taking $\alpha = 20°$.

Tip diameter
$$d_a = 2\,(r + m + 0.5m - ym) = 2m\,(z/2 + 1.5 - y) \tag{8.9}$$

Topping factor
$$y = \left[1 - \frac{z_1 - z_2}{2}\left(\frac{0.93969}{\cos \alpha_w} - 1\right)\right] \tag{8.10}$$

Root diameter $d_f = d - 2(1.25m - 0.5m) = m(z + 1 - 2.5) = m(z - 1.5)$ (8.11)

Centre distance
$$a = m\frac{z_2 + z_1}{2}\frac{0.93969}{\cos \alpha_w} \quad (8.12)$$

Working pressure angle α_w is given by

$$\text{inv}\,\alpha_w = \frac{2\tan 20^\circ(0.5 + 0.5)}{z_1 + z_2} + \text{inv}\,20^\circ = \frac{0.72794046}{z_1 + z_2} + 0.01490438 \quad (8.13)$$

Base tangent length for inspection is given by

$$W = m\cos 20^\circ[(z' - 0.5)\pi + z\ \text{inv}\,20^\circ] + (2 \times 0.5 \times m \times \sin 20^\circ)$$

$$= m \times 0.93969[(z' - 0.5)\pi + z \times 0.014904381 + (m \times 0.34202) \quad (8.14)$$

Chordal tooth thickness

$$\bar{s} = mz\left[\frac{1.93476656}{z} \times \frac{180}{\pi}\right] \quad (8.15)$$

Chordal tooth height (without topping)

$$\bar{h}_a = m \times 1.5 + m \times \frac{z}{2}\left[1 - \cos\left(\frac{1.93476656}{z} \times \frac{180^\circ}{\pi}\right)\right] \quad (8.16)$$

8.13 Shrink Fit Calculations for Power Transmission

When parts are fitted together by shrink fit, the surfaces of the components must have enough resistance against sliding or turning between mating parts. It is often required to design machine elements which are meant to transmit torque through shrink fit or to check whether a certain pair of shrink-fitted items are correctly designed to transmit a specified torque. One of such common cases is the torque transmission by a gear which is shrink-fitted on to a shaft. Example 8.3 illustrates such a case.

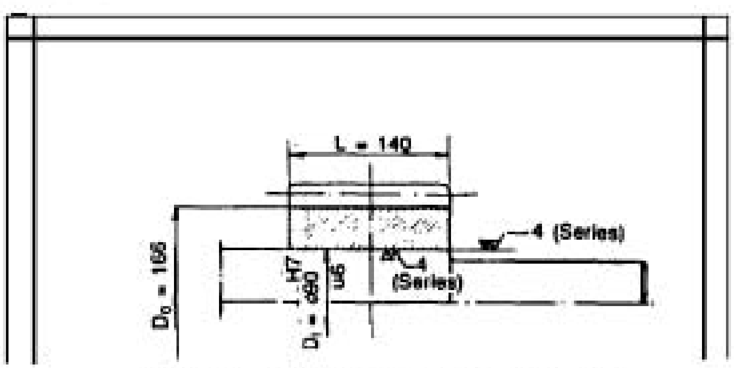


Fig. 8.14 Pinion shrink-fitted on a shaft
Based on Neue Festigkeitsberechnung fuer den Maschinenbau. Haenchen and Decker, 3rd Edition, 1967, Fig No 211, p 168 Carl Hanser Verlag. Munich

Example 8.3: Figure **8.14** shows a pinion shrink-fitted on a shaft. The material of pinion is **40** Cr **4**, and that of the shaft is **45** C8. Within the allowable tolerances specified on the drawing, what are the maximum and minimum contact pressures developed at the mating surfaces? What are the maximum and minimum compressive stresses which the shaft is subjected to? Determine whether the system is capable of transmitting a nominal torque of **2600** N m and the factor of safety thereof. Take a service factor of 1.25.

Solution: The shrink-fit chosen is 90 H7/u6. The tolerances have the following values,

$$90^{**} = 90^{+0.035}_{0} \qquad 90_{u6} = 90^{+0.146}_{+0.124}$$

The maximum and the minimum interferences are given by

$$Z_{max} = (146 - 0) = 146\ \mu m \text{ and } Z_{min} = (124 - 35\ \mu m = 89\ \mu m$$

As per Appendix V, the value of R_t for series **4** and **2** triangles finish of surfaces is 10 **pm**. The contact pressure at the mating surfaces is given by

$$p = \frac{Z}{(K_O + K_I)\ D_I\ 10^3}\ N/mm^2 \text{ (or megapascal, MPa)} \tag{8.17}$$

Factors K_O and Z are auxiliary values to be taken from Fig. 8.16. In the above equation and the subsequent equations, subscript O stands for the outer part or diameter and subscript I for the inner part or diameter.

For materials other than steel, the following relation is to be used to find K values

$$K_{material} = K_{steel} \times \frac{E_{steel}}{E_{material}} \tag{8.18}$$

where E is the modulus of elasticity

Diameter ratio is given by $Q_O = \frac{D_I}{D_O}$ and $Q_I = \frac{D_{IF}}{D_I}$

where D_O = Outside diameter of the outer part

D_I = Diameter at the joint = Inside diameter of the outer part = Outside diameter of the inner part

D_H = Inside diameter of the inner part in case of a hollow cylinder

In this case D_I = 90 mm and since it is a solid shaft, D_H = 0, therefore, Q_I = 0.

A generalised case of pressure distribution is shown in Fig. **8.15.**

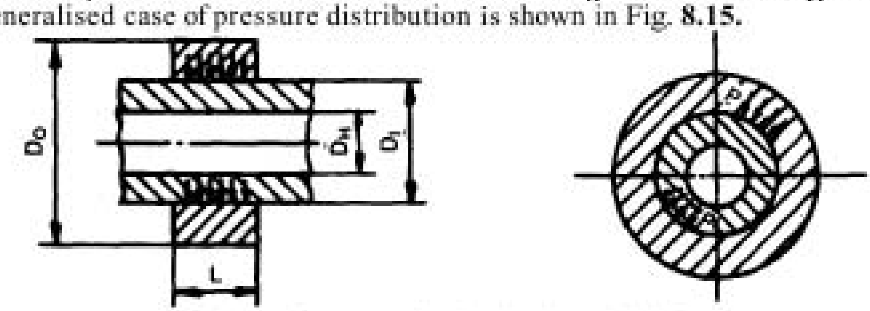


Fig. 8.15 Pressure distribution in a shrink-fitted system

Dimension of adhesion is given by

$$Z \text{ (in } \mu\text{m)} = I - 2(G_O + G_I) \tag{8.19}$$

where Z stands for interference, and G_O and G_I are magnitude of smoothness in case of outer and inner parts respectively. These are given by

$$G_O \approx 0.6 R_{tO} \text{ and } G_I \approx 0.6 R_{tI}$$

In this example both R_{tO} and R_{tI} are equal to 10 μm. Therefore, $G_O = G_I \approx 6$ μm.

Corresponding to the maximum and minimum values of interference, there will be two values of Z

$$Z_{max} = 146 - 2(6+6) = 122 \ \mu\text{m}$$

$$Z_{min} = 89 - 2(6+6) = 65 \ \mu\text{m}$$

Corresponding to $Q_O = 90/166 = \mathbf{0.54}$ and $Q_I = 0$, the values of K are found from the Fig. 8.16 to be

$$K_O \approx 0.1 \times 10^{-4} \text{ mm}^2/\text{N}$$

$$K_I = 0.03 \times 10^{-4} \text{ mm}^2/\text{N}$$

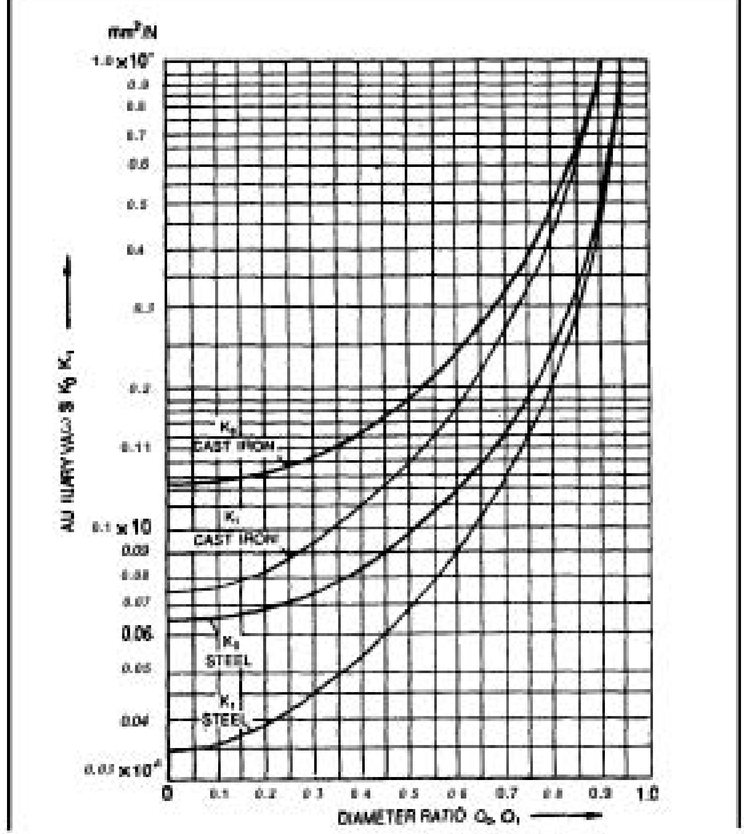


Fig. 8.16 Auxiliary values, K

Based on Neue Festigkeitsberechung fuer den Maschinenbau. Haenchen and Decker, 3rd Edition, 1967 Fig No. 210 p. 167. Carl Hanser Verlag, Munich.

The maximum and the minimum contact pressures are given by

$$P_{max} = \frac{122}{(0.1 + 0.03) \times 10^{-4} \times 90 \times 10^3} \approx 104 \text{ N/mm}^2$$

$$P_{min} = \frac{65}{(0.1 + 0.03) \times 10^{-4} \times 90 \times 10^3} \approx 56 \text{ N/mm}^2$$

The compressive stress in case of a solid shaft is equal to the contact pressure. Therefore

$$\sigma_{c\,(max)} = 104 \text{ N/mm}^2 \text{ and } \sigma_{c(min)} = 56 \text{ N/mm}^2$$

In this example, the maximum torque required to be transmitted is given by

$$T_{max} = \text{Nominal torque} \times \text{Service factor} \tag{8.20}$$

$$= 2600 \times 1.25$$

$$= 3250 \text{ Nm}$$

The area of contact is given by

$$A = D_I \times \pi L = 90 \times \pi \times 140 = 39584 \text{ mm}^2$$

The sliding torque, i.e. the torque required to overcome the frictional resistance at the surface of contact is given by

$$T_s = \tfrac{1}{2} \times D_I \times A \times p \times F \tag{8.21}$$

where F = The coefficient of adhesion found from the Table 8.10.

Table 8.10 Coefficient of adhesion F

Material	*Coefficient F*	*Shrink-fitted*
Steel/steel	0.10-0.15	in oil
Steel/steel (hardened)	0.12	dry
Steel (cast iron)	0.07-0.09	dry
Steel (hardened)/steel (cast)	0.10	in oil
Steel/brass or bronze	0.07-0.14	dry

Taking F = 0.12 and the minimum value of p, i.e. 56 N/mm², we have

$$T_s = \frac{1}{2} \times 90 \times 39584 \times 56 \times 0.12 \times 10^{-3} \text{ N m}$$

$$= 11970 \text{ N m}$$

Factor of safety or the safety against failure $T_s/T_{max} = \frac{11970}{3250} \simeq 3.7$

Since the factor of safety required in such cases is usually 1.5 to 1.8, the above shrink-fit system is adequate to transmit the required torque.

The preheating temperature of the outer part is given by

$$T\,(\text{in } K) = \frac{1.5 \times L_n}{\alpha \times D_l \times 10^3} + T_R \qquad (8.22)$$

where α = The coefficient of linear expansion

T_R = Room temperature in kelvin (K)

The factor 1.5 ensures sufficient expansion to facilitate the proper joining of the parts.

8.14 The SI Units

In science and technology the units and the systems of measurements of physical and other quantities have evolved from the crude systems of the past to the extremely sophisticated ones used in the modern world of high technology to meet the extremely high degree of accuracy, precision and exacting demands of measurements. The British system, or the FPS system, was followed in the industrial countries in initial stages. Later, this gave way slowly to the French or metric system or the CGS system. The CGS system was primarily used for scientific measurements. In Continental Europe, another slightly different system was followed in engineering and technology. This was known as the "technical system" as distinct from the "physical system" using CGS units. The technical system was again sub-divided into two forms: the old and the new. In the old technical system, the unit of force was kg and the unit of mass was kg/g, where g = the acceleration due to gravity = 9.81 m/sec^2. The unit of mass was thus left unnamed. This led to confusion as kg is the unit of mass in the physical system and not of force. To correct this ambiguity, the unit of force was designated as kilogramme force or kgf and that of mass as kilogramme-mass or kgm in the English speaking countries for calculations involving metric technical system. In the new technical system, the word kilopond (or kp) was coined to represent the unit of force. This was defined as

1 kp = The force required to impart to a mass of 1 kg an acceleration of 9.81 m/s^2

or 1 kp = 1 kg × 9.81 m/s^2

The unit of mass was again left unnamed as kg is the unit of mass in the physical system and not in the technical system—old or new. Later on, the symbol kp was discarded in many quarters because the word pond was phonetically similar to pound which might have again led to further misunderstanding. The symbol kp was largely replaced by kgf.

All these confusions and anomalies were set aright by the introduction of the international system of units (SI) which developed intially from three basic units: the unit of length as metre, the unit of mass as kilogramme and the unit of time as second. This was abbreviated as the **MKS** (metre-kilogramme-second) system of units. Later the unit of electric current or ampere (A) was added and the system came to be known as the **MKSA** system.

Further units were added and the present system units (SI) came into being. It now consists of seven base units, two supplementary units and a number of derived units as detailed in Appendix T. This international system of units is in fact an extension and refinement of the traditional metric system. "*his* system is known as: Systeme International d'Unites, for which the abbreviation is SI in all languages. It embodies features which make it logically superior to any other system as well as practically more convenient. It is rational, coherent and comprehensive.

The SI, like the traditional metric system, is based on decimal arthmetic, i.e, units of different sizes are formed by multiplying or dividing a single base value by powers of 10. Thus changes can be made very simply by adding zeros or shifting the decimal points.

The SI is a coherent system. A system of units is said to be coherent if the product or quotient of any two unit quantities in the system is the unit of the resultant quantity. For example, in coherent system in which the foot is a unit of length, then the square foot is the unit of area, whereas the acre is not. This coherence aspect of SI greatly simplifies technical calculations. For example, equations involving physical principles can be applied without introducing such numbers as 550 in power calculations, which in the English system have to be used to convert units. Thus conversion factors largely disappear from calculations carried out in SI units which leads to a great saving in time and labour. The SI being an absolute system and not a gravitational system (as the technical system), the factor g has been dispensed with. The unit of force is newton (N) and is defined as the unit of force required to impart to a mass of 1 kg an acceleration of 1 m/s². That is

$$1\ \text{N} = 1\ \text{kg m/s}^2$$

This has important consequences in calculations and is of particular interest to the scientists and engineers because the confusion regarding the units of force, mass etc. which often they had to face, have been totally removed. There are seven base units from which units for other quantities are derived. The unit horse power (hp) is no longer used and is replaced by the derived unit watt (W). Another great advantage which has ensued as a result of using the SI units is the attainment of the goal of international understanding between the scientists and the technologists. Units such as erg, calorie (cal) or horse power-hour (hp. h) which were used to measure energy in various disciplines have been now replaced by one unit only, viz. joule (J). This universality has now become even more pertinent because the scientists and the engineers of different countries have now to work in close cooperation in fields like control techniques, space technology, nuclear engineering and other techno-scientific areas.

Most of the industrially advanced countries using metric system are switching over to the SI units. In India, in many spheres such as schools, universities, industries and other allied fields, the adoption of SI units is being actively encouraged and this will help to end the confusion and wastefulness resulting from the present multiplicity of units. The Indian Standard Specifications are now practically all in SI units

Appendix T gives tables of the basic SI units, derived units and conversion factors. The units which are relevant to gear technology have been mainly included in these tables.

8.15 Preferred Numbers and Sizes

Preferred Numbers

In this age of global standardisation, increasing international cooperation and exchange of ideas and products in trade, commerce, science and industries, it stands to reason that along with so many other things, the numbers and sizes should be standardised too. Besides, within the country itself, there should be standardised numbers and sizes to effect rationalisation and to avoid multiplicity of product sizes produced by different manufacturing agencies of the country.

It is, therefore, logical that some sort of standardised values should be nationally and universally adopted. The question now is: What should be the basis of such standardisation? The most obvious solution which one may jump at is to employ a simple arithmetical progression to arrive at the standardised numerical values. But the drawback of this series lies in the fact that the ratio of successive terms in such a series varies widely. Consider the step from 1 to 2: the variation is 100%. The same difference, viz. 1 produces only 10% variation in case of 10 to 11 and an insignificant 1% in case of 100 to 101.

It is now generally accepted that such number series should be a geometrical progression because it offers several advantages. Here, each term is larger from the preceding term by a fixed percentage. Such a series provides small steps for small numbers and large steps for large numbers. An arrangement like that meets best the most requirements in the practical field and is also in conformity with the mode of variations found in nature (e.g. organic growth, radio active decay, and other natural laws which generally follow power curves). By studying the arithmetical and the geometrical series, we can see that the arithmetical series has the characteristic that the differences between adjacent values are equally large, while in geometrical series the percentage step between successive values are equally large.

Series of numbers which are standardised on a certain basis so that these numbers are used in preference to any other numbers are called "preferred numbers". The principal reason behind such concept is to provide a master series from which suitable terms can be selected to suit any needs.

When a product is to be manufactured, a thorough planning is required as to its number of sizes. Several factors come into consideration. If the sizes and types are many, this will lead to increased production costs, inventory control, difficulty in stocking, storage and distribution, besides other inconveniences. A rational approach, therefore, is necessary to limit the number of sizes which, at the same time, must also meet the consumer's demands covering a wide range of choice. The manufacturer can strike a balance between his constraints and the consumer's needs by following the standardised sizes for the manufacture of products.

In short, for variety reduction and dimensional standardisation, it is only a question of compiling a suitable series of numbers or sizes which, should be followed to achieve that goal.

Having decided the rationale behind the standardisation of numbers and sixes, we must now try to decide its basis. It has been already mentioned that the geometrical series is a suitable one. It has been found from experience that the consumer's requirements are generally satisfied when the range of sizes follows, more or less closely, a geometrical progression. Based on this concept, agreed series of preferred numbers have been carefully worked out which, while not restricting the liberty of choice, provide the designer with a guide to minimise unnecessary variations of articles marketed so that the requisite range is covered by a minimum number of different sizes with the resulting econom) to both the producer and the user. Experience shows that if an article is so marketable that every size can be produced in economically viable quantities, then there is no valied reason for severe standardisation. When the consumption figures are small, however, it is preferable to stick to standardised values or sizes.

The basis of the preferred numbers, therefore, is the geometrical progression. Besides the fact that in such a series, each term is larger than the preceding one by a fixed percentage, one more advantage of this series stems from the fact that if the linear dimensions are chosen in the series, then areas, volumes and other functions of powers of dimensions are also numbers of the same series.

The preferred numbers are classified into five principal series, namely, R5, R10, R20, R40 and R80, where the numeral indicates the particular root of 10 on which the series is based. The letter R stands as a tribute to Col. Charles Renard, a Frenchman who first conceived and proposed this system.

In the preferred number series, numbers become irrational with the exception of 10 and the integral powers of 10. That is why the terms must be conveniently rounded off in order to be of practical usage. Values given in Appendix U have been obtained after careful Consideration and agreement at international level.

After a most exhaustive study by the experts, the following system for preferred numbers has evolved.

The first number of a series is 10 and the other numbers are obtained by multiplying (or dividing) the first number by the constant factor relevant to this series and repeating this operation with each ensuing number. For example, in case of R5 series, the constant factor is

$$\sqrt[5]{10} = 1.5849 \ldots \approx 1.6$$

In the range of 10 to 100, the preferred numbers are found thus

$$10$$
$$10 \times 1.6 = 16$$
$$16 \times 1.6 \approx 25$$
$$25 \times 1.6 = 40$$
$$40 \times 1.6 \approx 63$$
$$63 \times 1.6 \approx 100$$

It can be seen that this series gives numbers which are, more or less, 60% apart. Similarly, for other series, we get

$$R\,10 = \sqrt[10]{10} = 1.2589 \approx 1.25$$
$$R\,20 = \sqrt[20]{10} = 1.1220 = 1.12$$
$$R\,40 = \sqrt[40]{10} = 1.0593 = 1.06$$
$$R\,80 = \sqrt[80]{10} = 1.0292 \approx 1.03$$

This indicates that the successive terms in the respective series from R10 to R 80 increase approximately by 25%, 12%, 6% and 3% respectively. For preferred numbers below 1, they are obtained by dividing the numbers between 1 and 10 by 10 or by 100, etc. as the case may be. For numbers above 10, they are obtained similarly by multiplying the numbers between 1 and 10 by 10 or 100, etc. For example, from the preferred number 6.3/10 = 0.63, 6.3, we can get 6.3 x 100 = 630, and so on.

The numbers of series R 5 should be preferred to the numbers of series R 10 which in turn should be preferred to the numbers of series R 20, and so on.

Preferred Sizes

Preferred sizes are a selection of sizes based on the preferred numbers. The numerical values more or less tally with the preferred numbers with a few minor variations for practical reasons.

The purpose of preferred sizes is to limit the number of arbitrarily chosen sizes. If preferred sizes are used, then there should be a continual reappearances of the same sizes. This in turn facilitates simple and economical production, use, assembly and above all, interchangeability of products. These sizes are finding increasing application for the preparation of product standards, rationalisation of number of tools and gauges, limiting the number of jigs and fixtures, and for limiting stocks to an orderly series of sizes. Use of these sizes leads to simplified practice for selecting sizes and ratings of machines. In short, the system affords to have a comprehensive plan in all fields of manufacture. The sizes of shafting, for example, would be so designed that they are in conformity with the sizes of bearings. The same argument applies to other items of design and production.

Appendix U gives tables of preferred numbers and preferred sizes. Preference for numbers have been emphasised by bold type. For preferred sizes above 1000, the basic preferred numbers may be used.

8.16 Limits, Fits and Tolerances

Selection of proper limits, fits and tolerances is an important aspect of mechanical design. For designing mechanical components in general or for assigning the correct fits and tolerances for specialised engineering items, the reader may refer to the appropriate books or reference material on those subjects. The discussion in this section is confined to only those aspects of fits and tolerances which are relevant as far as the gear design is concerned.

Values of dimensional tolerances are given in Appendix S. Surface quality symbols and their values are shown in Appendix V. There is a direct relationship between the dimensional tolerance of a part and the permissible surface roughness. Assigning of undue accuracy will only enhance the production cost of the piece unnecessarily. The designer should carefully select the tolerance, zone, and the surface quality required, keeping in mind the ultimate use to which the product will be subjected.

In this connection it should be borne in mind that the dimensional tolerance system has nothing to do with the gear quality and tolerance system discussed in Secs 2.27 and 2.28. The two systems are based on different considerations altogether and must not be confused. Discussed below are the general guidelines for the dimensional tolerance systems.

General Terminology

Fit: When two parts are assembled together, the relation resulting from the difference between their sizes is called a fit. A fit is normally classified into three undermentioned types:

(i) Clearance fit, where a clearance is always provided between the mating parts;

(ii) Interference fit, where an interference exists when the parts are assembled: and

(iii) Transition fit, where after assembly, either a clearance fit or an interference fit may result, depending upon the actual sizes of the mating parts.

Tolerance: Due to the inevitable inaccuracy caused by manufacturing methods and other factors, it is not possible to make a part precisely as per a given dimension. Even if it can be made, it will not be economically viable and the manufacturing cost of the part will be prohibitive. Hence, some minor "deviation" from the ideal dimension has to be acceded to. This deviation is called tolerance, which is the difference between the maximum allowable and the minimum allowable values, called limits.

Among the various methods of applying the concepts of fits, the principal ones are the "shaft basis" system and the "hole basis" system. The hole basis system is generally followed because of its inherent advantages in certain aspects. Normally, it is easier to produce a shaft with a specified tolerance than a hole with the same tolerance. It follows, therefore, that if the tolerance of the hole is standardised, shafts with different tolerances can be machined to achieve the desired fits more easily and economically. Consequently in modern engineering design, the hole basis system is most extensively used. The zone of hole, usually selected in the hole basis system, is H and for general engineering purposes, the tolerance is of 7 quality. Hence, most of the toleranced hole dimensions bear the symbol : H 7.

The quality of tolerance selected will depend upon the part and the type of its application. In IS: 919, the quality is categorised into 18 grades: 01, 0, 1 to 16 in decreasing order of fineness. For normal work-pieces, grades 5 to 11 are used for machined parts.

Some of the fits, based on the hole basis system, which are most commonly used and which are especially relevant to gear technology are given in Table 8.11.

Table 8.11 Types of fits and their application

Type of fit	Symbol	Example of application	Remark
Shrink fit	H 8/x 8 H8/u 8 H7/u 6	Gear and shaft where the torque is transmitted by fit only	Selection will depend upon the coefficient of thermal expansion
Press fit	H 7/r 6	Gear and shaft	Normally for light to medium duty
Light Press fit	H 7/n 6	Gear and worm-wheels where shafts is fitted with a parallel key	When shaft diameter is 50 mm or above, n 6 is normally chosen
Force fit	H 7/m 6	Gear and shaft with key	Easier dismantling
Push fit and easy push fit	H 7/k 6 H 7/j 6	Gear and shaft with key	When shaft diameter is below 50 mm, m 6 or k 6 is normally chosen; with j 6, an easy push fit ensues
Sliding fits	H 7/h 6 H 7/g 6 H 7/f 7 H8/e8 H 8/d 9	These fits are used for gears sliding on shafts, splines, and for movable gears in change-gear trains	Fit should be properly chosen depending on the magnitude of sliding required
Running fit	H8/h11	Oil seals with metallic housing. DIN recommends H8 for hub on which the seal is mounted and h 11 for shaft	See Sec. 8.2 for oil seal as per IS specifications

Besides those given in Table 8.11, some more fits and tolerances which are commonly used are given below.

Circlip: Many pinion shafts are fitted with circlips. Grooves for such circlips in the shaft are provided with tolerances, such as, H 12, H 13, and H 14. This is shown in Drawing 2.1 in Sec. 2.30.

Keyways: Tolerances for keyways in housings and shafts are given in appendices dealing with different types of keys. Tolerances for splines are also given in similar manner.

Seals: Radial oil seals and felt seals are used in bearing covers which are fitted on gear box housings. Tolerances for the above items are shown in Sec. 8.2 on gear box housing and accessories.

Bearings: Anti-friction bearings are generally used in gear boxes and shafts. In an antifriction bearing, all the components are manufactured to very close tolerances and to exacting material specifications. Depending on the type of bearing, magnitude and nature of load to which it is subjected, operating conditions, and other considerations, the fits and tolerances which are most suitable for the bearing selected have been internationally standardised by the bearing manufacturing companies. Since the types and sizes of such bearings have a vast range, their loading conditions, temperature considerations, expected life requirements and other allied factors vary according to their applications, the tolerances meant for anti-friction bearings are not elaborated here. The reader may refer to the catalogues of any standard bearing manufacturer from which the tolerance can be selected commensurate with the type and magnitude of gear forces, bearing loads, service conditions and other relevant parameters.

For journal bearings a suitable sliding or running fit should be chosen so that ample clearance is provided between the journal and the bearing. In many such cases, the maintenance of a film of lubricant is the deciding factor.

In case of both the anti-friction and the journal bearings, loads on the bearings can be purely radial, purely thrust or a combination of radial and axial forces. Calculation and selection procedures for the bearing for each type of loading condition are exhaustively dealt with in the catalogues and manuals or anti-friction bearing manufacturers. For such calculation and design procedures in case of journal bearings, the reader may consult the relevant books dealing on the subject.

APPENDICES

APPENDIX A

Construction of Involute Gear Tooth

Method 1

The involute tooth profile of standard 20° pressure angle can be drawn as explained below.

The values of the diameters of the tip circle d_a, the pitch circle d , and the root circle d_f are calculated as per the following relations.

$$d = m z$$

$$d_a = m z + 2m$$

$$d_f = m z - 2 \times 1.25m$$

The involute is generated from the base circle the diameter of which is given by

$$d_b = d \cos 20°$$

By geometrical construction, the base circle is found by first drawing a vertical central line, and with O as centre, the circles are laid out as per the values previously calculated [Fig. A. 1(a)]. The pitch circle intersects the vertical central line at C. This is the pitch point. The line of action is then drawn making an angle of 20° with the horizontal line and passing through C. A perpendicular from the centre is dropped on the line of action meeting it at A. The circle passing through A with O as centre is the base circle. On both sides of A, the line of action is divided into equal parts locating the points E_2', E_1', E, E_1, E_2, E_3, E_4, etc. Here, point A on the base circle and point E on the line of action coincide. Points E_4, E_3 etc. are then transferred on the base circle by means of circular arcs with E as centre, giving points A_4, A_3... A_2', etc. Tangents are then drawn from the points A_4, A_3 etc. On these tangents, points C_4, C_3, ... ,C_2' are, marked off making $A_4C_4 = E_4C, A_3C_3 = E_3C, A_2C_2 = E_2C, A_1C_1 = E_1C$ etc. Joining these points gives rise to the involute curve which, in case of a gear tooth, extends from the tip circle up to the base circle. For the sake of clarity and to avoid making the drawing cumbersome, many points, e.g. E_1, E_2, E_1', E_2', A_1, A_2, C_1, C_1', etc. have not been shown in the drawing.

From the base circle, the tooth profile is extended up to the root circle as a radial. At the root where the radial meets the root circle, a fillet is provided, the value of which is determined according to the basic rack profile discussed in Sec. 2.1.

The profile of the other side of the gear tooth can be similarly drawn, keeping in mind that the circular tooth thickness at the pitch circle is half of the circular pitch, i.e. $s = p/2 = \pi m/2$. This locates the point on the other side which corresponds to point C of that side, the profile of which has been drawn.

Method 2

An approximate and simplified method of drawing the involute tooth has been shown in Fig. A1 (b). This method is adequate for ordinary representation on a drawing when an accurate one is not needed. After drawing the necessary circles as before, the base circle is drawn as shown. Next, an arc is drawn with radius $(R) = AC$ with the centre at A. This arc C_1CC_2 gives a reasonable approximation of the involute profile which is adequate for all practical purposes as far as graphic representation is concerned. The same process is repeated for the other half of the tooth profile.

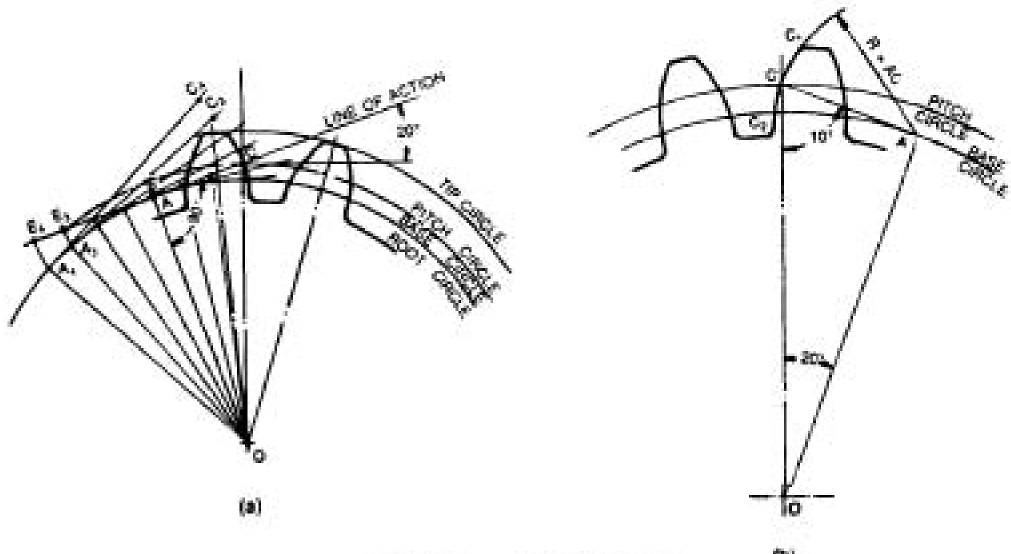


Fig. A.1 involute tooth

Method 3

The graphic method of drawing the involute curve is the most accurate one, though it is rather a laborious process. For both uncorrected and corrected gear profiles, this method gives the correct profiles. Sometimes this method is also used for making templates and form tools.

First, the values of the relevant circles are found by calculations, using the formulae given in Chaps 2 and 3. Recalling the parametric equations of the involute curve (Eqs 1.6 and 1.7), we start to plot from the base circle. Then assigning arbitrary values for the angle, we find the corresponding values of the x and y coordinates. Plotting these values on a graph paper or a white sheet of paper and passing a smooth curve from the base circle to the tip circle and joining these points, will trace the involute profile.

To find the other profile of the tooth, suitable changes are made in the equations since the curve now veers towards the left. The tooth thickness at the base circle is then calculated and marked on the base circle. The x and y axes should now be suitably rotated to correspond to the new location of the origin of the other half before applying the modified equations. The other profile is drawn in a similar way to complete the whole tooth. As a check, the tooth thicknesses at the pitch circle and the tip circle may be calculated by using the formulae given in Chaps 2 and 3. The calculated values can then be compared with the drawing just drawn to ensure accuracy. The rest of the tooth profile, i.e. the portion from the base circle to the root circle as well as the fillet, is drawn as before.

Since the left and the right hand profiles are mirror images, a much simplified method will be to draw first one profile on a transparent tracing paper, reverse it, and then re-trace the other profile after suitably adjusting the reversed profile to coincide with the previously located tooth-thickness points on the three circles as indicated above.

Method 4

By using the principle of gear cutting by generation method, the involute tooth profiles of a gear can be drawn on paper. The speciality of this method is that not only the face and flank comprising the tooth profile can be drawn, but the fillet curve portions of the teeth are also represented realistically. As indicated elsewhere in the book, this curve is a trochoid and not a circular arc. This aspect is not taken care of in the other methods described so far, which are only good approximations of tooth profiles. By using the generation method and depending upon the human and instrumental accuracy, a reasonably good replica of the tooth can be drawn, which can be used as a paper template for making a form tool or as a master profile to guide the tool in a copying machine. By this method, the trochoid is automatically generated along with other portions of the tooth profile. However, all the methods described so far produce sufficiently accurate tooth profiles, mainly in large gears with large modules (around 30 and above). With smaller modules, it becomes increasingly difficult to maintain the draughting accuracy and errors are likely to creep in. The generation method is described below step by step:

Step 1: On a transparent tracing paper, draw the profile of the gear-tooth cutting rack. This is the template of the tool for generating the tooth profiles. Fix it rigidly on the drawing board by adhesive tape or tack pins. The addendum of this cutting rack is 1.25 module with respect to its pitch line MM, which divides the rack into equal lengths of tooth thickness and tooth gap. This rack produces gears conforming to the Basic Rack as per IS: 2535, in which case the teeth have dedendum equal to 1.25 m. In this connection see Sec. 8.5 on cutter profiles.

Step 2: On a separate, loose transparent tracing paper, draw the pitch circle of the gear to be drawn. On this paper, the tooth profiles will be generated.

Step 3: The pitch line *MM* of the cutter rack is divided in short, equal parts, say 5 or 10 mm each, and numbered a, b, c,... as well as a', b', c', etc., on both sides of the vertical centre line. In a similar manner, the pitch circle of the gear is divided at equal intervals and numbered 1, 2, 3, ... and 1', 2', 3', etc. To facilitate correct alignment, draw thin vertical lines through these points parallel to the centre line in case of the rack and radial lines for the gear to be drawn. This is shown in Fig. A.2(a). Obviously, smaller the intervals, more realistic and accurate will be the ultimate result.

Step 4: Next, place the loose tracing paper over the template in such a manner, that the pitch line of the rack tool is tangent to the pitch circle of the gear at the pitch point P as shown in Fig. A.2 (a). The vertical through P is the centre line of the pitch circle.

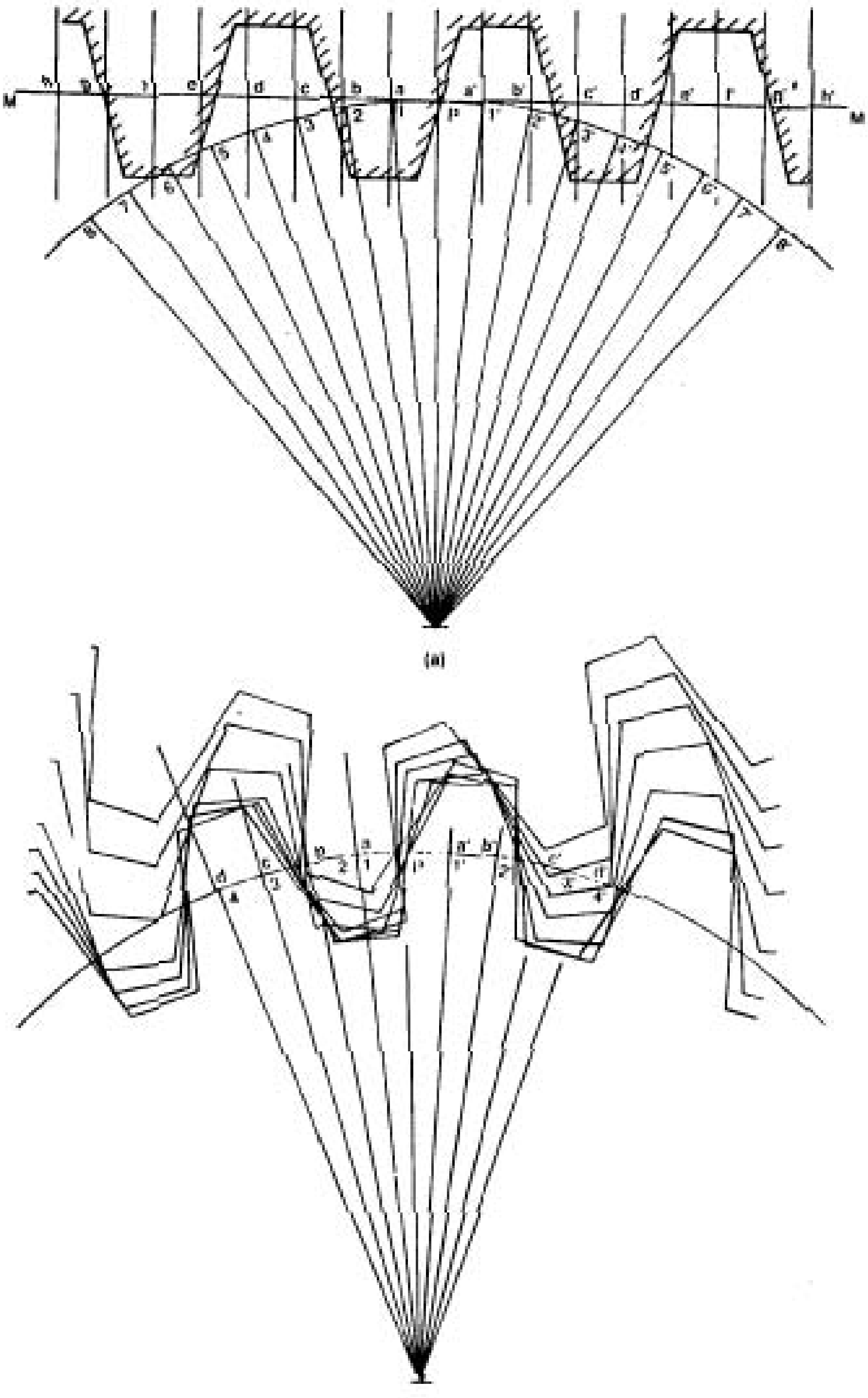


Fig. A.2 Generation of involute toothing

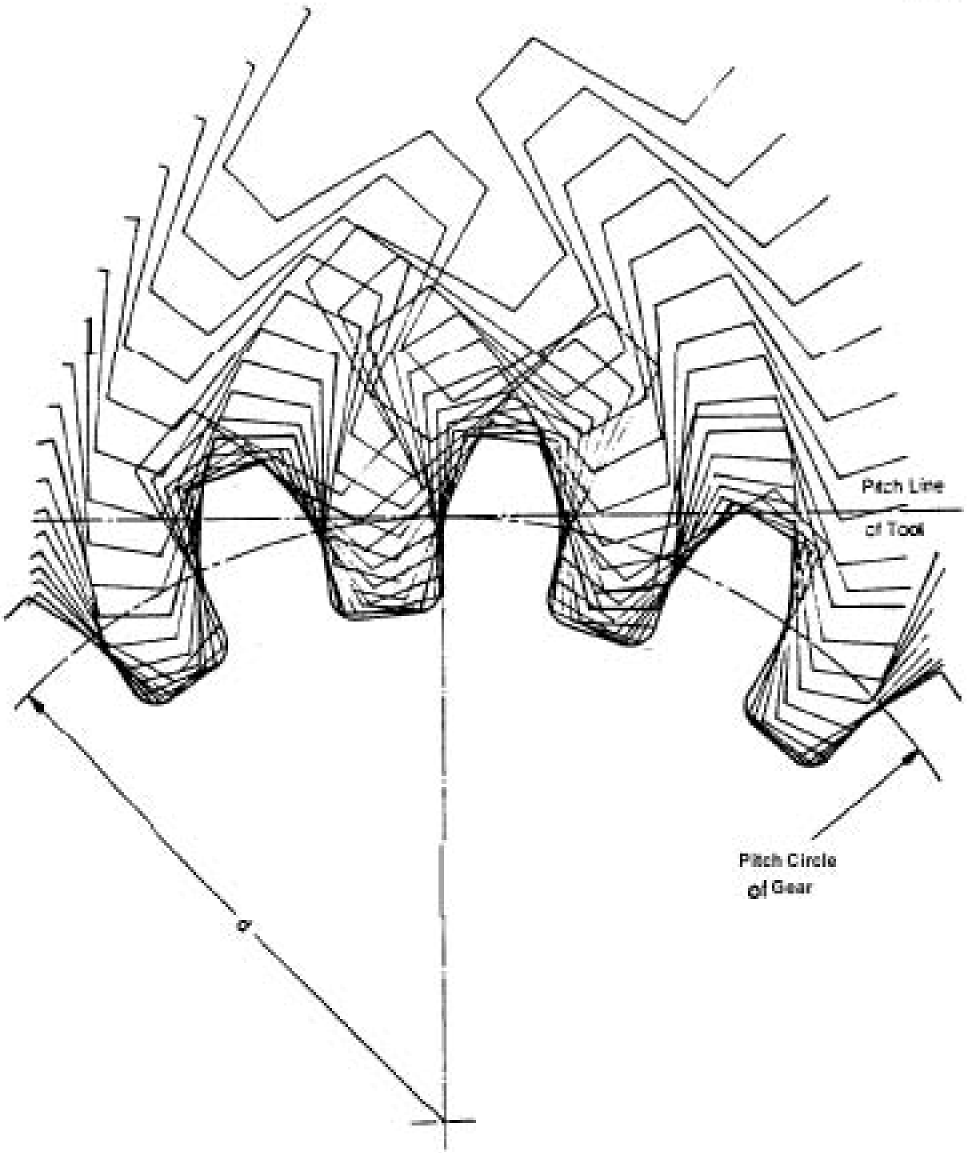


Fig. A.3 Completely generated involute toothing

Step 5: The rack, which is under the transparent tracing paper having the gear pitch circle, is visible though this paper. The profile of the rack is now traced on this paper by a sharp pencil.

Step 6: Now, shift and rotate the paper, so that the points a of rack and 1 of gear coincide. This means that the pitch line of the rack has now rolled on the pitch circle, so that this line is now tangent to the circle at point 1. In this condition, the radial line through 1 will be exactly superimposed on the parallel line through a of the rack. Draw the outline of the rack as before. Next, shift and rotate the paper so that points *b* and 2 coincide in the above manner and draw

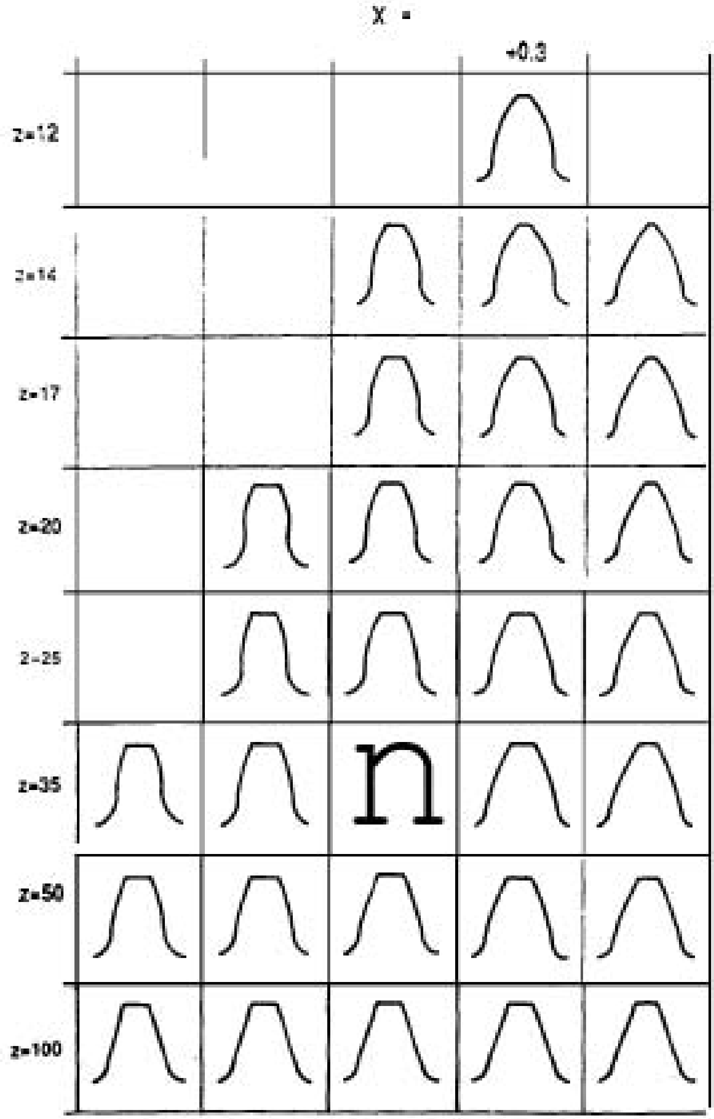


Fig. A4 Effect of number of teeth z and correction factor x on shape of teeth conforming to the Basic Rack as per IS: 2535.
Based on MAAG Taschenbuch of M/s. MAAG Zahnraeder AG, Zurich, Switzerland, 2nd edition 1985, Fig. no. 1.57, p. 85

the rack outline. Repeat the whole process for all the points on both sides of the centre line, pairing c − 3 ..., h − 8, a' − 1′, ... h′ − 8′. Care should be taken to ensure that the pitch line of the rack is always tangent to the pitch circle at all successive points.

Step 7: The desired profile is the envelope of the rack profiles thus drawn. Pass a smooth curve to delineate this envelope. It can be seen from Fig. A.2 (b) that the tooth profiles are gradually taking shape. It is particularly noticeable in case of the middle tooth in the figure. When the

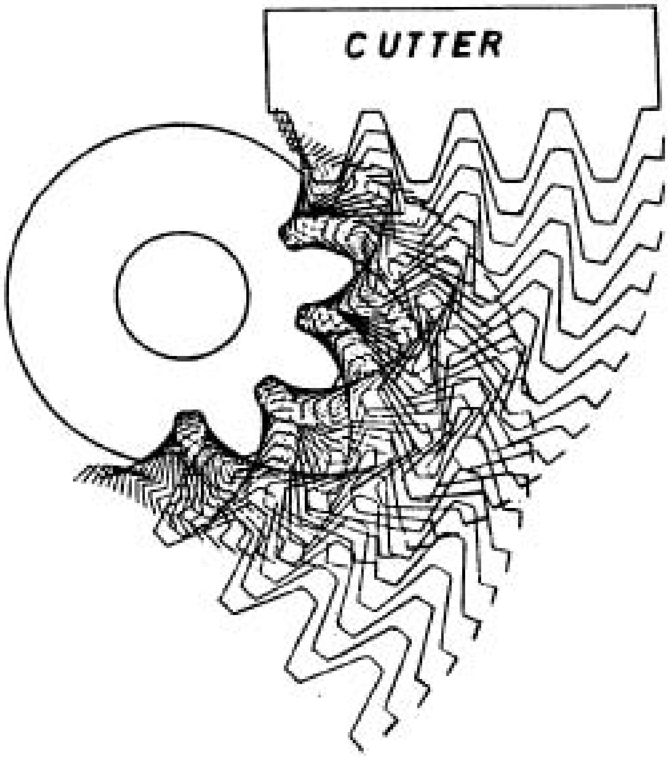


Fig. A.5 Corrected tooth profiles made by generation method

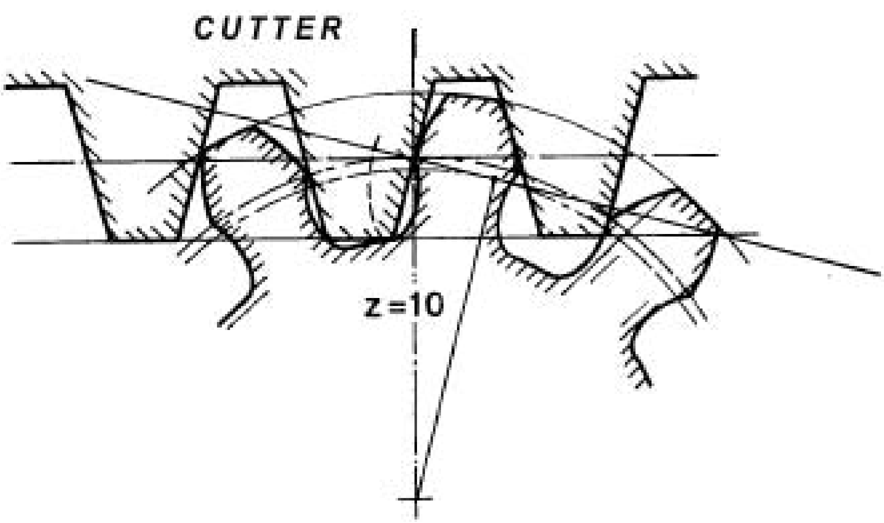


Fig. A.6 Undercut teeth

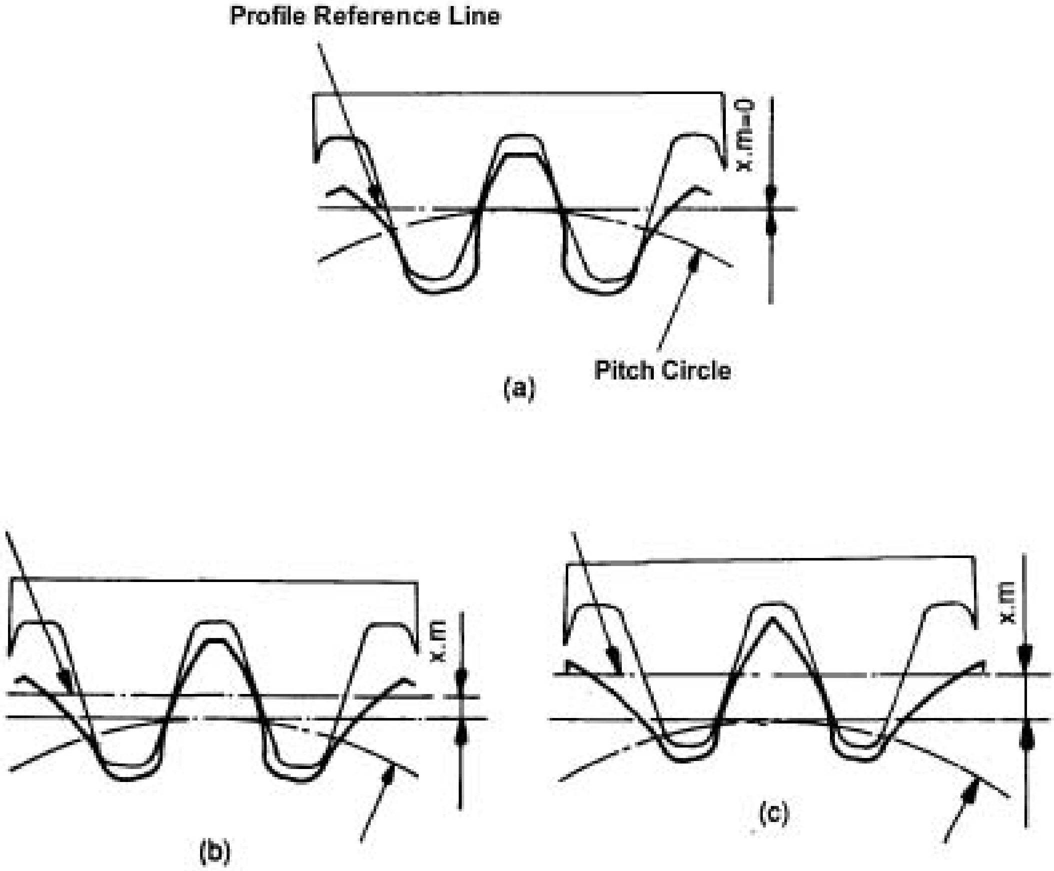


Fig. A.7 Tooth profileswith different correction factors

whole process iscompleted, the tooth profiles will look as shown in Fig. A.3. The tip circle is then drawn to mark off the boundary of the gear teeth.

By shifting the pitch line by an amount equal to $x \cdot m$ and proceeding as before, reasonably accurate profiles of corrected toothing can be generated on paper.

Theshapeof the toothisafunction ofthenumberofteeth ofthegearand thecorrectionfactor, if any. Figure A.4 shows the relation between the number of teeth vis-a-vis the correction factor, as far as the tooth shape is concerned. It will be noticed that the influence of the correction factor, positive or negative, in determining the ultimate shape of the tooth diminishes markedly with the increase of number of teeth of the gears. In this connection, see Sec. 2.12.

Figure A.5 shows the corrected tooth profiles made by generation method. In Fig. A.6, undercut teeth have been shown. In Fig. A.7, comparative tooth profiles of gears having the same module, number of teeth, etc. but with differentmagnitudes ofcorrection factor have been shown. These are given as an exercise for the reader, so that he can draw tooth profiles with different amounts of correction factor by means of the method described above and then compare those with the given figures. Figure A.7(a) shows a profile with correction factor zero, A.7(b) with the

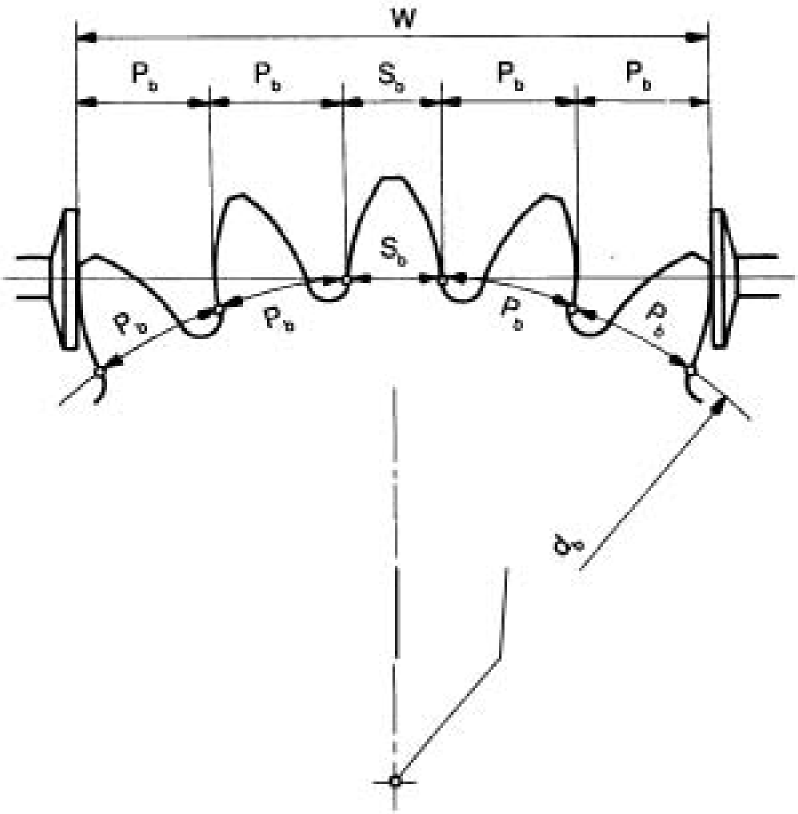


Fig. A.8 Tooth checking by base tangent length method

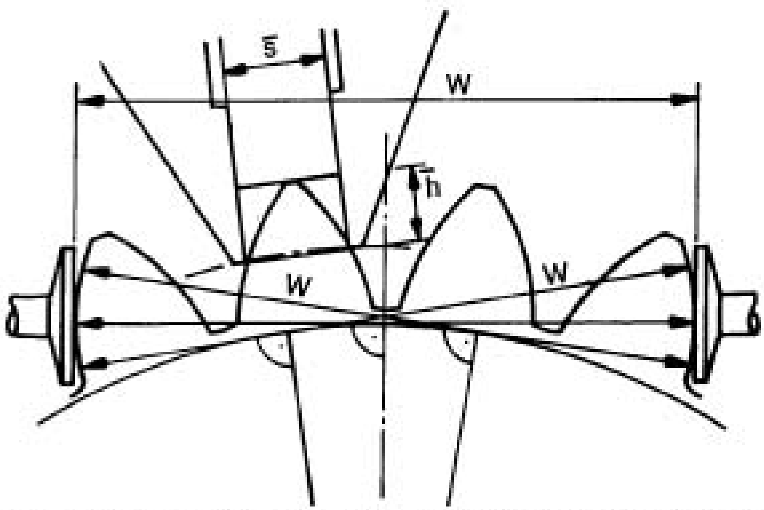


Fig. A.9 Tooth checking by chordal tooth thickness and span measurement methods right amount of correction factor relevant for the particular gear and A.7 (c) with too high a correction factor (peaked tooth).

A preliminary check as to the correctness of the drawn profile can be made on the drawing board by using the principle of "base tangent length" method of measurement described in Sec. 2.28. Having calculated the values of z' and W as per the formula given therein, the length W on the drawing can be checked against its calculated value as shown in Fig. A. 8. Here, the anvils represent the parallel scales, set squares or any other drawing instrument used for the purpose of measurement of the value of W.

In Sec. 2.28, we have seen that the base tangent length method of tooth checking has its own limitations. It cannot be employed in case of a helical gear having high helix angle coupled with comparatively narrow tooth width, which may not permit to span the required tooth distance W if z' is too high. In such cases, checking by measurement of chordal tooth thickness of individual tooth is resorted to. Besides, if the tooth size is very large, this method may be the only choice left for practicable measurement. Both of these methods have been represented in Fig. A.9. Depending upon his discretion and practical consideration, the designer has to decide upon the most convenient method and choose (or devise) a suitable gadget accordingly, so that tooth checking can be made on the drawing board.

APPENDIX B

Construction of Cycloidal Gear Tooth

Geometrical Method

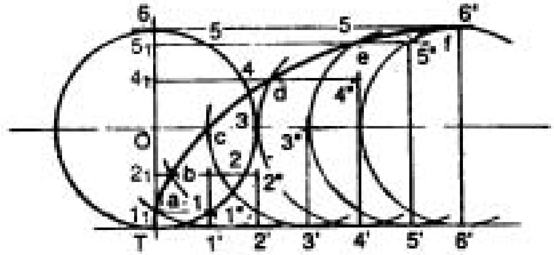
When a circle rolls on a straight line, a point on the circumference of the circle traces a path. This path is the cycloid curve. The curve can be drawn by solving its characteristic equations by assigning arbitrary values to the variables in the equations and then plotting the resulting values on paper and passing a smooth curve joining the points thus obtained. The curve can be drawn geometrically as explained below.

Cycloid

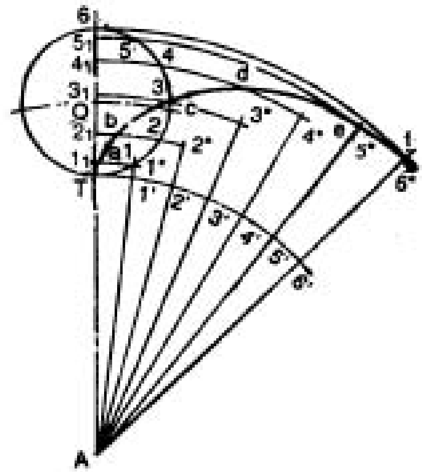
Let the circle of radius O-T in Fig. B.1 (a) roll on the straight line T-6′. One half of the circle is then divided into any number of equal parts. In this case, we have divided the semi-circle into six parts, denoted by the arcs T-1, 1-2, 2-3,..., 5-6. These arcs are then laid off on the straight line T-6′, so that arc T-1 = straight line T-1′, arc 1-2 = straight line 1′–2′, etc. (It is obvious that higher the number of arcs in which the semi-circle is divided, more accurate will be the resulting curve.) Next, perpendiculars are erected on the points 1′,2′,3′, etc of T-6′. Through the points 1,2,3, etc., which are already located on the semi-circle, lines parallel to T-6′ are drawn. These lines intersect the diameter of the circle at points 1_1, 2_1, 0, 4_1, etc. They also intersect the perpendiculars at points 1″,2″,3″, etc. From 1″, the horizontal line 1″—1, is cut in such a way that 1″—a = 1—1_1. Similarly, portions 2″—b = 2—2_1, etc., are laid off, locating points a,b,c,d,e, etc. A smooth curve is then drawn through these points. This curve T-a-b-c-d-e-f (6″) is the cycloid curve.

Epicycloid

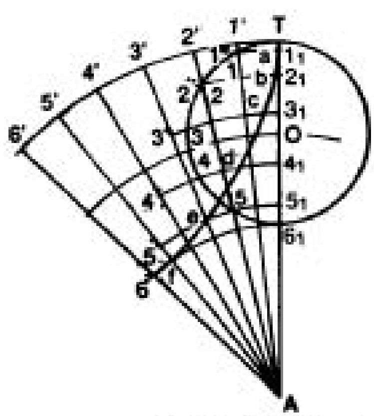
When the generating circle rolls on the outside periphery of another circle, then a point on the generating circle traces an epicycloid in similar fashion as the cycloid. As shown in Fig. B.1 (b), the epicycloid can be drawn thus as before, the semicircumference of the rolling circle being divided into six equal arcs, viz. T-1, 1-2, 2-3, etc. On the directing circle of radius AT, lay off arcs so that arc T-1′ = arc T-1, arc 1′-2 = arc 1-2, etc. as shown in the figure. Draw radial lines joining the centre A with 1′, 2′, 3′, etc. Next, draw arcs through points 1,2,3, etc. These arcs intersect the diameter T-6 at 1_1, 2_1, 3_1, etc., as well as the radial lines at 1″, 2″, 3″, etc. Then on the arc 1″ – 1_1, lay off a distance from 1″ so that 1″–a = 1–1_1, locating the point a. In similar manner, points a,b,c, etc., are obtained. The curve T-a-b-c-d-e-f(6′) is the epicycloid.



(a) CYCLOID



(b) EPICYCLOID



(c) HYPOCYCLOID

Fig. B.I Generation of cycloidal curves

Hypocycloid

When the generating circle rolls on the inside penphery of the directing circle, a point on the circumference of the generating circle traces a path known as the hypocycloid curve. As shown in Fig. B.2(c) the hypocycloid T-a-b-c-d-e-f(6") can be drawn in the similar manner as in the case of an epicycloid.

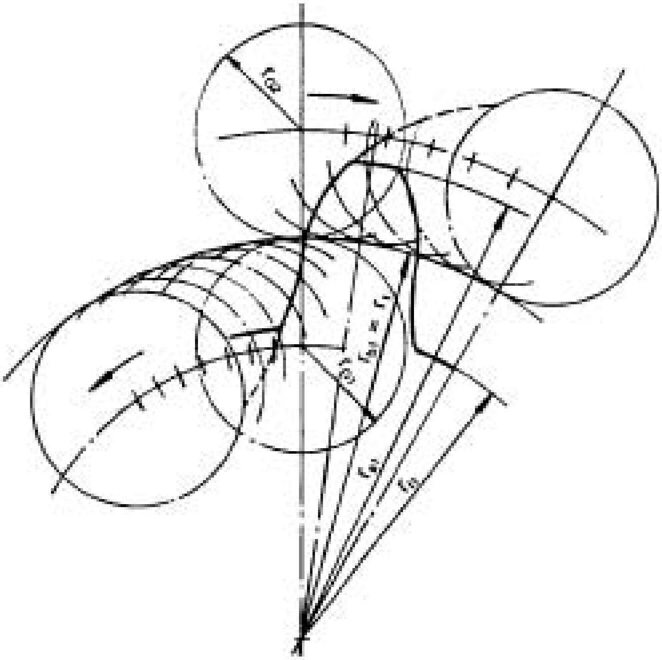


Fig. B.2 Cycloidal tooth

Graphical Method

For drawing the above curves accurately, graphical method may be adopted as indicated before. The relevant equations of the curves, which are given below, may be solved after assigning proper values for the variables and then the curves can be plotted on a graph paper or a white sheet of paper, choosing a proper scale.

Cycloid

The equations for the cycloid are

$$x = a\,(\phi - \sin\phi)$$

$$y = a\,(1 - \cos\phi)$$

where a = the radius of the rolling circle. The origin of the curve is at the intersection of the x-axis and the y-axis with $\phi = 0$.

Epicycloid

The equations for the epicycloid are

$$x = (a + b)\cos\phi - a\cos\left(\frac{a + b}{a}\,\phi\right)$$

$$y = (a + b)\sin\phi - a\sin\left(\frac{a+b}{a}\right. 41$$

where a = the radius of the rolling or generating circle

b = the radius of the directing or pitch circle

The origin of the curve is at the intersection of the directing circle and the x-axis in the 1st quadrant with $\phi = 0$.

Hypocycloid

The equations for the hypocycloid are

$$x = (a - b)\cos\phi + b\cos\left(\frac{a-b}{a}\phi\right)$$

$$y = (a - b)\sin\phi - b\sin\left(\frac{a-b}{a}\phi\right)$$

where a = the radius of the directing or pitch circle

b = the radius of the rolling or the generating circle

The origin of the curve is at the intersection of the directing circle and the x-axis in the 1st quadrant with $\phi = 0$.

APPENDIX *C*

Construction of Conjugate Profile

As explained in Chap. 1, conjugate profiles are those curves which conform to conjugate action, that is, the two profiles in mesh make it possible for the driving and the driven components (of which the two profiles are the mating curves) to have a constant angular velocity ratio. We have also seen that gears with teeth made of involute or cycloidal surfaces follow this rule.

Apart from gears, it is sometimes required to have a conjugate profile when one profile and other relevant data are given. With the help of the law of gearing, this can be achieved. As stated earlier, according to this law, the common normal to the two mating surfaces at any point of contact must always pass through a fixed point, called pitch point **P**, situated on the line of centres, irrespective of the position of the point of contact during the course of action. We shall now see how we can draw the second profile corresponding to the first one which is given.

In Fig. C-1(a) A_1F, is the profile for which the conjugate profile is required to be constructed. The pitch circles of the two rotating components are given as shown. They intersect the line of centres O_1O_2 at the pitch point P.

Consider any point B, on the given profile. Draw a tangent to the curve at B, . Drop a perpendicular $B_1B'_1$ on pitch circle 1. Now, B is the point where the two conjugate profiles originally met and B, is the point on profile **2** which met B_1 at B when the two profiles were in contact at that point, i.e. at B. In other words, the point B_1 and B_2 merged with B when the two profiles were in contact at B. The problem now is to find points B and B,.

To achieve this, profile 1 must be rotated back around centre O, till the point B'_1 comes to pitch point P. Point B_1 is now rotated back as shown by the arrow. With P as centre, draw an arc with radius $B_1B'_1$ which cuts the arc from B_1 at B so that $PB = B_1B'_1$. The point B is thus located.

Now, since the two pitch circles are in rolling contact (without slippage), if the point B'_1 of pitch circle 1 is rotated back to P, an equal amount of arc length of pitch circle 2 is also simultaneously brought back to **P**. In other words

$$\text{arc } B'_1P = \text{arc } B'_2P \quad \text{or} \quad \overset{\frown}{B'_1P} = \overset{\frown}{B'_2P}$$

Point B'_2 is thus located.

To find the required point B, on the second conjugate profile, draw an arc from B with O, as centre as shown by the arrow. With B', as centre, draw an arc with radius equal to $B_1B'_1$. This arc meets the arc from B at B, so that $B'_2B_2 = B_1B'_1$. Point ***B,*** is the desired point on the second conjugate curve corresponding to B, of the first curve.

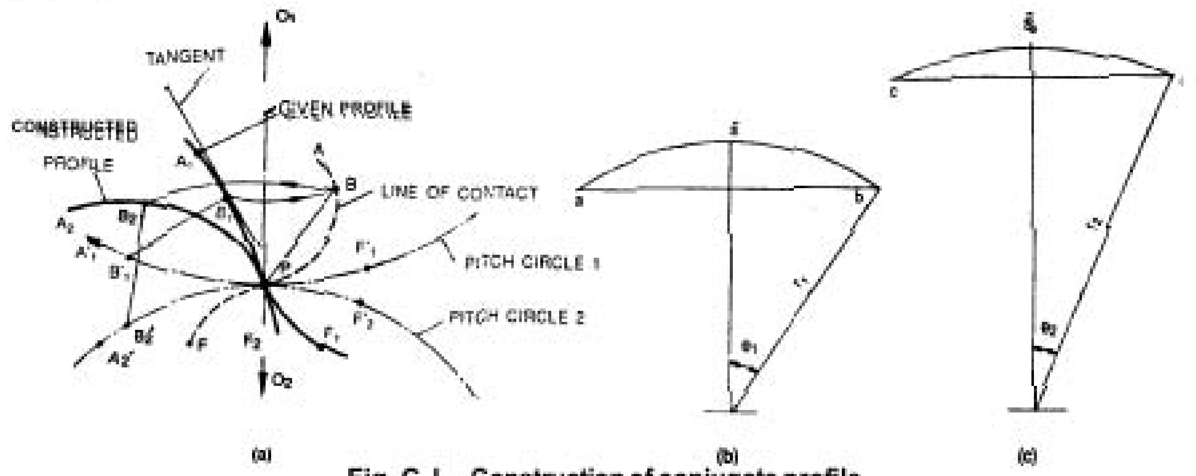


Fig. C.I Construction of conjugate profile

To summarize, B is the original point of contact of the two conjugate profiles. After mutual rotation, the separated points on the two profiles now occupy the positions B_1 and B_2. When in contact at B, the common normal to the two profiles passing through the pitch point is BP.

In a similar manner, the complete conjugate profile A_2F, can be constructed point by point. The curved line AF is the line of contact, i.e. the line on which all the points of contact at different stages lie.

It becomes sometimes difficult to lay equal arcs when the radii are different. A practical and more or less accurate method is described below with the aid of Fig. C.1 (b and c).

Arc $\hat{s}$ of Fig. C.1(b) is given. It is required to lay an arc equal in length on the circle in Fig. C.1(c).

Now, we join a with b to get the chord ab. Bisect ab. The radii r_1 and r_2 are known and ab can be measured off by a suitable scale.

$$\hat{s} = r_1 \times 2\theta_1 = r_2 \times 2\theta_2$$

or

$$\theta_2 = \frac{r_1}{r_2}\,\theta_2 \qquad \text{(all angles in radians)}$$

$$\frac{ab}{2} = r_1 \sin\theta_1 \quad \text{or } ab = 2r_1 \sin\theta_1$$

Therefore

$$\sin\theta_1 = \frac{ab}{2r_1} \quad \text{or } \theta_1 = \sin^{-1}\frac{ab}{2r_1}$$

Now, to find cd, we use the relations

$$\frac{cd}{2} = r_2 \sin\theta_2 \quad \text{or } cd = 2r_2 \sin\theta_2 = 2r_2 \sin\left(\frac{r_1}{r_2}\theta_1\right)$$

Hence

$$cd = 2r_2 \sin\left[\frac{r_1}{r_2} \sin^{-1}\frac{ab}{2r_1}\right]$$

Since, as stated earlier, r_1 and r_2 are known and the chord $\overline{ab}$ can be measured, the chord $\overline{cd}$ can be calculated. If this straight length cd is cut off on the circle in Fig. C.1(c), then the arc belonging to the chord $\overline{cd}$ is equal to $\bar{s}$.

APPENDIX D

Basic Dimensions of Standard Gear Tooth

(Basic Rack as per IS: 2535)

All dimensions in mm

Module	*Equivalent DP*	*Circular Pitch*	*Addendum*	*Dedendum*	*Whole depth*
0.30	84.667	0.943	0.30	0.375	0.675
0.40	63.500	1.257	0.40	0.500	0.900
0.50	50.800	1.571	0.50	0.625	0.125
0.60	42.300	1.885	0.60	0.750	1.350
0.70	36.286	2.199	0.70	0.875	1.575
0.80	31.750	2.513	0.80	1.000	1.800
0.90	28.222	2.827	0.90	1.125	2.025
1.00	25.400	3.142	1.00	1.250	2.250
1.25	20.320	3.927	1.25	1.562	2.812
1.50	16.933	4.712	1.50	1.875	3.375
1.75	14.514	5.498	1.75	2.188	3.938
2.00	12.700	6.283	2.00	2.500	4.500
2.25	11.289	7.069	2.25	2.812	5.062
2.50	10.160	7.854	2.50	3.125	5.625
2.75	9.236	8.639	2.75	3.438	6.188
3.00	8.466	9.425	3.00	3.750	6.750
3.25	7.816	10.218	3.25	4.062	7.312
3.50	7.257	10.996	3.50	4.375	7.875
3.75	6.773	11.781	3.75	4.688	8.438
4.00	6.350	12.566	4.00	5.000	9.000
4.50	5.644	14.137	4.50	5.625	10.125

(Contd.)

(Contd)

Module	Equivalent DP	Circular Pitch	Addendum	Dedendum	Whole depth
5.00	5.080	15.708	5.00	6.250	11.250
5.50	4.618	17.279	5.50	6.875	12.375
6.00	4.233	18.850	6.00	7.500	13.500
6.50	3.008	20.428	6.50	8.125	14.625
7.00	3.628	21.991	7.00	8.750	15.750
8.00	3.175	25.132	8.00	10.000	18.000
9.00	2.822	28.274	9.00	11.250	20.250
10.00	2.540	31.416	10.00	12.500	22.500
11.00	2.309	34.558	11.00	13.750	24.750
12.00	2.117	37.699	12.00	15.000	27.000
13.00	1.954	40.841	13.00	16.250	29.250
14.00	1.814	43.982	14.00	17.500	31.500
15.00	1.603	47.124	15.00	18.750	33.750
16.00	1.587	50.266	16.00	20.000	36.000
18.00	1.411	56.549	18.00	22.500	40.500
20.00	1.270	62.832	20.00	25.000	45.000
22.00	1.155	69.115	22.00	27.500	49.500
24.00	1.058	75.398	24.00	30.000	54.000
27.00	0.941	84.823	27.00	33.750	60.750
30.00	0.847	94.248	30.00	37.500	67.500
33.00	0.770	103.673	33.00	41.250	74.250
36.00	0.706	113.097	36.00	45.000	81.000
39.00	0.651	122.522	39.00	48.750	87.750
42.00	0.605	131.947	42.00	52.500	94.500
45.00	0.564	141.372	45.00	56.250	101.250
50.00	0.508	157.080	50.00	62.500	112.500
55.00	0.462	172.788	55.00	68.750	123.750
60.00	0.423	188.486	60.00	75.000	135.000
65.00	0.391	204.204	65.00	81.250	146.250
70.00	0.363	219.911	70.00	87.500	157.500
75.00	0.339	235.619	75.00	93.750	168.750

Note: Equivalent DP is given fir comparison only

APPENDIX E

Gear Materials

Sl. No.	Material	IS: No.	IS: Symbol (new)	IS: Symbol (old)	Typical uses
1 2	Cast Iron	210	FG 200 FG 260	Grade 20 Grade 25	Gears of light duty work, low speed (up to 0.8 m/s), hand-operated winches, etc.
3 4	Cast iron with spheroidal graphite	1865	SG 50017 SG 600/3		Gears with increased load capacity and resistance to contact stress
5 6	Cast steel	1030	Grade: 26-52 Grade: 30-57	Grade 2 Grade 1	Medium speed gears (up to 8 m/s), cranewheelgears. crane cross travel and long travel gears
7 8 9 10	Standard or structural steel	1570	Fe 410 Fe 490 Fe 620 Fe 690	St 42 St 50 St 63 ST 70	For general purpose gears
11 12 13 14 15	Heat treatable steel	1570	45 C8 60 C4	C 45 C 60 55Cr 70 37 Si 2 Mn 90 40 Cr 1Mo 28	Low speed, light duty gears Medium speed, medium duty gears. Gears for gear oil pumps. Gears for conventional gear boxes. Cross and long travelgearsfor cranes. Medium to high speed gears, machine tool gear boxes, etc.
16 17 18 19	Case carburising steel	1570	10 c4 15 C4	C10 C15 17 Mn 1 Cr 95 20 Mn Cr l	Heavy duty transmission gears General purpose gear boxes where hardened gear surface is imperative. Hoisting, crane, timing gears involving medium to heavy duty

(Contd)

(Contd)

Sl. No.	Material	IS: No.	IS: Symbol *(new)*	*IS:Symbol (old)*	Typical uses
20 21 22	Flame or induction hardening steel	1570	45 C 8 40Cr 4	C 45 40 Cr 1 37 Si 2 Mn 90	For gears of heavy duty For gears with high surface hardness and where the stresses are high and exacting
23 24	Steel hardened in cynide bath	1570	40Cr 4	40 Cr l 37 Si 2 Min 90	Gears for superficially hard tooth surfaces

Sl. No.	Ult.ten St. σ_{ut}	Yield st. $\sigma_y(0.2)$	End limit σ_e	Perm. Ben. St. σ_{bp}	Sur. fat. Str. p_{sc}	Br. Hardness HB	Heat Treatment
	Average values (N/mm²)						
1	200	—	90	35	295	1670	—
2	260	—	115	45	375	2060	—
3	500	320	—	120	470	1960	
4	600	370	—	150	550	2260	
5	520	260	205	90	365	1470	Annealed
6	570	300	235	110	490	1720	
7	410	250	220	110	335	1230	
8	490	290	255	120	390	1470	
9	620	380	305	125	440	1770	Annealed
10	690	410	370	160	510	2040	
11	600	380	320	130	490	1810	
12	700	430	370	150	600	2060	Quenched
13	900	650	400	170	745	2550	hardened and
14			375	190	640	2160	tempered
15	700	500	500	185	745	3340	
16	500	300	260	130	1630	5790	
17	500	300	270	155	1730	6250	Hardened
18	800	590		250	1790	6380	
19	980	685		260	1790	6380	
20	640	390		180	1640	5840	
21	880	640		200	1620	5760	Hardened
22	880	640			1550	5490	
23	1370				1640	5840	Hardened
24	1470	980			1520	5400	

Notes:
1. In most *cases*, the strength values given are the average ones and are for guidance only. In some cases, where a range is specified in the relevant codes from which the strength values have been taken, the lower values are given here for safety.
2. Values of σ_e, σ_{bp}, p_{sc} and HB for Indian steels are mainly based on those of equivalent or near-equivalent foreign steels. This is due to lack of sufficient experimental data on Indian steels. These values, however, can be used for calculation.
3. Except surface fatigue strength (p_{sc}), the strength values in case of case-hardened steels refer to core properties. For case-hardened steels, the values of p_{sc} and HB shown here are typical values obtained after surface hardening.
4. The values of permissible bending stress σ_{bp} are guiding values only. They can be changed, depending on loading conditions.

APPENDIX F

Table of Contents Ratios (CR) of Spur Gears in Mesh

Based on Grundzuege der Verzahnung, Thomas, 1957 Edition, table no. 11.10, p.247.
Carl Hanser Verlag, Munich.

Addendum = m, pressure angle = 20°

Z_2	Z_1				
	12	13	14	15	16
12	1.095				
13	1.181	1.269			
14	1.257	1.355	1.442		
15			1.452	1.481	
16				1.490	1.498
17				1.497	1.506
18				1.505	1.514
19				1.512	1.521
20				1.519	1.527
21				1.525	1.533
22				1.531	1.539
23				1.536	1.545
24				1.542	1.550
25				1.547	1.555
26				1.548	1.559
27					1.564
28					1.568
29					1.572
30					1.576
∞	1.257	1.355	1.452	1.548	1.643

Z_2	Z_1								
	17	18	19	20	25	30	50	100	200
17	1.514								
18	1.522	1.529							
19	1.529	1.536	1.543						
20	1.535	1.542	1.549	1.556					
21	1.541	1.548	1.555	1.562					
22	1.547	1.554	1.561	1.568					
23	1.553	1.560	1.567	1.574					
24	1.558	1.565	1.572	1.579					
25	1.563	1.570	1.577	1.584	1.611				
26	1.567	1.574	1.581	1.588	1.615				
27	1.572	1.579	1.586	1.593	1.620				
28	1.576	1.583	1.590	1.597	1.624				
29	1.580	1.587	1.594	1.601	1.628				
30	1.584	1.591	1.598	1.605	1.632	1.656			
40	1.616	1.622	1.629	1.636	1.664	1.686			
50	1.637	1.643	1.650	1.657	1.685	1.707	1.758		
100	1.683	1.689	1.696	1.703	1.731	1.753	1.804	1.850	
200	1.714	1.720	1.727	1.734	1.762	1.784	1.835	1.881	1.912
500	1.733	1.739	1.746	1.753	1.781	1.803	1.854	1.900	1.931
∞	1.748	1.754	1.761	1.768	1.796	1.818	1.867	1.916	1.946

APPENDIX G

Tables of Trigonometrical Functions

Sine 0′ ....45′

Deg.	0′	10′	20′	30′	40′	50′	60′	
	Minutes for sine							
0	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145	0.0175	89
1	0.0175	0.0204	0.0233	0.0262	0.0291	0.0320	0.0349	88
2	0.0349	0.0378	0.0407	0.0436	0.0465	0.0494	0.0523	87
3	0.0523	0.0552	0.0581	0.0610	0.0640	0.0669	9.0698	86
4	0.0698	0.0727	0.0756	0.0785	0.0814	0.0843	0.0872	85
5	0.0872	0.0901	0.0929	0.0958	0.0987	0.1016	0.1045	84
6	0.1045	0.1074	0.1103	0.1132	0.1161	0.1190	0.1219	83
7	0.1219	0.1248	0.1276	0.1305	0.1334	0.1363	0.1392	82
8	0.1392	0.1421	0.1449	0.1478	0.1507	0.1536	0.1564	81
9	0.1564	0.1593	0.1622	0.1650	0.1679	0.1708	0.1736	80
10	0.1736	0.1765	0.1794	0.1822	0.1851	0.1880	0.1908	79
11	0.1908	0.1937	0.1965	0.1994	0.2022	0.2051	0.2079	78
12	0.2079	0.2108	0.2136	0.2164	0.2193	0.2221	0.2250	77
13	0.2250	0.2278	0.2306	0.2334	0.2363	0.2391	0.2419	76
14	0.2419	0.2447	0.2476	0.2504	0.2532	0.2560	0.2588	75
15	0.2588	0.2616	0.2644	0.2672	0.2700	0.2728	0.2756	74
16	0.2756	0.2784	0.2812	0.2840	0.2868	0.2896	0.2924	73
17	0.2924	0.2952	0.2979	0.3007	0.3035	0.3062	0.3090	72
18	0.3090	0.3118	0.3145	0.3173	0.3201	0.3228	0.3256	71
19	0.3256	0.3283	0.3311	0.3338	0.3365	0.3393	0.3420	70
20	0.3420	0.3448	0.3475	0.3502	0.3529	0.3557	0.3584	69
21	0.3584	0.3611	0.3638	0.3665	0.3692	0.3719	0.3746	68
22	0.3746	0.3773	0.3800	0.3827	0.3854	0.3881	0.3907	67
23	0.3907	0.3934	0.3961	0.3987	0.4014	0.4041	0.4067	66
24	0.4067	0.4094	0.4120	0.4147	0.4173	0.4200	0.4226	65
25	0.4226	0.4253	0.4279	0.4305	0.4331	0.4358	0.4384	64
26	0.4384	0.4410	0.4436	0.4432	0.4488	0.4514	0.4540	63
27	0.4540	0.4566	0.4592	0.4617	0.4643	0.4669	0.4695	62
28	0.4695	0.4720	0.4746	0.4772	0.4797	0.4823	0.4848	61
29	0.4848	0.4874	0.4899	0.4924	0.4950	0.4975	0.5000	60

(Contd.)

(Contd)

Sine 0' ....45'

Deg.	0'	10'	20'	30'	40'	50'	60'	
				Minutes for sine				
30	0.5000	0.5025	0.5050	0.5075	0.5100	0.5125	0.5150	59
31	0.5150	0.5175	0.5200	0.5225	0.5250	0.5275	0.5299	58
32	0.5299	0.5324	0.5348	0.5373	0.5398	0.5422	0.5446	57
33	0.5446	0.5471	0.5495	0.5519	0.5544	0.5568	0.5592	56
34	0.5592	0.5616	0.5640	0.5664	0.5688	0.5712	0.5736	55
35	0.5736	0.5760	0.5783	0.5807	0.5831	0.5854	0.5878	54
36	0.5878	0.5901	0.5925	0.5948	0.5972	0.5995	0.6018	53
37	0.6018	0.6041	0.6065	0.6088	0.6111	0.6134	0.6157	52
38	0.6157	0.6180	0.6202	0.6225	0.6248	0.6271	0.6293	51
39	0.6293	0.6316	0.6338	0.6361	0.6383	0.6406	0.6428	50
40	0.6428	0.6450	0.6472	0.6494	0.6517	0.6539	0.6561	49
41	0.6561	0.6583	0.6604	0.6626	0.6648	0.6670	0.6691	48
42	0.6691	0.6713	0.6734	0.6756	0.6777	0.6799	0.6820	47
43	0.6820	0.6841	0.6862	0.6884	0.6905	0.6926	0.6947	46
44	0.6947	0.6967	0.6988	0.7009	0.7030	0.7050	0.7071	45
	60'	50'	40'	30'	20'	10'	0'	Deg.
				Minutes for cosine				

Cosine 45'..90'

Sine 45' ....90'

Deg.	0'	10'	20'	30'	40'	50'	60'	
				Minutes for sine				
45	0.7071	0.7092	0.7112	0.7133	0.7153	0.7173	0.7193	44
46	0.7193	0.7214	0.7234	0.7254	0.7274	0.7294	0.7314	43
47	0.7314	0.7333	0.7353	0.7373	0.7392	0.7412	7.7431	42
48	0.7431	0.7451	0.7470	0.7490	0.7509	0.7528	0.7547	41
49	0.7547	0.7566	0.7585	0.7604	0.7623	0.7642	0.7660	40
50	0.7660	0.7679	0.7698	0.7716	0.7735	0.7753	0.7771	39
51	0.7771	0.7790	0.7808	0.7826	0.7844	0.7862	0.7880	38
52	0.7880	0.7898	0.7916	0.7934	0.7951	0.7969	0.7986	37
53	0.7986	0.8004	0.8021	0.8039	0.8056	0.8073	0.8090	36
54	0.8090	0.8107	0.8124	0.8141	0.8158	0.8175	0.8192	35
55	0.8192	0.8208	0.8225	0.8241	0.8258	0.8274	0.8290	34
56	0.8290	0.8307	0.8323	0.8339	0.8355	0.8371	0.8387	33
57	0.8387	0.8403	0.8418	0.8434	0.8450	0.8465	0.8480	32
58	0.8480	0.8496	0.8511	0.8526	0.8542	0.8557	0.8572	31
59	0.8572	0.8587	0.8601	0.8616	0.8631	0.8646	0.8660	30

(Contd)

(Contd)

Sine 45° ....90°

Deg.	0'	10'	20'	30'	40'	50'	60'	
	Minutes for sine							
60	0.8660	0.8675	0.8689	0.8704	0.8718	0.8732	0.8746	29
61	0.8746	0.8760	0.8774	0.8788	0.8802	0.8816	0.8829	28
62	0.8829	0.8843	0.8857	0.8700	0.8884	0.8897	0.8910	27
63	0.8910	0.8923	0.8936	0.8949	0.8962	0.8975	0.8988	26
64	0.8988	0.9001	0.9013	0.9026	0.9038	0.9051	0.9063	25
65	0.9063	0.9075	0.9088	0.9100	0.9112	0.9124	0.9135	24
66	0.9135	0.9147	0.9159	0.9171	0.9182	0.9194	0.9205	23
67	0.9205	0.9216	0.9228	0.9239	0.9250	0.9261	0.9272	22
68	0.9272	0.9283	0.9293	0.9304	0.9315	0.9325	0.9336	21
69	0.9336	0.9346	0.9356	0.9367	0.9377	0.9387	0.9397	20
70	0.9397	0.9407	0.9417	0.9426	0.9436	0.9446	0.9455	19
71	0.9455	0.9465	0.9474	0.9483	0.9492	0.9502	0.9511	18
72	0.9511	0.9520	0.9528	0.9537	0.9546	0.9555	0.9563	17
73	0.9563	0.9572	0.9580	0.9588	0.9596	0.9605	0.9613	16
74	0.9613	0.9621	0.9628	0.9636	0.9644	0.9652	0.9659	15
75	0.9659	0.9667	0.9674	0.9681	0.9689	0.9696	0.9703	14
76	0.9703	0.9710	0.9717	0.9724	0.9730	0.9737	0.9744	13
77	0.9744	0.9750	0.9757	0.9763	0.9769	0.9775	0.9781	12
78	0.9781	0.9787	0.9793	0.9799	0.9805	0.9811	0.9816	11
79	0.9816	0.9822	0.9827	0.9833	0.9838	0.9843	0.9848	10
80	0.9848	0.9853	0.9858	0.9863	0.9868	0.9872	0.9877	9
81	0.9877	0.9881	0.9886	0.9890	0.9894	0.9899	0.9903	8
82	0.9903	0.9907	0.9911	0.9914	0.9918	0.9922	0.9925	7
83	0.9925	0.9929	0.9932	0.9936	0.9939	0.9942	0.9945	6
84	0.9945	0.9948	0.9951	0.9954	0.9957	0.9959	0.9962	5
85	0.9962	0.9964	0.9967	0.9969	0.9971	0.9974	0.9976	4
86	0.9976	0.9978	0.9980	0.9981	0.9983	0.9985	0.9986	3
87	0.9986	0.9988	0.9989	0.9990	0.9992	0.9993	0.9994	2
88	0.9994	0.9995	0.9996	0.9997	0.9997	0.9998	0.99985	1
89	0.99985	0.99989	0.99993	0.99996	0.99998	0.99999	1.0000	0
		60'	50'	30'	20	10'	0'	Deg.
		Minutes for cosine						

Cosine 0°...45°

(Contd)

Tangent 0' ...45'

Deg.	Minutes for tangent 0'	10'	20'	30'	40'	50'	60'	
0	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145	0.0175	89
1	0.0175	0.0204	0.0233	0.0262	0.0291	0.0320	0.0349	88
2	0.0349	0.0378	0.0407	0.0437	0.0466	0.0495	0.0524	87
3	0.0524	0.0553	0.0582	0.0612	0.0641	0.0670	0.0699	86
4	0.0699	0.0729	0.0758	0.0787	0.0816	0.0846	0.0875	85
5	0.0875	0.0904	0.0934	0.0963	0.0992	0.1022	0.1051	84
6	0.1051	0.1080	0.1110	0.1139	0.1169	0.1198	0.1228	83
7	0.1228	0.1257	0.1287	0.1317	0.1346	0.1376	0.1405	82
8	0.1405	0.1435	0.1465	0.1495	0.1524	0.1554	0.1584	81
9	0.1584	0.1614	0.1644	0.1673	0.1703	0.1733	0.1763	80
10	0.1763	0.1793	0.1823	0.1853	0.1883	0.1914	0.1944	79
11	0.1944	0.1974	0.2004	0.2035	0.2065	0.2095	0.2126	78
12	0.2126	0.2156	0.2186	0.2217	0.2247	0.2278	0.2309	77
13	0.2309	0.2339	0.2370	0.2401	0.2432	0.2462	0.2493	76
14	0.2493	0.2524	0.2555	0.2586	0.2617	0.2648	0.2679	75
15	0.2679	0.2711	0.2742	0.2773	0.2805	0.2836	0.2867	74
16	0.2867	0.2899	0.2931	0.2962	0.2994	0.3026	0.3057	73
17	0.3057	0.3089	0.3121	0.3153	0.3185	0.3217	0.3249	72
18	0.3249	0.3281	0.3314	0.3346	0.3378	0.3411	0.3443	71
19	0.3443	0.3476	0.3508	0.3541	0.3574	0.3607	0.3640	70
20	0.3640	0.3673	0.3706	0.3739	0.3772	0.3805	0.3839	69
21	0.3839	0.3872	0.3906	0.3939	0.3973	0.4006	0.4040	68
22	0.4040	0.4074	0.4108	0.4142	0.4176	0.4210	0.4245	67
23	0.4245	0.4279	0.4314	0.4348	0.4383	0.4417	0.4452	66
24	0.4452	0.4487	0.4522	0.4557	0.4592	0.4628	0.4663	65
25	0.4663	0.4699	0.4734	0.4770	0.4806	0.4841	0.4877	64
26	0.4877	0.4913	0.4950	0.4986	0.5022	0.5059	0.5095	63
27	0.5095	0.5132	0.5169	0.5206	0.5243	0.5280	0.5317	62
28	0.5317	0.5354	0.5392	0.5430	0.5467	0.5505	0.5543	61
29	0.5543	0.5581	0.5619	0.5658	0.5696	0.5735	0.5774	60
30	0.5774	0.5812	0.5851	0.5890	0.5930	0.5969	0.6009	59
31	0.6009	0.6048	0.6088	0.6128	0.6168	0.6208	0.6249	58
32	0.6249	0.6289	0.6330	0.6371	0.6412	0.6453	0.6494	57
33	0.6494	0.6536	0.6577	0.6619	0.6661	0.6703	0.6745	56
34	0.6745	0.6787	0.6830	0.6873	0.6916	0.6959	0.7002	55
35	0 7002	0.7046	0.7089	0.7133	0.7177	0.7221	0.7265	54
36	0.7265	o 7310	0.7355	0.7400	0.7445	0.7490	0.7536	53
37	0.7536	o 7581	07627	07673	0.7720	0.7766	0.7813	52
38	o 7813	o 7860	07907	07954	0.8002	0.8050	0.8098	51
39	o 8098	o 8146	08195	08243	0.8292	0.8342	0.8391	50

(Contd)

(Contd)

Tangent 45'….90'

Deg.	0'	10'	20'	30'	40'	50'	60'	
	Minutes for tangent							
40	0.8391	0.8441	0.8491	0.0541	0.0591	0.8642	0.8693	49
41	0.8693	0.8744	0.8796	0.8847	0.8899	0.8952	0.9004	48
42	0.9004	0.9057	0.9110	0.9163	0.9217	0.9271	0.9325	47
43	0.9325	0.8380	0.9435	0.9490	0.9545	0.9601	0.9657	46
44	0.9657	0.9713	0.9770	0.9827	0.9884	0.9942	1.0000	45
	60'	50'	40'	30'	20'	10'	0'	Deg.
	Minutes for cotangent							

Cotangent 45'. .90'

Tangent 45'….90'

Deg.	0'	10'	20'	30'	40'	50'	60'	
	Minutes for tangent							
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0416	1.0477	1.0538	1.0599	1.0661	1.0724	43
47	1.0724	1.0786	1.0850	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1237	1.1303	1.1369	1.1436	1.1504	41
49	1.1504	1.1571	1.1640	1.1708	1.1778	1.1847	1.1918	40
50	1.1918	1.1988	1.2059	1.2131	1.2203	1.2276	1.2349	39
51	1.2349	1.2423	1.2497	1.2572	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3597	1.3680	1.3764	36
54	1.3764	1.3848	1.3934	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4460	1.4550	1.4641	1.4733	1.4826	34
56	1.4826	1.4919	1.5013	1.5108	1.5204	1.5301	1.5399	33
57	1.5399	1.5497	1.5597	1.5697	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6213	1.6318	1.6426	1.6534	1.6643	31
59	1.6643	1.6753	1.6864	1.6977	1.7090	1.7205	1.7321	30
60	1.7321	1.7438	1.7556	1.7675	1.7796	1.7917	1.8041	29
61	1.8041	1.8165	1.8291	1.8418	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9210	1.9347	1.9486	1.9626	27
63	1.9626	1.9768	1.9912	2.0057	2.0204	2.0353	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1775	2.1943	2.2113	2.2286	2.2460	24
66	2.2460	2.2637	2.2817	2.2998	2.3183	2.3369	2.3559	23
67	2.3559	2.3750	2.3945	2.4142	2.4342	2.4545	2.4751	22
68	2.4751	2.4960	2.5172	2.5387	2.5605	2.5826	2.6051	21
69	2.6051	2.6279	2.6511	2.6746	2.6985	2.7228	2.7475	20

(Contd)

(Contd)

Tangent 45° ....90°

Deg.	0'	10'	20'	30'	40'	50'	60'	
	Minutes for tangent							
70	2.7475	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9319	2.9600	2.9887	3.0178	3.0475	3.0777	18
72	3.0777	3.1084	3.1397	3.1716	3.2041	3.2371	3.2709	17
73	3.2709	3.3052	3.3402	3.3759	3.4124	3.4495	3.4874	16
74	3.4874	3.5261	3.5656	3.6059	3.6470	3.6891	3.7321	15
75	3.7321	3.7760	3.8208	3.8667	3.9136	3.9617	4.0108	14
76	4.0100	4.0611	4.1126	4.1653	4.2193	4.2747	3.3315	13
77	4.3315	4.3897	4.4494	4.5107	4.5736	4.6383	4.7046	12
78	4.7046	4.7729	4.8430	4.9152	4.9894	5.0658	5.1446	11
79	5.1446	5.2257	5.3093	5.3955	5.4845	5.5764	5.6713	10
80	5.6713	5.7694	5.8708	5.9758	6.0844	6.1970	6.3138	9
81	6.3138	6.4348	6.5605	6.6912	6.8269	6.9682	7.1154	8
82	7.1154	7.2687	7.4287	7.5958	7.7704	7.9530	8.1444	7
83	8.1444	8.3450	8.5556	8.7769	9.0098	9.2553	9.5144	6
84	9.5144	9.7882	10.0780	10.3854	10.7119	11.0594	11.4301	5
85	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267	14.3007	4
86	14.3007	14.9244	15.6048	16.3499	17.1693	18.0750	19.0811	3
87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316	28.6363	2
88	28.6363	31.2416	34.3678	38.1885	42.9641	49.1039	57.2900	1
89	57.2900	68.7501	85.9398	114.5887	171.8854	343.7737	∞	0
	60'	50'	40'	30'	20'	10'	0'	Deg
	Minutes for cotangent							

Cotangent 0° ... 45°

APPENDIX H

Involute Functions

Angle α	inv α = tan α − α (radians)				
Degrees / Minutes	10	11	12	13	14
0	0.00179 41	0.00239 41	0.00311 71	0.00397 54	0.00498 19
1	0.00180 31	0.00240 51	3 02	9 09	0.00500 00
2	1 22	1 61	4 34	0.00400 65	1 82
3	2 13	2 72	5 67	2 21	3 64
4	3 05	3 83	6 99	3 77	5 46
5	3 97	4 95	8 32	5 34	7 29
6	4 89	6 07	9 66	6 92	9 12
7	5 81	7 19	0.00321 00	8 49	0.00510 96
8	6 74	8 31	2 34	0.00410 08	2 80
9	7 67	9 44	3 69	1 66	4 65
10	8 60	0.00250 57	5 04	3 25	6 50
11	0 00189 54	0 00251 71	0 00326 39	0 00414 85	0.00518 35
12	0 00190 48	2 85	7 75	6 44	0.00520 22
13	1 42	3 99	9 11	8 05	2 08
14	2 37	5 13	0 00330 40	9 65	3 95
15	3 32	6 28	1 85	0 00421 26	5 82
16	4 27	7 44	3 22	2 88	7 70
17	5 23	8 59	4 60	4 50	9 58
18	6 19	9 75	5 98	6 12	0 00531 47
19	7 15	0 00260 91	7 36	7 75	3 36
20	8 12	2 08	8 75	9 38	5 26
21	0 00199 09	0 00263 25	0 00340 14	0.00431 02	0.00537 16
22	0 00200 06	4 43	1 54	2 66	9 07
23	1 03	5 60	2 94	4 30	0.00540 98
24	2 01	6 78	4 34	5 95	2 90
25	2 99	7 97	5 75	7 60	4 82
26	3 98	9 16	7 16	9 26	6 74
27	4 97	0 00270 35	8 58	0 00440 92	8 67
28	5 96	1 54	0 00350 00	2 59	0 00550 60
29	6 95	2 74	1 42	4 26	2 54
30	7 95	3 94	2 85	5 93	4 48

(Contd)

(Contd)

Angle a	inv a = tan α − a (radians)				
Degrees / *Minutes*	10	11	12	13	14
31	0.00208 95	0.00275 15	0.00354 28	0.00447 61	0.00556 43
32	9 95	6 36	5 72	9 29	8 38
33	0.00210 96	7 57	7 16	0.00450 98	0.00560 34
34	1 97	8 79	8 60	2 67	2 30
35	2 99	0.00280 01	0.00360 05	4 37	4 27
36	4 00	1 23	1 50	6 07	6 24
37	5 02	2 46	2 96	7 77	8 22
38	6 05	3 69	4 41	9 48	0.00570 20
39	7 07	4 93	5 88	0.00461 20	2 18
40	8 10	6 16	7 35	2 91	4 17
41	0.00219 14	0.00287 41	0.00368 82	0.00464 64	0.00576 17
42	0.00220 17	8 65	0.00370 29	6 36	8 17
43	1 21	9 90	1 77	8 09	0.00580 17
44	2 26	0.00291 15	3 26	9 83	2 18
45	3 30	2 41	4 74	0.00471 57	4 20
46	4 35	3 67	6 23	3 31	6 22
47	5 41	4 94	7 73	5 06	8 24
48	6 47	6 20	9 23	6 81	0.00590 28
49	7 53	7 47	0.00380 73	8 57	2 30
50	8 59	8 75	22 24	0.00480 33	4 34
51	0.00229 66	0.00300 03	0.00383 75	0.00482 10	0.00596 38
52	0.00230 73	1 31	5 27	3 87	8 43
53	1 80	2 60	6 79	5 64	0.00600 48
54	2 88	3 89	8 31	7 42	2 54
55	3 96	5 18	9 84	9 21	4 60
56	5 04	6 48	0.00391 37	0.00490 99	6 67
57	6 13	7 78	2 91	2 79	8 74
58	7 22	9 08	4 45	4 58	0.00610 81
59	8 31	0.00310 39	5 99	6 39	2 89
60	0.00239 41	0.00311 71	0.00397 54	0.00498 19	0.00614 98

Angle a	inv α = tan α − a (radians)				
Degrees / *Minutes*	15	16	17	18	19
0	0.00614 98	0.00749 3	0.00902 5	0.01076 0	0.01271 5
1	7 07	51 7	05 2	79 1	75 0
2	9 17	54 1	07 9	82 2	78 4
3	0.00621 27	56 5	10 7	85 3	81 9
4	3 37	58 9	13 4	88 4	85 4
5	5 48	61 3	16 1	91 5	88 8

(Contd)

(Contd)

Angle a	inv a = tan a − a (radians)				
Degrees / *Minutes*	15	16	17	18	19
6	7 60	63 7	18 9	94 6	92 3
7	9 72	66 1	21 6	97 7	95 8
8	0.00631 84	68 6	24 4	0.01100 8	99 3
9	3 97	71 0	27 2	03 9	0.01302 8
10	6 11	73 5	29 9	07 1	06 3
11	0.00638 25	0.00775 9	0.00932 7	0.01110 2	0.01309 8
12	0.00640 39	78 4	35 5	13 3	13 4
13	2 54	80 8	38 3	16 5	16 9
14	4 70	83 3	41 1	19 6	20 4
15	6 86	85 7	43 9	22 8	24 0
16	9 02	88 2	46 7	26 0	27 5
17	0.00651 19	90 7	49 5	29 1	31 1
18	3 37	93 2	52 3	32 3	34 6
19	5 55	95 7	55 2	35 5	38 2
20	7 73	98 2	58 0	38 7	41 8
21	0.00659 92	0.00800 7	0.00960 8	0.01141 9	0.01345 4
22	0.00662 11	03 2	63 7	45 1	49 0
23	4 31	05 7	66 5	48 3	52 6
24	6 52	08 2	69 4	51 5	56 2
25	8 73	10 7	72 2	54 7	59 8
26	0.00670 94	13 3	75 1	58 0	63 4
27	3 16	15 8	78 0	61 2	67 0
28	5 39	18 3	80 8	64 4	70 7
29	7 62	20 9	83 7	67 7	74 3
30	9 85	23 4	86 6	70 9	77 9
31	0.00682 09	0.00826 0	0.00989 5	0.01174 2	0.01381 6
32	4 34	28 5	92 4	77 5	85 2
33	6 59	31 1	95 3	80 7	88 9
34	8 84	33 7	98 2	84 0	92 6
35	0.00691 10	36 2	0.01001 2	87 3	96 3
36	3 37	38 8	04 1	90 6	99 9
37	5 64	41 4	07 0	93 9	0.01403 6
38	7 91	44 0	09 9	97 2	07 3
39	0.00700 19	46 6	12 9	0.01200 5	11 0
40	2 48	49 2	15 8	03 8	14 8
41	0.00704 77	0.00851 8	0.01018 8	0.01207 1	0.01418 5
42	7 06	54 4	21 7	10 5	22 2
43	9 36	57 1	24 7	13 8	25 9
44	0.00711 67	59 7	27 7	17 2	29 7
45	3 98	62 3	30 7	20 5	33 4
46	2 30	65 0	33 6	23 9	37 2
47	8 62	67 6	36 6	27 2	40 9
48	0.00720 95	70 2	39 6	30 6	44 7
49	3 28	72 9	42 6	34 0	48 5
50	5 61	75 6	45 6	37 3	52 3

(Contd)

(Contd)

Angle a	inv *a* = tan *a* − *a* (radians)				
Degrees / *Minutes*	15	16	17	18	19
51	0.00727 96	0.00878 2	0.01048 6	0.01240 7	0.01456 0
52	0.00730 30	80 9	51 7	44 1	59 8
53	2 66	83 6	54 7	47 5	63 6
54	5 01	86 3	57 7	50 9	67 4
55	7 38	88 9	60 8	54 3	71 3
56	9 75	91 6	63 8	57 8	75 1
57	0.00742 12	94 3	66 9	61 2	78 9
58	4 50	97 0	69 9	64 6	82 7
59	6 88	99 8	73 0	68 1	86 6
60	0.00749 27	0.00902 5	0.010760	0.012715	0.014904

Angle a	inv *a* = tan *a* − *a* (radians)				
Degrees / *Minutes*	20	21	22	23	24
0	0.014904	0.017345	0.02005 4	0.023049	0.02635 0
1	94 3	38 8	10 1	10 2	40 7
2	98 2	43 1	14 9	15 4	46 5
3	0.015020	47 4	19 7	20 7	52 3
4	05 9	51 7	24 4	25 9	58 1
5	09 8	56 0	29 2	31 2	63 9
6	13 7	60 3	34 0	36 5	69 7
7	17 6	64 7	38 8	41 8	75 6
8	21 5	69 0	43 6	47 1	81 4
9	25 4	73 4	48 4	52 4	87 2
10	0.015293	0.017777	0.02053 3	0.023577	0.02693 1
11	33 3	82 1	58 1	63 1	98 9
12	37 2	86 5	62 9	68 4	0.027048
13	41 1	90 8	67 8	73 8	10 7
14	45 1	95 2	72 6	79 1	16 6
15	49 0	99 6	77 5	84 5	22 5
16	53 0	0.01804	82 4	89 9	28 4
17	57 0	08 4	87 3	95 2	34 3
18	60 9	12 9	92 1	0.024006	40 2
19	64 9	17 3	97 0	06 0	46 2
20	0.01568 9	0.01821 7	0.02101 9	0.02411 4	0.027521
21	72 9	26 2	06 9	16 9	58 1
22	76 9	30 6	11 8	22 3	64 1
23	80 9	35 1	16 7	27 7	70 0
24	85 0	39 5	21 7	33 2	76 0

(Contd)

(Contd)

Angle a	inv a = tan a − a (radians)				
Degrees / Minutes	20	21	22	23	24
25	89 0	440	266	386	82 0
26	93 0	485	31 6	441	88 0
27	97 1	53 0	365	49 5	940
28	0.01601 1	57 5	41 5	55 0	0.02800 0
29	05 2	62 0	46 5	605	06 0
30	0.01609 2	0.01866 5	0.02151 4	0.02466 0	0.02812 1
31	13 3	71 0	56 4	71 5	18 1
32	17 4	75 5	61 4	77 0	24 2
33	21 5	80 0	665	82 5	302
34	25 5	846	71 5	88 1	363
35	296	89 1	76 5	93 6	42 4
36	337	937	81 5	992	48 5
37	37 9	983	866	0.02504 7	54 6
38	42 0	0.01902 8	91 6	10 3	607
39	46 1	07 4	967	15 9	668
40	0.01650 2	0.01912 0	0.02201 8	0.02521 4	0.028729
41	54 4	16 6	068	27 0	79 1
42	58 5	21 2	11 9	32 6	85 2
43	62 7	25 8	170	382	91 4
44	66 9	30 4	22 1	43 9	97 6
45	7 . 0	350	27 2	49 5	0.02903 7
46	75 2	39 7	32 4	55 1	09 9
47	79 4	44 3	37 5	608	16 1
48	836	49 0	42 6	664	22 3
49	87 8	53 6	47 8	72 1	285
50	0.01692 0	0.01958 3	0.02252 9	0.02577 7	0.02934 8
51	962	630	58 1	034	41 0
52	0.01700 4	67 6	633	89 1	47 2
53	047	72 3	684	94 8	53 5
54	08 9	77 0	73 6	0.02600 5	59 8
55	13 2	81 7	78 8	062	660
56	17 4	864	84 0	12 0	72 3
57	21 7	91 2	892	17 7	78 6
58	25 9	959	944	235	849
59	30 2	0.02000 7	997	292	91 2
60	0.01734 5	0.02005 4	0.02304 9	0.02635 0	0.02997 5

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(Contd)

Angle a	inv a = tan a − a (radians)				
Degrees / Minutes	25	26	27	28	29
0	0.02997 5	0.03394 7	0.03828 7	0.04301 7	0.04816 4
1	0.03003 9	0.03401 6	36 2	10 0	25 3
2	10 2	08 6	43 8	18 2	34 3
3	16 6	15 5	51 4	26 4	43 2
4	22 9	22 5	59 0	34 7	52 2
5	29 3	29 4	66 6	43 0	61 2
6	35 7	36 4	74 2	51 3	70 2
7	42 0	43 4	81 8	59 6	79 2
8	48 4	50 4	89 4	67 9	88 3
9	54 9	57 4	97 1	76 2	97 3
10	61 3	64 4	0.03904 7	84 5	0.04906 4
11	0.03067 7	0.03471 4	0.03912 4	0.04392 9	0.04915 4
12	74 1	78 5	20 1	0.04401 2	24 5
13	80 6	85 5	27 8	09 6	33 6
14	87 0	92 6	35 5	18 0	42 7
15	93 5	99 7	43 2	26 4	51 8
16	0.03100 0	0.03506 7	50 9	34 8	60 9
17	06 5	13 8	58 6	43 2	70 1
18	13 0	20 9	66 4	51 6	79 2
19	19 5	28 0	74 1	60 1	88 4
20	26 0	35 2	81 9	68 5	97 6
21	0.03132 5	0.03542 3	0.03989 7	0.04477 0	0.05006 8
22	39 0	49 4	97 4	85 5	16 0
23	45 6	56 6	0.04005 2	93 9	25 2
24	52 1	63 7	13 1	0.04502 4	34 4
25	58 7	70 9	20 9	11 0	43 7
26	65 3	78 1	28 7	19 5	52 9
27	71 8	85 3	36 6	28 0	62 2
28	78 4	92 5	44 4	36 6	71 5
29	85 0	99 7	52 3	45 1	80 8
30	91 7	0.03606 9	60 2	53 7	90 1
31	0.03198 3	0.03614 2	0.04068 0	0.04562 3	0.05099 4
32	0.03204 9	21 4	75 9	70 9	0.05108 7
33	11 6	28 7	83 9	79 5	18 1
34	18 2	35 9	91 8	88 1	27 4
35	24 9	43 2	99 7	96 7	36 8
36	31 5	50 5	0.04107 6	0.04605 4	46 2
37	38 2	57 8	15 6	14 0	55 6
38	44 9	65 1	23 6	22 7	65 0
39	51 6	72 4	31 6	31 3	74 4
40	58 3	79 8	39 5	40 0	83 8

(Contd)

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Angle a	inv a = tan a − a (radians)				
Degrees / Minutes	25	26	27	28	29
41	0.03265 1	0.03687 1	0.04147 5	0.04648 7	0.05193 3
42	71 8	94 5	55 6	57 5	0.05202 7
43	78 5	0.03701 8	63 6	66 2	12 2
44	85 3	09 2	71 6	74 9	21 7
45	92 0	16 6	79 7	83 7	31 2
46	98 8	24 0	87 7	92 4	40 7
47	0.03305 6	31 4	95 8	0.04701 2	50 2
48	12 4	38 8	0.04203 9	10 0	59 7
49	19 2	46 2	12 0	18 8	69 3
50	26 0	53 7	20 1	27 6	78 8
51	0.03332 8	0.03761 1	0.04228 2	0.04736 4	0.05288 4
52	39 7	68 6	36 3	45 2	98 0
53	46 5	76 1	44 4	54 1	0.05307 6
54	53 4	83 5	52 6	63 0	17 2
55	60 2	91 0	60 7	71 8	26 8
56	67 1	98 5	68 9	80 7	36 5
57	74 0	0.03806 0	77 1	89 6	46 1
58	80 9	13 6	85 3	98 5	55 8
59	87 8	· 21 1	93 5	0.04807 4	65 5
60	0.03394 7	0.03828 7	0.04301 7	0.04816 4	0.05375 1

Angle α	inv a = tan a − a (radians)				
Degrees / Minutes	30	31	32	33	34
0	0.05373 1	0.05980 9	0.06636 4	0.07344 9	0.08109 7
1	84 9	91 4	47 8	57 2	22 9
2	94 6	0.06001 9	59 1	69 5	36 2
3	0.05404 3	12 4	70 5	81 8	49 4
4	14 0	23 0	81 9	94 1	62 7
5	23 8	33 5	93 4	0.07406 4	76 0
6	33 6	44 1	0.06704 8	18 8	89 4
7	43 3	54 7	16 3	31 2	0.08202 7
8	53 1	65 3	27 7	43 5	16 1
9	62 9	75 9	39 2	55 9	29 4
10	72 8	86 6	50 7	68 4	42 8
11	0.05482 6	0.06097 2	0.06762 2	0.074808	0.08256 2
12	92 4	0 06107 9	73 8	93 2	69 7
13	0.05502 3	18 6	85 3	0.07505 7	83 1
14	12 2	29 2	96 9	18 2	96 6
15	22 1	40 0	0.06808 4	30 7	0 08310 0
16	32 0	50 7	20 0	43 2	23 5

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Angle α	inv $\alpha = \tan\alpha - \alpha$ (radians)				
Degrees / *Minutes*	30	31	32	33	34
17	41 9	61 4	31 6	55 7	37 1
18	51 8	72 1	43 2	68 3	50 6
19	61 7	82 9	54 9	80 8	64 1
20	71 7	93 7	66 5	93 4	77 7
21	0.05581 7	0.06204 5	0.06878 2	0.07606 0	0.08391 3
22	91 6	15 3	89 9	18 6	0.08404 9
23	0.05601 6	26 1	0.06901 6	31 2	18 5
24	11 6	36 9	13 3	43 9	32 1
25	21 7	47 8	25 0	56 5	45 7
26	31 7	58 6	36 7	69 2	59 4
27	41 7	69 5	48 5	81 9	73 1
28	51 8	80 4	60 2	94 6	86 8
29	61 9	91 3	72 0	0.07707 3	0.08500 5
30	72 0	0.06302 2	83 8	20 0	14 2
31	0.05682 1	0.06313 1	0.06995 1	0.07732 8	0.08528 0
32	92 2	24 1	0.07007 5	45 5	41 8
33	0.05702 3	35 0	19 3	58 3	55 5
34	12 4	46 0	31 2	71 1	69 3
35	22 6	57 0	43 0	83 9	83 2
36	32 8	68 0	54 9	96 8	97 0
37	42 9	79 0	66 8	0.07809 6	0.08610 8
38	53 1	90 1	78 7	22 5	24 7
39	63 3	0.06401 1	90 7	35 4	38 6
40	73 6	12 2	0.07102 6	48 3	52 5
41	0.05783 8	0.06423 2	0.07114 6	0.07861 2	0.08666 4
42	94 0	34 3	26 6	74 1	80 4
43	0.05804 3	45 4	38 6	87 1	94 3
44	14 6	56 5	50 6	0.07900 0	0.08708 3
45	24 9	67 7	62 6	13 0	22 3
46	35 2	78 8	74 7	26 0	36 3
47	45 5	90 0	86 7	39 0	50 3
48	55 8	0.06501 2	98 8	52 0	64 4
49	66 2	12 3	0.07210 9	65 1	78 4
50	76 5	23 6	23 0	78 1	92 5
51	0.05886 9	0.06534 8	0.07235 1	0.07991 2	0.08806 6
52	97 3	46 0	47 3	0.08004 3	20 7
53	0.05907 7	57 3	59 4	17 4	34 8
54	18 1	68 5	71 6	30 6	49 0
55	28 5	79 8	83 8	43 7	63 1
56	39 0	91 1	95 9	56 9	77 3
57	49 4	0.06602 4	0.07308 2	70 0	91 5
58	59 9	13 7	20 4	83 2	0.08905 7
59	70 4	25 0	32 6	96 4	20 0
60	0.05980 9	0.06636 4	0.07344 9	0.08109 7	0.08934 2

Angle α	inv α = tan α − α (radians)				
Degrees / Minutes	35	36	37	38	39
0	0.08934 2	0.09822	0.10778	0.11806	0.12911
1	48 5	838	795	824	930
2	62 8	853	811	842	949
3	77 1	869	828	859	968
4	91 4	884	844	877	987
5	0.09005 8	899	061	895	0.13006
6	20 1	915	878	913	025
7	34 5	930	894	931	045
8	48 9	946	911	949	064
9	63 3	961	928	957	083
10	77 7	977	944	985	102
11	0.09092 2	0.09992	0.10961	0.12003	0.13122
12	0.09106 7	0.10008	978	021	141
13	21 1	024	995	039	160
14	35 6	039	0.11011	057	180
15	50 2	055	028	075	199
16	64 7	070	045	093	219
17	79 3	086	062	111	238
18	93 8	102	079	129	258
19	0.09208 4	118	096	147	277
20	23 0	133	113	165	297
21	0 09237 7	0.10149	0.11130	0.012184	0.13316
22	52 3	165	146	202	336
23	67 0	181	163	220	355
24	81 6	196	180	238	375
25	96 3	212	197	257	395
26	0.09311 1	228	215	275	414
27	25 8	244	232	293	434
28	40 6	260	249	312	454
29	55 3	276	266	330	473
30	70 1	292	283	348	493
31	0.09384 9	0.10308	0.10300	0.12367	0.13513
32	99 8	323	317	385	533
33	0.09414 6	339	334	404	553
34	29 5	355	352	422	572
35	44 3	371	369	441	592
36	59 2	388	386	459	612
37	74 2	404	403	478	632
38	89 1	420	421	496	652
39	0.09504 1	436	438	515	672
40	19 0	452	455	534	692

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(Contd)

Angle a	inv a = tan a − a (radians)				
Degrees / *Minutes*	35	36	37	38	39
41	0.09534 0	0.10468	0.11473	0.12552	0.13712
42	49 0	484	490	571	732
43	64 1	500	507	590	752
44	79 1	516	525	608	772
45	94 2	533	542	627	792
46	0.09609 3	549	560	646	812
47	24 4	565	577	664	833
48	39 5	581	595	683	853
49	54 6	598	612	702	873
50	69 8	614	630	721	893
51	0.09685 0	0.10630	0.11647	0.12740	0.13913
52	0.09700 2	647	665	759	934
53	15 4	663	682	778	954
54	30 6	679	700	797	974
55	45 9	696	718	815	995
56	61 1	712	735	834	0.14015
57	76 4	729	753	853	035
58	91 7	745	771	872	056
59	0.09807 1	762	788	891	076
60	0.09822 4	0.10778	0.11806	0.12911	0.14097

Angle a	inv a = tan a − a (radians)				
Degrees / *Minutes*	40	41	42	43	44
0	0.14097	0.15370	0.16737	0.18202	0.19774
1	117	392	760	228	802
2	138	414	784	253	829
3	158	436	807	278	856
4	179	458	831	304	883
5	200	480	855	329	910
6	220	503	879	355	938
7	241	525	902	380	965
8	261	547	926	406	992
9	282	569	950	431	0.20020
10	303	591	974	457	047
11	0.14324	0.15614	0.16998	0.18482	0.20075
12	344	636	0.17022	508	102
13	365	658	045	534	130
14	386	680	069	559	157
15	407	703	093	585	185

(Contd)

(Contd)

Angle a	inv a = tan a − a (radians)				
Degrees / Minutes	40	41	42	43	44
16	428	725	117	611	212
17	448	748	142	637	240
18	469	770	166	662	268
19	490	793	190	688	296
20	511	815	214	714	323
21	0.14532	0 15838	0 17238	0 18740	0.20351
22	553	860	262	766	379
23	574	883	28E	792	407
24	595	905	31•	818	435
25	616	928	335	844	463
26	638	950	359	870	490
27	659	973	383	896	518
28	680	996	408	922	546
29	701	0.16019	432	948	575
30	722	041	457	975	603
31	0.14743	0.16064	0.17481	0.19001	0.20631
32	765	087	506	027	659
33	786	110	530	053	687
34	807	133	555	080	715
35	829	156	579	106	743
36	850	178	604	132	772
37	871	201	628	159	800
38	893	224	653	185	828
39	914	247	678	212	857
40	936	270	702	238	885
41	0.14957	0.16293	0.17727	0.19265	0.20914
42	979	317	752	291	942
43	0.15000	340	777	318	971
44	022	363	801	344	999
45	043	386	826	371	0 21028
46	065	409	851	398	056
47	087	432	876	424	085
40	108	456	901	451	114
49	130	479	926	478	142
50	152	502	951	505	171
51	0.15173	0.16525	0.17976	0.19532	0.21200
52	195	549	0.18001	558	229
53	217	572	026	585	257
54	239	596	051	612	2d6
55	261	619	076	639	315
56	282	642	101	666	344
57	304	666	127	693	373
58	326	689	152	720	402
59	348	713	177	747	431
60	015370	016737	018202	019774	012460

(Contd.)

(Contd)

(Contd)Angle a				inv a = tan a − a (radians)
Degrees / Minutes	45	46	47	48	49

Degrees / Minutes	45	46	47	48	49
0	0.21460	0.23268	0.25206	0.27285	0.29516
1	489	299	240	321	554
2	518	330	273	357	593
3	548	362	307	393	631
4	577	393	341	429	670
5	606	424	374	465	709
6	635	456	408	501	747
7	665	487	442	538	786
8	694	519	475	574	825
9	723	550	509	610	864
10	753	582	543	646	903
11	0.21782	0.23613	0.25577	0.27683	0.29942
12	812	645	611	719	981
13	841	676	645	755	0.30020
14	871	708	679	792	059
15	900	740	713	828	098
16	930	772	747	865	137
17	960	803	781	902	177
18	989	835	815	938	216
19	0.22019	867	849	975	255
20	049	899	883	0.28012	0.30295
21	0.22079	0.23931	0.25918	0.28048	0.30334
22	108	963	952	085	374
23	138	995	986	122	413
24	168	0.24027	0.26021	159	453
25	198	059	055	196	492
26	228	091	089	233	532
27	258	123	124	270	572
28	288	156	159	307	611
29	318	188	193	344	651
30	348	220	228	381	691
31	0.22378	0.24253	0.26262	0.28418	0.30731
32	409	285	297	455	771
33	439	317	332	493	811
34	469	350	368	530	851
35	499	382	401	567	891
36	530	415	436	605	931
37	560	447	471	642	971
38	590	480	506	680	0.31012
39	621	512	541	717	052
40	651	545	576	755	092

(Contd)

(Contd)

Angle a	inv a = tan a − a (radians)				
Degrees / Minutes	45	46	47	48	49
41	0.22682	0.24578	0.26611	0.28792	0.31133
42	712	611	646	830	173
43	743	643	682	868	214
44	773	676	717	906	254
45	804	709	752	943	295
46	835	742	787	981	335
47	865	775	823	0.29019	376
48	896	808	858	057	417
49	927	841	893	095	457
50	958	874	929	133	498
51	0.22989	0.24907	0.26964	0.29171	0.31539
52	0.23020	940	0.27000	209	580
53	050	973	035	247	621
54	081	0.25006	071	286	662
55	112	040	107	324	703
56	143	073	142	362	744
57	174	106	178	400	785
58	206	140	214	439	826
59	237	173	250	477	868
60	0.23268	0.25206	0.27285	0.29516	0.31909

Angle a	inv a = tan a − a (radians)				
Degrees / Minutes	50	51	52	53	54
0	0.31909	0.34478	0.37237	0.40202	0.43390
1	950	522	285	253	446
2	992	567	332	305	501
3	0.32033	611	380	356	556
4	075	656	428	407	611
5	116	700	476	459	667
6	158	745	524	511	722
7	199	790	572	562	778
8	241	834	620	614	833
9	283	879	668	666	889
10	324	924	716	717	945
11	0.32366	0.34969	0.37765	0.40769	0.44001
12	408	0.35014	813	821	057
13	450	059	861	873	113
14	492	104	910	925	169

(Contd)

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Angle a	inv *a* = tan *a* − *a* (radians)				
Degrees / *Minutes*	50	51	52	53	54
15	534	149	958	977	225
16	576	194	038007	0 41030	281
17	618	240	055	082	337
18	661	285	104	134	393
19	703	330	153	187	450
20	745	376	202	239	506
21	0 32787	0 35421	0 38251	0 41292	0 44563
22	830	467	299	344	619
23	872	512	348	397	676
24	915	558	397	450	733
25	957	604	446	502	789
26	0 33000	649	496	555	846
27	042	695	545	608	903
28	085	741	594	661	960
29	128	787	643	714	0.45017
30	171	833	693	767	047
31	0 33213	0 35879	0 38742	0 41820	0.45132
32	256	925	792	874	189
33	299	971	841	927	246
34	342	0 36017	891	980	304
35	385	063	941	0 42034	361
36	428	110	990	087	419
37	471	156	0 39040	141	476
38	515	202	090	194	534
39	558	249	140	248	592
40	601	295	190	302	650
41	0.33645	0.36342	0.39240	0.42355	0.45708
42	688	388	290	409	766
43	731	435	340	463	824
44	775	482	390	517	882
45	818	529	441	571	940
46	862	575	491	625	998
47	906	622	541	680	0.46057
48	949	669	592	734	115
49	993	716	642	788	173
50	0.34037	763	693	843	232
51	0.34081	0.36810	0.39743	0.42897	0.46291
52	125	858	794	952	349
53	169	905	845	0.43006	408
54	215	952	896	061	467

(Contd)

(Contd)

Angle α	inv α = tan α − α (radians)				
Degrees / *Minutes*	50	51	52	53	54
55	257	999	947	116	526
56	301	0.37047	998	171	585
57	345	094	0.40049	225	644
58	389	142	100	280	703
59	434	189	151	335	762
60	0.34478	0.37237	0.40202	0.43390	0.46822

Angle α	inv α = tan α − α (radians)				
Degrees / *Minutes*	55	56	57	58	59
0	0.46822	0.50518	0.54503	058804	0.63454
1	881	582	572	879	534
2	940	646	641	954	615
3	0.47000	710	710	0.59028	696
4	060	774	779	103	777
5	119	838	849	178	858
6	179	903	918	253	939
7	239	967	988	328	0.64020
8	299	0.51032	0.55057	403	102
9	359	096	127	479	183
10	419	161	197	554	265
11	0.47479	0.51226	0.55267	0.59630	0.64346
12	539	291	337	705	428
13	599	356	407	781	510
14	660	421	477	857	592
15	720	486	547	933	674
16	780	551	618	0.60009	756
17	841	616	688	085	839
18	902	682	759	161	921
19	962	747	829	237	0.65004
20	0.48023	813	900	314	086
21	0.48084	0.51878	0.55971	0.60390	0.65160
22	145	944	0.56042	467	252
23	206	0.52010	113	544	335
24	267	076	184	620	418
25	328	141	255	697	501

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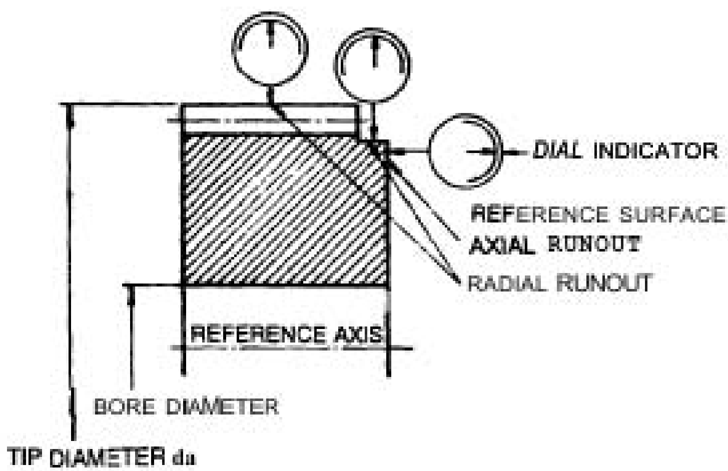
(Contd)

Angle α	inv α = tan α − α (radians)				
Degrees / Minutes	55	56	57	58	59
26	389	207	326	774	585
27	451	274	398	851	668
28	512	340	469	929	752
29	574	406	541	0.61006	835
30	635	472	612	083	919
31	0.48697	0.52539	0.56684	0.61161	0.66003
32	758	605	756	239	087
33	820	672	828	316	171
34	882	739	900	394	255
35	944	805	972	472	340
36	0.49006	872	0.57044	550	424
37	068	939	116	628	509
38	130	0.53006	188	706	593
39	192	073	261	785	670
40	255	141	333	863	763
41	0.49317	0.53208	0.57406	0.61942	0.66848
42	380	275	479	0.62020	933
43	442	343	552	099	0.67019
44	505	410	625	178	104
45	568	478	698	257	189
46	630	546	771	336	275
47	693	613	844	415	361
48	756	681	917	494	447
49	819	749	991	574	532
50	882	817	0.58064	653	618
51	0.49945	0.53885	0.58138	0.62733	0.67705
52	0.50009	954	211	812	791
53	072	054022	285	892	877
54	135	090	359	972	964
55	199	159	433	0.63052	0.68050
56	263	228	507	132	137
57	326	296	581	212	224
58	390	365	656	293	311
59	454	434	730	273	398
60	0.50518	0.54503	0.58804	0.63454	0.68485

APPENDIX I

Tolerances on Gear Blanks



Tolerances on Gear Blanks

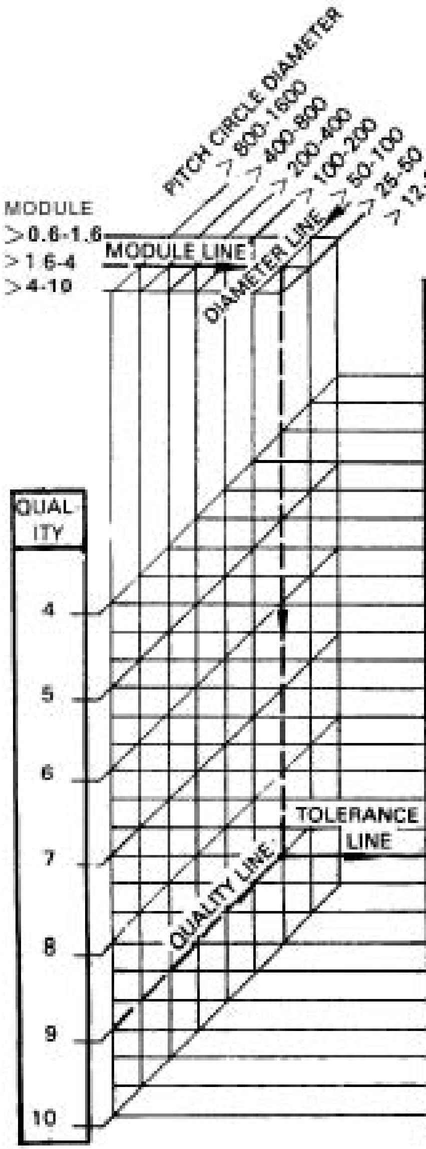
(All values in microns except tip diameter in mm)

Quality	3	4	5	6	7	8	9	10	11	12
Bore size tolerance	IT4	IT4	IT5	IT6	IT7	IT7	IT8	IT8	IT8	IT8
Shaft size tolerance	IT4	IT4	IT5	IT5	IT6	IT6	IT7	IT7	IT8	IT8
Radial run-out of tip cylinder Radial run-out of reference surface Axial run-out of reference surface	$0.01\,d_a + 5$		$0.016\,d_a + 10$		$0.025\,d_a + 15$		$0.04\,d_a + 25$		$0.04\,d_a + 25$	
Blank diameter (d_a)	h7	h7	h7	h8	h8	h6	h9	h9	h11	h11

APPENDIX J

Table of Tooth Thickness Tolerances

Based on Zahnraeder, Zirpke, 11th edition, 1980, table no. 10, p. 210, VEB Fachbuchverlag, Leipzig.



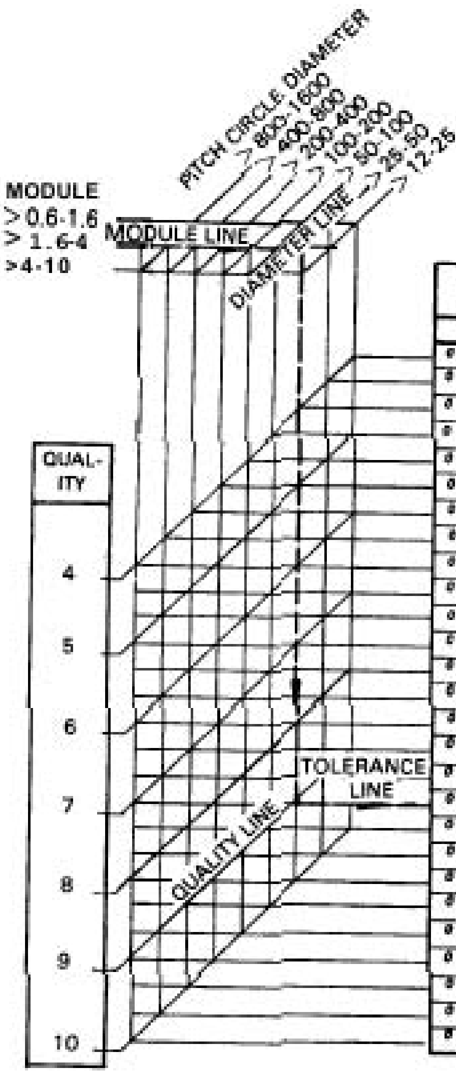
TOOTH THICKNESS TOLERANCE, A_s µm

h	g	f	e	d	c	b	a
0 / −7	−3.5 / −10	−7 / −14	−10 / −17	−14 / −21	−21 / −28	−28 / −35	−35 / −42
0 / −7	−3.5 / −10	−7 / −14	−10 / −17	−14 / −21	−21 / −28	−28 / −35	−35 / −42
0 / −8	−4 / −12	−8 / −16	−12 / −20	−16 / −24	−24 / −32	−32 / −40	−40 / −48
0 / −9	−4.5 / −14	−9 / −18	−13 / −22	−18 / −27	−27 / −36	−36 / −45	−45 / −54
0 / −10	−5 / −15	−10 / −20	−15 / −25	−20 / −30	−30 / −40	−40 / −50	−50 / −60
0 / −11	−5.5 / −16	−11 / −22	−16 / −27	−22 / −33	−33 / −44	−44 / −55	−55 / −66
0 / −12	−6 / −18	−12 / −24	−18 / −30	−24 / −36	−36 / −48	−48 / −60	−60 / −72
0 / −14	−7 / −21	−14 / −28	−21 / −35	−28 / −42	−42 / −56	−56 / −70	−70 / −84
0 / −16	−8 / −24	−16 / −32	−24 / −40	−32 / −48	−48 / −64	−64 / −80	−80 / −96
0 / −18	−9 / −27	−18 / −36	−27 / −45	−36 / −54	−54 / −72	−72 / −90	−90 / −108
0 / −20	−10 / −30	−20 / −40	−30 / −50	−40 / −60	−60 / −80	−80 / −100	−100 / −120
0 / −22	−11 / −33	−22 / −44	−33 / −55	−44 / −66	−66 / −88	−88 / −110	−110 / −132
0 / −25	−12 / −37	−25 / −50	−37 / −62	−50 / −75	−75 / −100	−100 / −125	−125 / −150
0 / −28	−14 / −42	−28 / −56	−42 / −70	−56 / −84	−84 / −112	−112 / −140	−140 / −168
0 / −32	−16 / −48	−32 / −64	−48 / −80	−64 / −96	−96 / −128	−128 / −160	−160 / −192
0 / −36	−18 / −54	−36 / −72	−54 / −90	−72 / −108	−108 / −144	−144 / −180	−180 / −216
0 / −40	−20 / −60	−40 / −80	−60 / −100	−80 / −120	−120 / −160	−160 / −200	−200 / −240
0 / −45	−22 / −67	−45 / −90	−67 / −112	−90 / −135	−135 / −180	−180 / −225	−225 / −270
0 / −50	−25 / −75	−50 / −100	−75 / −125	−100 / −150	−150 / −200	−200 / −250	−250 / −300
0 / −56	−28 / −84	−56 / −112	−84 / −140	−112 / −168	−168 / −224	−224 / −280	−280 / −336
0 / −63	−32 / −95	−63 / −126	−95 / −158	−126 / −189	−189 / −252	−252 / −315	−315 / −378
0 / −71	−36 / −107	−71 / −142	−107 / −178	−142 / −213	−213 / −284	−284 / −355	−355 / −426
0 / −80	−40 / −120	−80 / −160	−120 / −200	−160 / −240	−240 / −320	−320 / −400	−400 / −480
0 / −90	−45 / −135	−90 / −180	−135 / −225	−180 / −270	−270 / −360	−360 / −450	−450 / −540
0 / −100	−50 / −150	−100 / −200	−150 / −250	−200 / −300	−300 / −400	−400 / −500	−500 / −600
0 / −110	−55 / −165	−110 / −220	−165 / −275	−220 / −310	−330 / −440	−440 / −550	−550 / −660
0 / −125	−63 / −188	−125 / −250	−188 / −313	−250 / −375	−375 / −500	−500 / −625	−625 / −750

APPENDIX K

Table of Tooth Distance Tolerances

Based on Zahnraeder, Zirpke, 11th edition, 1980, table no. 10, p. 210, VEB Fachbuchverlag, Leipzig.

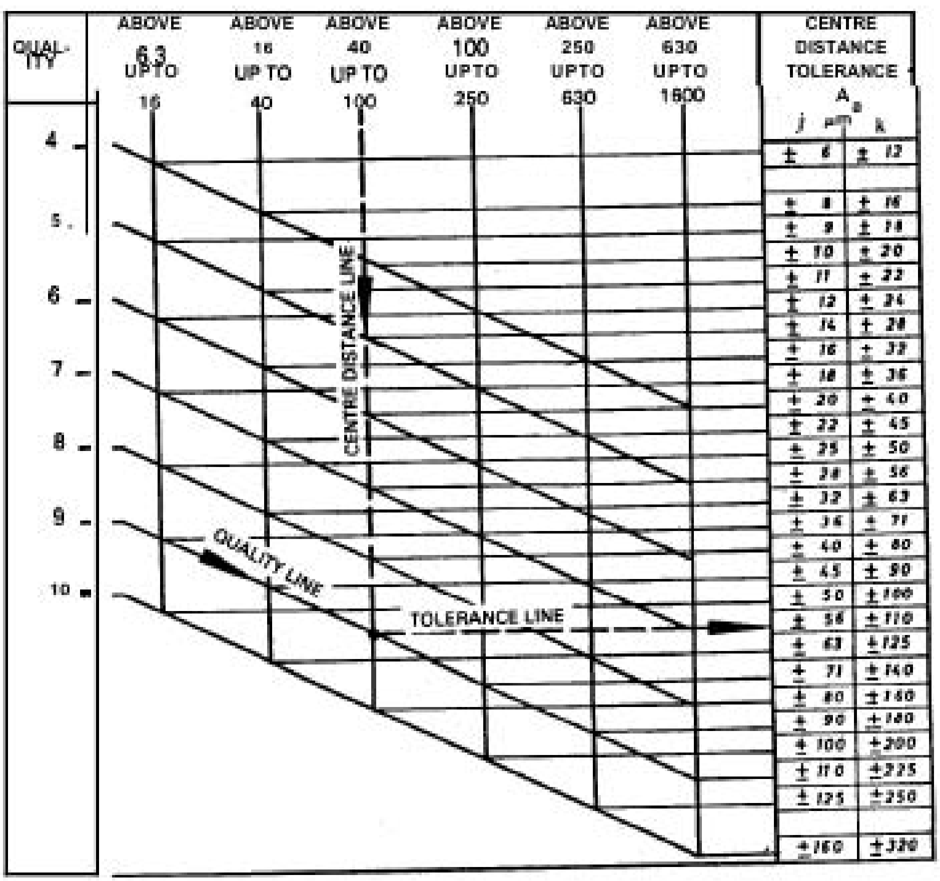


TOOTH DISTANCE TOLERANCE TOLERANCE, A_w μm

h	g	f	e	d	c	b	a
0 / -6.5	-3.5 / -10	-6.5 / -13	-10 / -19	-13 / -20	-20 / -26	-24 / -33	-33 / -38
0 / -6.5	-3.5 / -10	-6.5 / -13	-10 / -17	-13 / -20	-20 / -26	-24 / -33	-33 / -38
0 / -7.5	-4 / -11	-7.5 / -15	-11 / -18	-15 / -23	-23 / -30	-30 / -38	-38 / -45
0 / -8.5	-4 / -13	-8.5 / -17	-13 / -22	-17 / -25	-26 / -34	-34 / -42	-43 / -51
0 / -9	-4.5 / -14	-9 / -19	-14 / -23	-19 / -28	-28 / -38	-38 / -47	-47 / -56
0 / -10	-5 / -16	-10 / -21	-16 / -26	-21 / -31	-31 / -41	-41 / -52	-52 / -62
0 / -11	-5.5 / -17	-11 / -23	-17 / -28	-23 / -34	-34 / -45	-45 / -56	-56 / -68
0 / -13	-6.5 / -20	-13 / -26	-20 / -33	-26 / -39	-39 / -53	-53 / -66	-66 / -79
0 / -15	-7.5 / -23	-15 / -30	-23 / -38	-30 / -45	-45 / -60	-60 / -75	-75 / -90
0 / -17	-8.5 / -25	-17 / -34	-25 / -42	-34 / -51	-51 / -68	-68 / -85	-85 / -101
0 / -19	-9 / -28	-19 / -38	-28 / -47	-38 / -56	-56 / -75	-75 / -94	-94 / -113
0 / -21	-10 / -31	-21 / -41	-31 / -52	-41 / -62	-62 / -83	-83 / -103	-103 / -124
0 / -23	-12 / -35	-23 / -47	-35 / -59	-47 / -70	-70 / -94	-94 / -117	-117 / -141
0 / -26	-13 / -39	-26 / -53	-39 / -66	-53 / -79	-79 / -105	-105 / -132	-132 / -158
0 / -30	-15 / -45	-30 / -60	-45 / -75	-60 / -90	-90 / -120	-120 / -150	-150 / -180
0 / -34	-17 / -51	-34 / -68	-51 / -85	-68 / -101	-101 / -135	-135 / -169	-169 / -203
0 / -38	-19 / -56	-38 / -75	-56 / -94	-75 / -113	-113 / -150	-150 / -188	-188 / -226
0 / -42	-21 / -63	-42 / -85	-63 / -105	-85 / -127	-127 / -169	-169 / -211	-211 / -254
0 / -47	-23 / -70	-47 / -94	-70 / -117	-94 / -141	-141 / -188	-188 / -235	-235 / -282
0 / -53	-26 / -79	-53 / -105	-79 / -132	-105 / -158	-158 / -210	-210 / -263	-263 / -316
0 / -59	-30 / -89	-59 / -118	-89 / -148	-118 / -178	-178 / -237	-237 / -296	-296 / -355
0 / -67	-34 / -101	-67 / -133	-101 / -169	-133 / -200	-200 / -267	-267 / -334	-334 / -400
0 / -75	-38 / -113	-75 / -150	-113 / -188	-150 / -226	-226 / -301	-301 / -376	-376 / -451
0 / -85	-42 / -127	-85 / -169	-137 / -211	-169 / -254	-254 / -338	-338 / -423	-423 / -507
0 / -94	-47 / -141	-94 / -188	-141 / -235	-188 / -282	-282 / -376	-376 / -470	-470 / -564
0 / -103	-52 / -155	-103 / -207	-155 / -258	-207 / -310	-310 / -413	-413 / -517	-517 / -620
0 / -117	-59 / -177	-117 / -235	-177 / -294	-235 / -352	-352 / -470	-470 / -587	-587 / -705

APPENDIX L

Table of Centre Distance Tolerances



APPENDIX M

Standard Shaft Diameters

If no other values for special applications are mentioned, the figures in this table are valid for toleranced diameters for shafts, shaft steps and shaft ends including the truncated cones which run in journal or anti-friction bearings or which are meant for receiving couplings, gears, travelling wheels and other wearing parts.

Dimensions in mm

10					(38)	100				380	
			40				110		400		
12					(42)	120				420	
								(125)			(440)
				45			130		450		
		" 4)				140					(460)
	15				(48)		150			480	
16			50			160			500		
		(17)						(170)		530	
		(18)		55		180					
								(190)	560		
20						200					
			60					(210)		600	
		(22)				220			630		
					(65)			(230)			
							240			670	
25				70		250					
							260		710		
		(28)			(75)	280				750	
30			80				300		800		
		(32)			(85)	320				850	
				90			340		900		
	35				(95)					950	
						360			1000		

Bracketed sizes are not preferred.
Diameters in bold types are preferred.

APPENDIX N

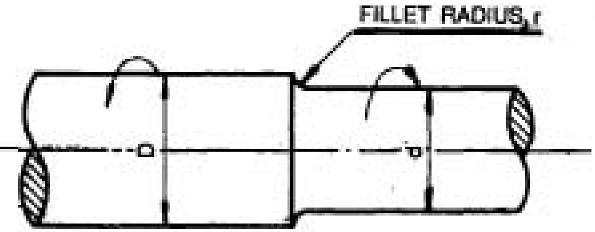
Table of Fillet Radii for Stepped Shafts in Torsion

d \ D	12	16	20	25	30	40	50	60	80	100	120	140	160	180	200	220	250	280	(300)	320	(340)	360	(380)	400
10	0.5	1.0																						
12		1.0	1.5																					
14		0.5	1.0																					
16			1.0	1.5																				
18				1.5																				
20				1.5	2.0																			
22				1.0	1.5																			
25					1.5	2.0																		
28					1.0	2.0																		
30						2.0																		
(32)						1.5	2.5																	
35						1.5	2.5																	

(Contd)

(Contd)

d \ D	12	16	20	25	30	40	50	60	80	100	120	140	160	180	200	220	250	280	(300)	320	(340)	360	(380)	400
38						1.0	2.5																	
40							2.5	3.0																
(42)							2.0	3.0																
45							1.5	3.0																
(48)							1.0	2.5																
50								2.5																
55								1.5	4.0															
60									4.0															
(65)									3.5															
70									2.5	5.0														
(75)									1.5	5.0														
80										5.0	6.0													
(85)										3.5	6.0													
90										2.5	6.0													
(95)										2.0	5.5	6.5												
100											5.0	6.5												
(105)											3.5	6.5												
110											2.5	6.0	8.0											
120												5.0	8.0											
(125)												3.0	7.5	8.5										



(Contd)

(Contd)

d \ D	12	16	20	25	30	40	50	60	80	100	120	140	160	180	200	220	250	280	(300)	320	(340)	360	(380)	400
130												2.5	7.0	8.5										
140													5.0	8.5	10.0									
150													3.0	7.5	10.0									
160														5.0	9.5	11.0								
(170)														3.0	7.5	11.0								
180															5.5	9.5	12.0							
(190)															3.0	8.0	12.0							
200																5.5	11.0	14.0						
(210)																3.0	10.0	14.0						
220																	8.0	13.0	16.0					
(230)																	6.0	12.0	15.0					
240																	5.0	11.0	15.0	17.0				
250																		7.5	13.0	16.0	17.0			
260																		5.5	11.0	14.0	16.0	17.0		
280																			5.5	10.0	13.0	16.0	18.0	20.0
300																				6.0	10.0	15.0	18.0	20.0

The fillet radii (r) given in the above table are meant for ordinary applications only. For applications where stress concentration and notch effect are predominant design criteria, the shapes of fillets should be decided by the relevant design codes. In such cases, undercuts or fillet curves comprising two or more radii may be recommended.

APPENDIX *O*

Key for Power ismissio

Range of shaft dia, *d*		Key	Keyway					
		b x h	Tolerance on *b*					
				Running fit		Light drive fit		Force fit
Above	*Up to*		*b*	Shaft *H9*	Hub *D10*	Shaft *N9*	Hub *JS9*	Shaft and hub *P9*
6	8	2 x 2	2	+0.025	+0.060	-0.004	+0.0125	-0.006
8	10	3 x 3	3	0	+0.020	-0.029	-0.0125	-0.031
10	12	4 x 4	4	+0.030	+0.078	0	+0.0150	-0.012
12	17	5 x 5	5	0	+0.030	-0.030	-0.0150	-0.042
17	22	6 x 6	6					
22	30	8 x 7	8	+0.036	4.098	0	+0.0180	-0.015
30	38	10 x 8	10	0	+0.040	-0.030	-0.0180	-0.051
38	44	12x8	12					
44	50	14x9	14	+0.043	+0.120	0	+0.0215	-0.018
50	58	16x10	16	0	+0.050	-0.043	-0.0215	-0.061
58	65	18x11	18					
65	75	20x12	20					
75	85	22x14	22	+0.052	+0.149	0	+0.0260	-0.022
85	95	25 x 14	25	0	+0.065	-0.052	-0.0260	-0.074
95	110	28 x 16	28					
110	130	32x18	32					
130	150	36x20	36	+0.062	4.180	0	+0.0310	-0.026
150	170	40x22	40	0	+0.080	-0.062	-0.0310	-0.088
170	200	45x25	45					
200	230	50 x 28	50					
230	260	56 x 32	56					
260	290	63x32	63	+0.074	+0.220	0	+0.0370	-0.032
290	330	70x36	70	0	+0.100	-0.074	-0.0370	-0.106
330	380	80x40	80					
380	440	90x45	90	+0.087	+0.260	0	+0.0435	-0.037
440	500	100 x 50	100	0	+0.120	-0.087	-0.0435	-0.124

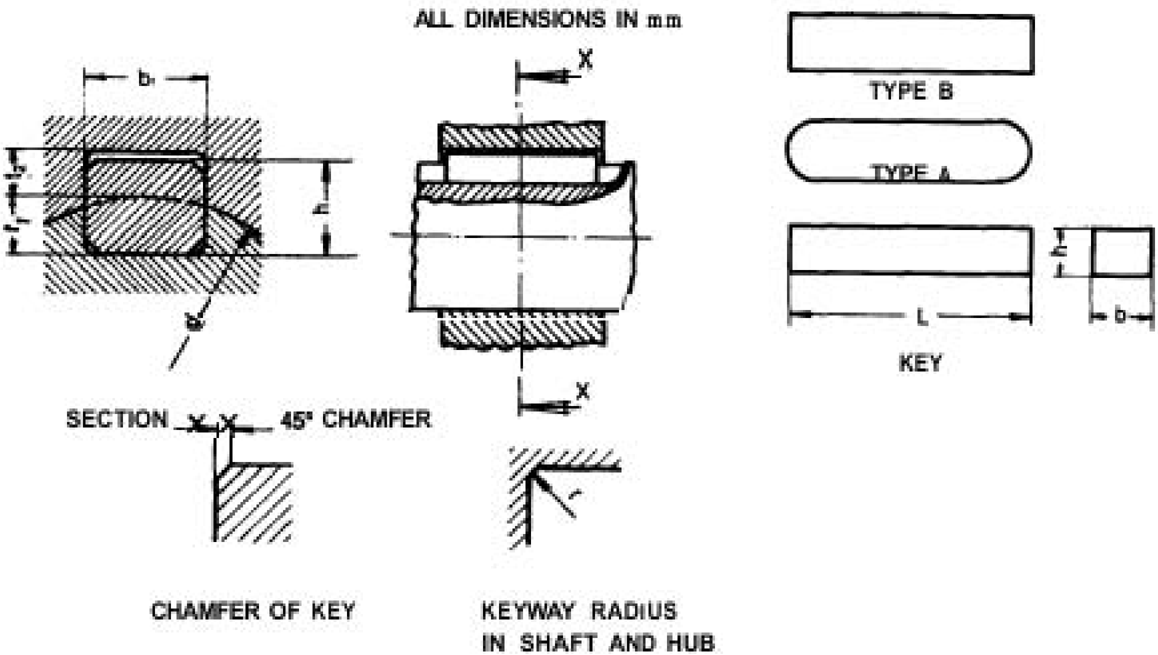
Parallel key, also known as feather key, is the most commonly used type of key. It is usually used in the case of unidirectional torque. Where axial movement of the hub is required, a clearance fit is provided between the shaft and the hub and between the hub and the key. The key is usually screwed to the shaft key-way for such type of application. Parallel keys are not suitable for the transmission of reversible or fluctuating torques.

Parallel keys given here are based on IS : 2048-1975. The relevant data are given below.

Types Types range from A to J. Only types A and B are shown in the adjoining table. These are usually meant for non-sliding applications and the remaining types for sliding applications. Further details for the remaining types may be obtained from the relevant IS code.

Tolerances : h_9 for both width and height in case of square section keys,
h_9 on width and h_{11} on height in case of rectangular section keys.

Keyway						Range of key length L	
t_1	Tolerance on t_1	t_2	Tolerance on t_2	r Min	r Max	Min	Max
1.2		1.0		0.08	0.16	6	20
1.8	+0.1	1.4	+0.1	0.08	0.16	6	36
2.5	0	1.8	0	0.08	0.16	8	45
3.0		2.3		0.16	0.25	10	56
3.5		2.8		0.16	0.25	14	70
4.0		3.3		0.16	0.25	18	90
5.0		3.3		0.25	0.40	22	110
5.0		3.3		0.25	0.40	28	140
5.5		3.8		0.25	0.40	36	160
6.0		4.3		0.25	0.40	45	180
7.0	+0.2	4.4	+0.2	0.25	0.40	50	200
7.5	0	4.9	0	0.40	0.60	56	220
9.0		5.4		0.40	0.60	63	250
9.0		5.4		0.40	0.60	70	280
10.0		6.4		0.40	0.60	80	320
11.0		7.4		0.40	0.60	90	360
12.0		8.4		0.70	1.00	100	400
13.0		9.4		0.70	1.00	110	400
15.0		10.4		0.70	1.00	125	400
17.0		11.4		0.70	1.00	140	400
20.0	+0.3	12.4	+0.3	1.20	1.60	160	400
20.0	0	12.4	0	1.20	1.60	180	400
22.0		14.4		1.20	1.60	200	400
25.0		15.4		2.00	2.50	220	400
28.0		17.4		2.00	2.50	250	400
31.0		19.5		2.00	2.50	280	400



PARALLEL KEY

Tolerances on width of key-way (b) are given in the table.
Tolerances on length of keys and key-ways are as follows.

Length of key (mm)	Tol. on key length (mm)	Tol. on key-way length (mm)
up to 28	0 to − 0.2	0 to + 0.2
32 to 80	0 to − 0.3	0 to + 0.3
90 and above	0 to − 0.5	0 to + 0.5

Material of key Steel of tensile strength not less than 600 N/mm^2.

Designation A parallel key of type A, having dimensions width 12 mm, height 8 mm and length 50 mm, shall be designated as

Parallel Key A, 12 x 8 x 50, IS: 2048

Calculation

Calculation is usually not necessary for normal applications as the key size along with its key-way is simultaneously decided once the value of the shaft diameter becomes fixed after the usual calculations for determining the shaft diameter have been carried out. The key can be selected according to the shaft diameter as per the given table.

However, calculation may become necessary in some cases. The main disadvantage of a key type joint is the reduction of the effective cross-section as well as tress concentration leading to high localised stresses. To reduce this effect, the shaft diameter within the hub length may be increased by about 30% or it may also be strengthened by hardening. The following relations may be used for hub using a steel shaft

$$L_h = x \sqrt[3]{\frac{T}{10}} \qquad s = y \sqrt[3]{\frac{T}{10}}$$

where L_h (cm) = Length of hub, s (cm) = Thickness of hub, (OD-ID of hub)/2
T(N cm) = Torque, and x and y are coefficients as follows

x = 0.53 to 0.70 for CI hub = 0.35 to 0.46 for steel hub

y = 0.18 to 0.21 for CI hub = 0.14 to 0.18 for steel hub

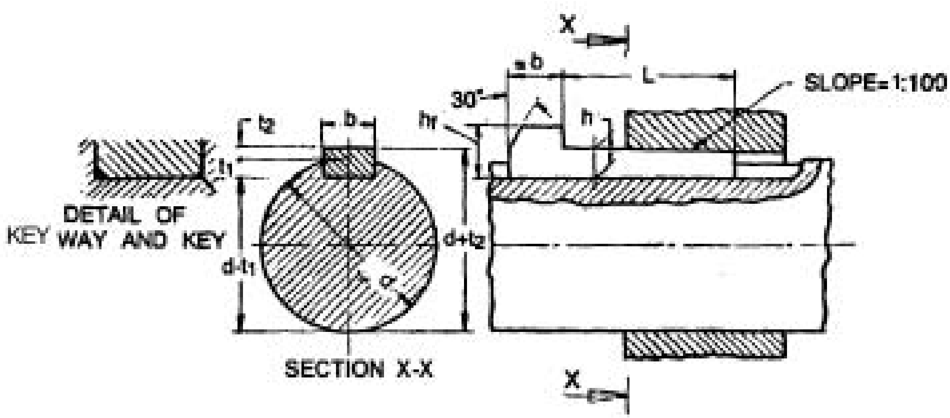
Circumferential force $F(N)$, transmitted by key = $2T/d = p(h - t_1) Li$
where d (cm) = Diameter of shaft, h (cm) = Height of key, t_1 (cm) = Depth of key inside shaft only, L(cm) = Length of key, i = Number of keys, p (N/cm²) = Bearing pressure = 5000 for CI hub = 9000 for steel hub.

APPENDIX P

Gib-Head Key for Power Transm'ission

Dimensions of gib-head keys are as per the following table based on IS: 2293-1974. Tolerances on key length and material for the key are the same as those of parallel keys. Designation system is also similar. For gib-head keys, the shaft key-way must be longer than the key so that enough space is provided for driving the key tight while fitting. Compared to parallel keys, the gib-head keys can transmit more torque.

ALL DIMENSIONS IN mm



GIB HEAD KEY

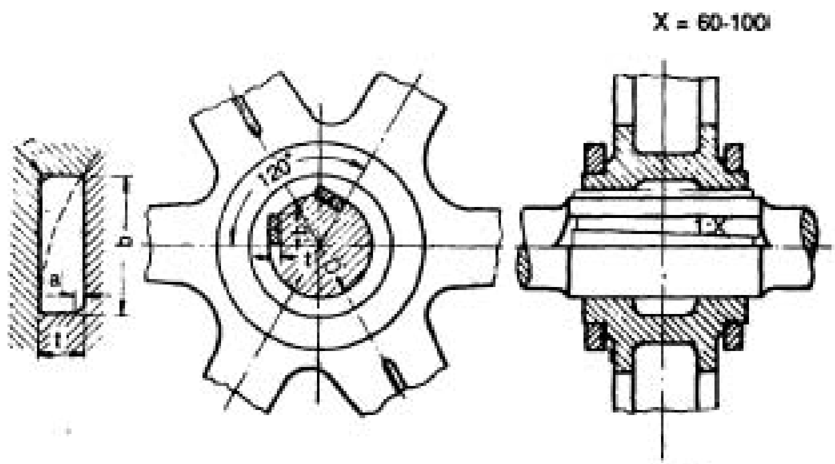
All dimensions in millimetres

Range of shaft Dia. d		Key	Keyway								Range of key length L		h_1
Above	Up to	b x h	b	Tolerance on b D10	t_1	Tolerance on t_1	t_2	Tolerance on t_2	r Min	r Max	Min	Max	
10	12	4 x 4	4		2.5		1.2		0.08	0.16	14	45	7
12	17	5 x 5	5	+0.078	3.0	io.1	1.7	+0.1	0.16	0.25	14	56	8
17	22	6 x 6	6	+0.030	3.5	0	2.2	0	0.16	0.25	16	70	10
22	30	8 x 7	8	+0.098	4.0		2.4		0.16	0.25	20	90	11
30	38	10 x 8	10	+0.040	5.0		2.4		0.25	0.40	25	110	12
38	44	12 x 8	12		5.0		2.4		0.25	0.40	32	140	12
44	50	14 x 9	14	+0.120	5.5		2.9		0.25	0.40	40	160	14
50	58	16 x 10	16	+0.050	6.0		3.4		0.25	0.40	45	180	16
58	65	18 x 11	18		7.0	+0.2	3.4	+0.2	0.25	0.40	50	200	18
65	75	20 x 12	20		7.5	0	3.9	0	0.40	0.60	56	220	20
75	85	22 x 14	22	+0.149	9.0		4.4		0.40	0.60	63	250	22
85	95	25 x 14	25	+0.065	9.0		4.4		0.40	0.60	70	280	22
95	110	28 x 16	28		10.0		5.4		0.40	0.60	80	320	25
110	130	32 x 18	32		11.0		6.4		0.40	0.60	90	360	28
130	150	36 x 20	36		12.0		7.1		0.70	1.00	100	400	32
150	170	40 x 22	40	+0.180	13.0		8.1		0.70	1.00	110	400	36
170	200	45 x 25	45	+0.080	15.0		9.1		0.70	1.00	125	400	40
200	230	50 x 28	50		17.0		10.1		0.70	1.00	140	400	45
230	260	56 x 32	56		20.0	+0.3	11.1	+0.3	1.20	1.60			50
260	290	63 x 32	63	+0.220	20.0	0	11.1	0	1.20	1.60			50
290	330	70 x 36	70	+0.100	22.0		13.1		1.20	1.60			56
330	380	80 x 40	80		25.0		14.1		2.00	2.50			63
380	440	90 x 45	90	t0.260	28.0		16.1		2.00	2.50			70
440	500	100 x 50	100	t0.120	31.0		18.1		2.00	2.50			80

APPENDIX Q

Tangent Key for Power Transmission

All Dimensions in mm



Tangent key is meant for service conditions where a large amount of impact type of load is to be transmitted in both directions of rotations as in the case of fly-wheels, rolling mills, etc. The angle between the keys can be 180' also, if necessary.

Tangent Key

Shaft diameter *D*	*Keyway*		*Key*	
	Depth *t*	*Width* *b*	*Radius* *r*	*Chamfer* *a*
100	10	30	2	3
110	11	33	2	3
120	12	36	2	3
130	13	39	2	3
140	14	42	2	3
150	15	45	2	3
160	16	48	2	3
170	17	51	2	3
180	18	54	2	3

(Conrd)

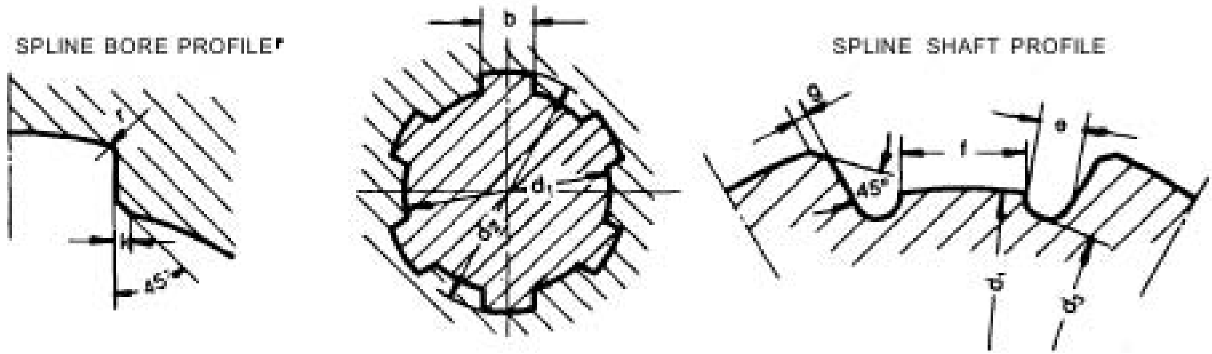
(Contd)

Shaft diameter D	Keyway		Key	
	Depth t	Width b	Radius r	Chamfer a
190	19	57	2	3
200	20	60	2	3
210	21	63	2	3
220	22	66	2	3
230	23	69	2	3
240	24	72	3	4
250	25	75	3	4
260	26	78	3	4
270	27	81	3	4
280	28	84	3	4
290	29	87	3	4
300	30	90	3	4
320	32	96	3	4
340	34	102	3	4
360	36	108	3	4
380	38	114	4	5
400	40	120	4	5
420	42	126	4	5
440	44	132	4	5
460	46	138	4	5
480	48	144	5	6
500	50	150	5	6
520	52	156	5	6
540	54	162	5	6
560	56	168	5	6
580	58	174	5	6
600	60	180	6	7
620	62	186	6	7
640	64	192	6	7
660	66	198	6	7
680	68	204	6	7
700	70	210	6	7
720	72	216	6	7
740	74	222	6	7
760	76	228	6	7
780	78	234	6	7
800	80	240	6	7
820	82	246	6	7
840	84	252	6	7
860	86	258	6	7
880	88	264	8	9
900	90	270	8	9
920	92	276	8	9
940	94	282	8	9
960	96	288	8	9
980	98	294	8	9
1000	100	300	8	9

APPENDIX R

Straight-Sided Splines for Power Transmission

ALL DIMENSIONS IN mm



DETAILS NOT SHOWN TO BE SELECTED ACCORDING TO SUITABILITY

THE FLANK OF EACH SPLINE KEY MUST BE PARALLEL UP TO THE MEETING POINT WITH THE INSIDE DIA.

(NO.OF SPLINE-KEYS SHOWN HERE = 6)

STRAIGHT-SIDEDSPLINES

Light Duty Series

No. of spline keys	Centering	d_1	d_2	b	d_3 min	e max	f	g max	k max	r max
	On	23	25	6	22.1	1,25	3,54	0,3	0,3	0,2
6	inside	26	30	6	24.6	1,84	3.85	0,3	0,3	0,2
	diameter	28	32	7	26.7	1,77	4,03	0,3	0,3	0,2
		32	36	6	30.42	1.89	2.71	0,4	0,4	0,3
	On inside	36	40	7	34,5	1,78	3,46	0,4	0,4	0,3
	diameter	42	46	8	40,4	1,68	5,03	0,4	0,4	0,3
8	or flanks	46	50	9	44,62	1,61	5,75	0,4	0,4	0,3

Ught Duty Series(*Contd*)

No. of spline keys	Centering	d_1	d_2	b	d_3 min	e max	f	g max	k max	r max
		52	58	10	49,7	2,72	4,89	0,5	0.5	0,5
		56	62	10	53.6	2.76	6,38	0,5	0.5	0,5
	On	62	68	12	59,82	2.48	7,31	0,5	0,5	0,5
	inside									
	diameter	72	78	12	69,6	2.54	5,45	0.5	0,5	0,5
	or	82	88	12	79.32	2.67	8,62	0,5	0,5	0.5
	flanks	92	98	14	89,44	2,36	10,08	0,5	0,5	0,5
10		102	108	16	99,9	2,23	11,49	0,5	0.5	0,5
		112	120	18	108.8	3,23	10.72	0,5	0,5	0,5

Designation Designation of a spline with number of spline keys = 6, inner diameter = 28 mm, outer diameter = 32 mm, is given as

Spline 6 ×28×32, IS:2327

Medium Duty **Series**

No. of spline keys	Centering	d_1	d_2	b	d_3 min	e max	f	g max	k max	r max
		11	14	3	9.9	1.55	—	0.3	0.3	0.2
		13	16	3.5	12.0	1.5	0.32	0.3	0.3	0.2
		16	20	4	14.54	2.1	0.16	0.3	0.3	0.2
	On inside	18	22	5	16.7	1.95	0.45	0.3	0.3	0.2
6	diameter	21	25	5	19.5	1.98	1.95	0.3	0.3	0.2
		23	28	6	21.3	2.3	1.34	0.3	0.3	0.2
		26	32	6	23.4	2.94	1.65	0.4	0.4	0.3
		28	34	7	25.9	2.94	1.70	0.4	0.4	0.3
		32	38	6	29.4	3.3	0.15	0.4	0.4	0.3
		36	42	7	33.5	3.01	1.02	0.4	0.4	0.3
		42	48	8	39.5	2.91	2.57	0.4	0.4	0.3
8		46	54	9	42.7	4.1	0.86	0.5	0.5	0.5
	On inside	52	60	10	48.7	4.0	2.44	0.5	0.5	0.5
	diameter	56	65	10	52.2	4.74	2.5	0.5	0.5	0.5
	or	62	72	12	57.8	5.0	2.4	0.5	0.5	0.5
	flanks									
		72	82	12	67.4	5.43	—	0.5	0.5	0.5
		82	92	12	77.1	5.4	3.0	0.5	0.5	0.5
10		92	102	14	87.3	5.2	4.5	0.5	0.5	0.5
		102	112	16	97.7	4.9	6.3	0.5	0.5	0.5
		112	125	18	106.3	6.4	4.4	0.5	0.5	0.5

Heavy Duty Series

No. of spline keys	Centering	d_1	d_2	b	d_3 min	g max	k max	r max
10	On inside diameter or flanks	16	20	2.5	14	0.3	0.3	0.15
		18	23	3	15.6	0.3	0.3	0.15
		21	26	3	18.44	0.3	0.3	0.15
		23	29	4	20.3	0.3	0.3	0.15
		26	32	4	23	0.4	0.4	0.15
		28	35	4	24.4	0.4	0.4	0.25
		32	40	5	28	0.4	0.4	0.25
		36	45	5	31.3	0.4	0.4	0.25
		42	52	6	36.9	0.5	0.4	0.4
		46	56	7	40.9	0.5	0.5	0.4
16	On flanks	52	60	5	47	0.5	0.5	0.4
		56	65	5	50.6	0.5	0.5	0.4
		62	72	6	56.1	0.5	0.5	0.4
		72	82	7	65.9	0.5	0.5	0.4
20		82	92	6	75.6	0.5	0.5	0.4
		92	102	7	85.5	0.5	0.5	0.4
		102	115	8	93.7	0.5	0.5	0.4
		112	125	9	103.7	0.5	0.5	0.4

Tolerances for Spline Hub and Shaft

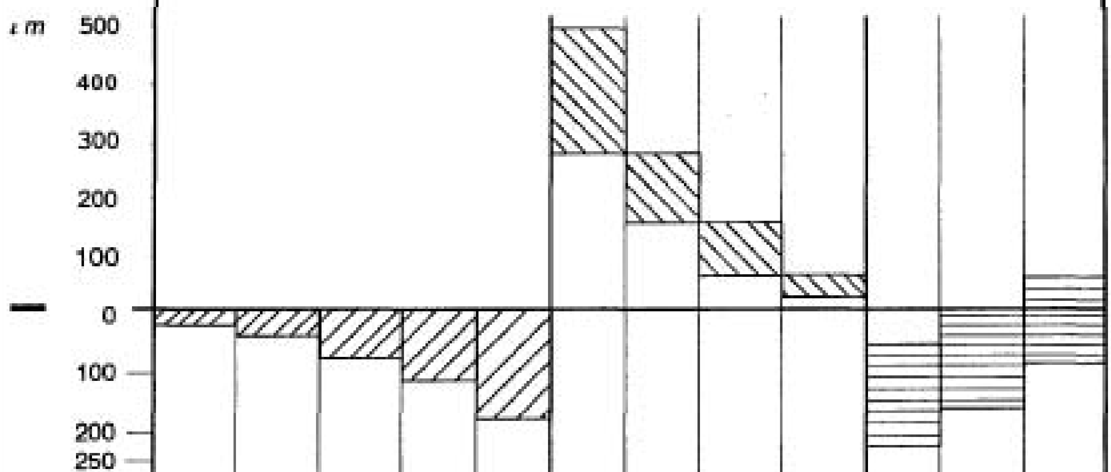
			b Hub unhardened	b Hub hardened	d_1 Hub hardened or unhardened	d_2 Hub hardened or unhardened
Spline hub	For centering on inside diameter or on flanks		D9	F10	H7	H11
Spline shaft	For centering on inside diameter	Shaft sliding inside hub	h8	e8	f7	a11
		Shaft fixed inside hub	p6	h6	j6	
	For centering on inside flanks	Staff sliding inside hub	h8	e8	—	a11
		Shaft fixed inside hub	u6	k6	—	

APPENDIX S

Tables of Dimensional Tolerances

TOLERANCE VALUES IN μm

Tolerances for Nominal Dimensions 450 mm pictorially represented (as an example)



RANGE OF NOMINAL DIMENSION mm		SHAFT					BORE				KEYWAY		
		h5	h6	h7	h8	h9	D10	E8	F8	G6	P9	N9	JS9
1 TO	3	0	0	0	0	0	+ 60	+ 28	+ 20	+ 8	− 6	− 4	+ 12.
		− 4	− 6	− 10	− 14	− 25	+ 20	+ 14	+ 6	+ 2	− 31	− 29	− 12.
OVER	3	0	0	0	0	0	+ 78	+ 38	+ 28	+ 12	− 12	− 0	+ 15
UPTO	6	− 5	− 8	− 12	− 18	− 30	+ 30	+ 20	+ 10	+ 4	− 42	− 30	− 15
OVER	6	0	0	0	0	0	+ 98	+ 47	+ 35	+ 14	− 15	0	+ 18
UPTO	10	− 6	− 9	− 15	− 22	− 36	+ 40	+ 25	+ 13	+ 5	− 51	− 36	− 18
OVER	10	0	0	0	0	0	+ 120	+ 59	+ 43	+ 17	− 18	0	+ 21..
UPTO	18	− 8	− 11	− 18	− 27	− 43	+ 50	+ 32	+ 16	+ 6	− 61	− 43	− 21..
OVER	18	0	0	0	0	0	+ 149	+ 73	+ 53	+ 20	− 22	0	+ 26
UPTO	30	− 9	− 13	− 21	− 33	− 52	+ 65	+ 40	+ 20	+ 7	− 74	− 52	− 26
OVER	30	0	0	0	0	0	+ 180	+ 89	+ 64	+ 25	− 26	0	+ 31
UPTO	50	− 11	− 16	− 25	− 39	− 62	+ 80	+ 50	+ 25	+ 9	− 88	− 62	− 31
OVER	50	0	0	0	0	0	+ 220	+ 106	+ 76	+ 29	− 32	0	+ 37
UPTO	80	− 13	− 19	− 30	− 46	− 74	+ 100	+ 60	+ 30	+ 10	− 106	− 74	− 37
OVER	80	0	0	0	0	0	+ 260	+ 126	+ 90	+ 34	− 37	0	+ 43.
UPTO	120	− 15	− 22	− 35	− 54	− 87	+ 120	+ 72	+ 36	+ 12	− 124	− 87	− 43.
OVER	120	0	0	0	0	0	+ 305	+ 148	+ 106	+ 39	− 43	0	+ 50
UPTO	180	− 18	− 25	− 40	− 63	− 100	+ 145	+ 85	+ 43	+ 14	− 143	− 100	− 50
OVER	180	0	0	0	0	0	+ 355	+ 172	+ 122	+ 44	− 50	0	+ 57..
UPTO	250	− 20	− 29	− 46	− 72	− 115	+ 170	+ 100	+ 50	+ 15	− 165	− 115	− 57..
OVER	250	0	0	0	0	0	+ 400	+ 191	+ 137	+ 49	− 56	0	+ 65
UPTO	315	− 23	− 32	− 52	− 81	− 130	+ 190	+ 110	+ 56	+ 17	− 186	− 130	− 65
OVER	315	0	0	0	0	0	+ 440	+ 214	+ 151	+ 54	− 62	0	+ 70
UPTO	400	− 25	− 36	− 57	− 89	− 140	+ 210	+ 125	+ 62	+ 18	− 202	− 140	− 70
OVER	400	0	0	0	0	0	+ 480	+ 232	+ 165	+ 60	− 68	0	+ 77.
UPTO	500	− 27	− 40	− 63	− 97	− 155	+ 230	+ 135	+ 68	+ 20	− 223	− 155	− 77.

RANGE OF NOMINAL DIMENSION mm	BORE				SHAFT							
	P6	F7	N7	C11	j5	g6	r6	u7	x7	f8	d9	u9
1 TO 3	− 6 − 12	+ 16 + 6	− − 14	+120 + 60	+ 2 − 2	− 2 − 8	+ 16 + 10	+ 28 + 18	+ 30 + 20	− 6 − 20	− 20 − 45	—
OVER 3 UPTO 6	− 9 − 17	+ 22 + 10	− 4 − 16	+145 + 70	+ 3 − 2	− 4 − 12	+ 23 + 15	+ 35 + 23	+ 40 + 28	− 10 − 28	− 30 − 60	+ 53 + 23
OVER 6 UPTO 10	− 12 − 21	+ 28 + 13	− 4 − 19	+170 + 80	+ 4 − 2	− 5 − 14	+ 28 + 19	+ 43 + 28	+ 49 + 34	− 13 − 35	− 40 − 76	+ 64 + 28
OVER 10 UPTO 14	− 15 − 26	+ 34 + 16	− 5 − 23	+205 + 95	+ 5 − 3	− 6 − 17	+ 34 + 23	+ 51 + 33	+ 56 + 40	− 16 − 43	− 50 − 93	+ 76 + 33
OVER 14 UPTO 18									+ 63			
OVER 18 UPTO 24	− 18 − 31	+ 41 + 20	− 7 − 28	+240 +110	+ 5 − 4	− 7 − 20	+ 41 + 28	+ 62 + 41	+ 75 + 54	− 20 − 53	− 65 − 117	—
OVER 24 UPTO 30									+ 85 + 64			+ 100 48
OVER 30 UPTO 40	− 21 − 37	+ 50 + 25	− 8 − 33	+280 +120	+ 6 − 5	− 9 − 25	+ 50 + 34	+ 85 + 60	+ 105 + 80	− 25 − 64	− 80 − 142	+ 122 + 60
OVER 40 UPTO 50				+290 +130				+ 95 + 70	+ 122			+ 132 + 70
OVER 50 UPTO 65	− 25 − 45	+ 60 + 30	− 9 − 39	+330 +140	+ 6 − 7	− 10 − 29	+ 60 + 41	+ 117	+ 152 + 122	− 30 − 76	− 100 − 174	+ 161 + 87
OVER 65 UPTO 80				+340 +150			+ 62 + 43	+ 132 + 102	+ 176			+ 176 + 102
OVER 80 UPTO 100	− 30 − 52	+ 71 + 36	− 10 − 45	+390 +170	+ 6 − 9	− 12 − 34	+ 73 + 51	+ 159 + 124	+ 213 + 178	− 36 − 90	− 120 − 207	+ 211 + 124
OVER 100 UPTO 120				+400 +180			+ 76 + 54	+ 175 + 144	+ 245			+ 231 + 144
OVER 120 UPTO 140	− 36 − 61	+ 83 + 43	− 12 − 52	+450 +200	+ 7 − 11	− 14 − 39	+ 88 + 63	+ 210 + 170	+ 200 + 240	− 42 − 106	− 145 − 245	+ 270 + 170
OVER 140 UPTO 160				+460 +210			+ 90 + 65	+ 230 + 190	+ 320 + 260			+ 290 + 190
OVER 160 UPTO 180				+480 +230			+ 93 + 68	+ 250	—	—		+ 310 + 210
OVER 180 UPTO 200	− 41 − 70	+ 96 + 50	− 14 − 60	+530 +240	+ 7 − 13	− 15 − 44	+ 106 + 77	+ 282 + 236	—	− 50 − 122	− 170 − 285	+ 351 + 236
OVER 200 UPTO 225				+550 +260			+ 109 + 80	+ 304 + 258				+ 373 + 258
OVER 225 UPTO 250				+570 +280			+ 113 + 84	+ 330 + 284				+ 399 + 284
UPTO 280	− 47 − 79	+ 108 + 56	− 14 − 66	+620 +300	+ 7 − 16	− 17 − 49	+ 126 + 94	+ 367 + 315	—	− 56 − 137	− 190 − 320	+ 445 + 315
UPTO 315				+650 +330			+ 130 + 98	+ 402 + 350				+ 480 + 350
OVER 315 UPTO 355	− 51 − 87	+ 119 + 62	− 16 − 73	+720 +360	+ 7 − 18	− 18 − 54	+ 144 + 106	+ 447 + 390	—	− 62 − 151	− 210 − 350	+ 530 + 390
OVER 355 UPTO 400				+760 +400			+ 150 + 114	+ 492 + 435				+ 575 + 435
OVER 400 UPTO 450	− 55 − 95	+ 131 + 68	− 17 − 80	+840 +440	+ 7 − 20	− 20 − 60	+ 166 + 126	+ 553 + 490	—	− 68 − 165	− 230 − 385	+ 645 + 490
OVER 450 UPTO 500				+880 +480			+ 172 + 132	+ 603 + 540				+ 696 + 540

RANGE OF NOMINAL DIMENSION mm	BORE											
	D9	E6	E9	H5	H6	H9	H10	H12	H13	H14	u8	x8
1 TO 3	+45 +20	+20 +14	+39 +14	+4 0	+6 0	+25 0	+40 0	+100 0	+140 0	+250 0	+32 +18	+34 +20
OVER 3 UPTO 6	+60 +30	+28	+50 +20	+5 0	+8 0	+30 0	+48 0	+120 0	+180 0	+300 0	+41 +23	+46 +28
OVER 6 UPTO 10	+76 +40	+34 +25	+61 +25	+6 0	+9 0	+36 0	+58 0	+150 0	+220 0	+360 0	+50 +28	+56 +34
OVER 10 UPTO 14	+93 +50	+43 +32	+75 +32	+8 0	+11 0	+43 0	+70 0	+180 0	+270 0	+430 0	+60 +33	+87 +40
OVER 14 UPTO 18												+72 +45
OVER 18 UPTO 24	+117 +65	+53 +40	+92 40	+9 0	+13 0	+52 0	+84 0	+210 0	+330 0	+520 0	+74 +41	+87 +54
OVER 24 UPTO 30											+81 +48	+97 +64
OVER 30 UPTO 40	+142 +80	+66 +50	+112 +50	+11 0	+16 0	+62 0	+100 0	+250 0	+390 0	+620 0	+99 +60	+119 +80
OVER 40 UPTO 50											+109 +70	+136 +97
OVER 50 UPTO 65	+174 +100	+79 +60	+134 +60	+13 0	+19 0	+74 0	+120 0	+300 0	+460 0	+740 0	+133 +87	+168 +122
OVER 65 UPTO 80											+148 +102	+192 +146
OVER 80 UPTO 100	+207 +120	+94 +72	+159 +72	+15 0	+22 0	+87 0	+140 0	+350 0	+540 0	+870 0	+178 +124	+232 +178
OVER 100 UPTO 120											+198 +144	+264 +210
OVER 120 UPTO 140	+245 +145	+110 +85	+185 +85	+18 0	+25 0	+100 0	+160 0	+400 0	+630 0	+1000 0	+233	+311
OVER 140 UPTO 160												
OVER 160 UPTO 180												
OVER 180 UPTO 200	+285 +170	+129 +100	+215 +100	+20 0	+29 0	+115 0	+185 0	+460 0	+720 0	+1150 0		
OVER 200 UPTO 225												
OVER 225 UPTO 250												
OVER 250 UPTO 280	+320 +190	+142 +110	+240 +110	+23 0	+32 0	+130 0	+210 0	+520 0	+810 0	+1300 0		
OVER 280 UPTO 315												
OVER 315 UPTO 355	+350 +210	+161 +125	+265 +125	+25 0	+36 0	+140 0	+230 0	+570 0	+890 0	+1400 0		
OVER 355 UPTO 400												
OVER 400 UPTO 450	+385 +230	+175 +135	+290 +135	+27 0	+40 0	+155 0	+250 0	+630 0	+970 0	+1550 0	+490	—
OVER 450 UPTO 500											+637 +540	—

RANGE OF NOMINAL DIMENSION mm	BORE			SHAFT								
	H7	H8	H11	j6	k6	m6	n6	p6	f7	e8	h11	a11
1 TO 3	+ 10 0	+ 14 0	+ 60 0	+ 4 − 2	+ 6 0	+ 8 + 2	+ 10 + 4	+ 12 + 6	− 6 − 16	− 14 − 28	0 − 60	− 270 − 330
OVER 3 UPTO 6	+ 12 0	+ 18 0	+ 75 0	+ 6 − 2	+ 9 + 1	+ 12 + 4	+ 16 + 8	+ 20 + 12	− 10 − 22	20 − 38	0 − 75	− 270 − 345
OVER 6 UPTO 10	+ 15 0	+ 22 0	+ 90 0	+ 7 − 2	+ 10 + 1	+ 15 + 6	+ 19 + 10	+ 24 + 15	− 13 − 28	− 25 − 47	0 − 90	− 280 − 370
OVER 10 UPTO 18	+ 18 0	+ 27 0	+ 110 0	+ 8 − 3	+ 12 + 1	+ 18 + 7	+ 23 + 12	+ 29 + 18	− 16 − 34	− 32 − 59	0 − 110	− 290 − 400
OVER 18 UPTO 30	+ 21 0	+ 33 0	+ 130 0	+ 9 − 4	+ 15 + 2	+ 21 + 8	+ 28 + 15	+ 35 + 22	− 20 − 41	− 40 − 73	0 − 130	− 300 − 430
OVER 30 UPTO 40	+ 25 0	+ 39 0	+ 160 0	+ 11 − 5	+ 18 + 2	+ 25 + 9	+ 33 + 17	+ 42 + 26	− 25 − 50	− 50 − 89	0 − 160	− 310 − 470
OVER 40 UPTO 50	+ 25 0	+ 39 0	+ 160 0	+ 11 − 5	+ 18 + 2	+ 25 + 9	+ 33 + 17	+ 42 + 26	− 25 − 50	− 50 − 89	0 − 160	− 320 − 480
OVER 50 UPTO 65	+ 30 0	+ 46 0	+ 190 0	+ 12 − 7	+ 21 + 2	+ 30 + 11	+ 39 + 20	+ 51 + 32	− 30 − 60	− 60 − 106	0 − 190	− 340 − 530
OVER 65 UPTO 80	+ 30 0	+ 46 0	+ 190 0	+ 12 − 7	+ 21 + 2	+ 30 + 11	+ 39 + 20	+ 51 + 32	− 30 − 60	− 60 − 106	0 − 190	− 360 − 550
OVER 80 UPTO 100	+ 35 0	+ 54 0	+ 220 0	+ 13 − 9	+ 25 + 3	+ 35 + 13	+ 45 + 23	+ 59 + 37	− 36 − 71	− 72 − 126	0 − 220	− 380 − 600
OVER 100 UPTO 120	+ 35 0	+ 54 0	+ 220 0	+ 13 − 9	+ 25 + 3	+ 35 + 13	+ 45 + 23	+ 68 + 43	− 36 − 71	− 72 − 126	0 − 220	− 410 − 630
OVER 120 UPTO 140	+ 40 0	+ 63 0	+ 250 0	+ 14 − 11	+ 28 + 3	+ 40 + 15	+ 52 + 27	+ 68 + 43	− 43 − 83	− 85 − 148	0 − 250	− 460 − 710
OVER 140 UPTO 160	+ 40 0	+ 63 0	+ 250 0	+ 14 − 11	+ 28 + 3	+ 40 + 15	+ 52 + 27	+ 68 + 43	− 43 − 83	− 85 − 148	0 − 250	− 520 − 770
OVER 160 UPTO 180	+ 40 0	+ 63 0	+ 250 0	+ 14 − 11	+ 28 + 3	+ 40 + 15	+ 52 + 27	+ 68 + 43	− 43 − 83	− 85 − 148	0 − 250	− 580 − 830
OVER 180 UPTO 200	+ 46 0	+ 72 0	+ 290 0	+ 16 − 13	+ 33 + 4	+ 46 + 17	+ 60 + 31	+ 79 + 50	− 50 − 96	− 100 − 172	0 − 290	− 660 − 950
OVER 200 UPTO 225	+ 46 0	+ 72 0	+ 290 0	+ 16 − 13	+ 33 + 4	+ 46 + 17	+ 60 + 31	+ 79 + 50	− 50 − 96	− 100 − 172	0 − 290	− 740 − 1030
OVER 225 UPTO 250	+ 46 0	+ 72 0	+ 290 0	+ 16 − 13	+ 33 + 4	+ 46 + 17	+ 60 + 31	+ 79 + 50	− 50 − 96	− 100 − 172	0 − 290	− 820 − 1110
OVER 250 UPTO 280	+ 52 0	+ 81 0	+ 320 0	+ 16 − 16	+ 36 + 4	+ 52 + 20	+ 66 + 34	+ 88 + 56	− 56 − 108	− 110 − 191	0 − 320	− 920 − 1240
OVER 280 UPTO 315	+ 52 0	+ 81 0	+ 320 0	+ 16 − 16	+ 36 + 4	+ 52 + 20	+ 66 + 34	+ 88 + 56	− 56 − 108	− 110 − 191	0 − 320	− 1050 − 1370
OVER 315 UPTO 355	+ 57 0	+ 89 0	+ 360 0	+ 18 − 18	+ 40 + 4	+ 57 + 21	+ 73 + 37	+ 98 + 62	− 62 − 119	− 125 − 214	0 − 360	− 1200 − 1560
OVER 355 UPTO 400	+ 57 0	+ 89 0	+ 360 0	+ 18 − 18	+ 40 + 4	+ 57 + 21	+ 73 + 37	+ 98 + 62	− 62 − 119	− 125 − 214	0 − 360	− 1350 − 1710
OVER 400 UPTO 450	+ 63 0	+ 97 0	+ 400 0	+ 20 − 20	+ 45 + 5	+ 63 + 23	+ 80 + 40	+ 108 + 68	− 68 − 131	− 135 − 232	0 − 400	− 1500 − 1900
OVER 450 UPTO 500	+ 63 0	+ 97 0	+ 400 0	+ 20 − 20	+ 45 + 5	+ 63 + 23	+ 80 + 40	+ 108 + 68	− 68 − 131	− 135 − 232	0 − 400	− 1650 − 2050

APPENDIX T

Tables of SI Units and Conversion Factors

Base Units

Sl. No.	Quantity	Unit	Symbol
1	Length	metre	m
2	Mass	kilogram	kg
3	Time	second	s
4	Electric current	ampere	A
5	Thermodynamic temperature	kelvin	K
6	Luminous intensity	candela	cd
7	Amount of substance	mole	mol

Definitions

1. The metre is the length equal to 1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transition between the energy levels 2_{p10} and 5_{d5} of the krypton-86 atom.

2. The kilogram is the mass of the international prototype of the kilogram which is in the custody of the Bureau International des Poids et. Mesures (BIPM) at Serves, near Paris.

3. The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

4. The ampere is the constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed at a distance of 1 metre apart in a vacuum, would produce between the conductors a force equal to 2×10^{-7} newton per metre of length.

5. The kelvin, unit of thermodynamic temperature, is the fraction $\frac{1}{273.16}$ of the thermo-dynamic temperature of the triplex point of water.

6. The candela is the luminous intensity in the perpendicular direction, of a surface of $\frac{1}{600,000}$ square metre of a black body at the temperature of freezing plantinum under a pressure of 101,325 newtons per square metre.

7. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.

Supplementary Units

In addition to the main units discussed above, there are two supplementary units measurement covering plane angle and solid angle. *

1. The radian is the plane angle which, having its vertex at the centre of a circle, cuts off a length on the circumference of the circle equal to the radius of the circle.
2. The steradian is the solid angle which, having its vertex at the centre of a sphere, cuts off an area on the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

Derived Units with Special Units Symbol

The SI units are augmented by the derived units. Special names have been adopted for some of the derived SI units, together with special unit symbols, which are given below.

The symbols appearing in the Quantity column below correspond to the recommendations of the ISO. It is preferred that these symbols are used in equations and formulae to effect overall uniformity in scientific and technical writings. For example, such symbol for force is F and is no longer P or Q or K or any other letter or symbol.

ISO Symbols end Definitions

Quantity	*Unit*	*Definition*
Force (F)	N	1 newton is that force which, when applied to a body having a mass of 1 kilogram gives it an acceleration of 1 metre per second squared 1 N = 1kg m/s^2
Work (W) **Energy (E)** **Quantity of heat (Q)**	J	1 joule is the work done when the point of application of a force of 1 newton is displaced through a distance of 1 metre in the direction of the force 1 J = 1 N m = W.s
Power (P)	W	1 Watt is one Joule per second 1 W = 1 J/s
Pressure (P) Normal stress (σ) Shear stress (τ)	Pa	1 pascal is the pressure equally exerted on an area of 1 m^2 by a force of 1 newton acting vertically 1 Pa= 1 N/m^2
Frequency (f)	Hz	1 hertz is equal to the frequency of one cycle per second 1 Hz = 1/s

Derived Units with Complex Names

Some derived SI units with complex names are in use.

SI. No	Quantity		SI unit	Unit symbol
1	Area	A	Square metre	m^2
2	Volume	V	Cubic metre	m3
3	Density (mass density)	ρ	Kilograms per cubic metre	kg/m^3
4	Velocity	v	Metres per second	m/s
5	Angular velocity	ω	Radians per second	rad/s
6	Acceleration	a	Metres per second squared	m/s^2
7	Angular acceleration	a	Radians per second squared	rad/s^2

Decimal fractions and multiples of SI units along with the relevant prefixes are given in the following table. These are approved factors by which the base units are to be multiplied to arrive at a higher or a smaller new unit convenient for the purpose of calculation.

Multiples of SI Units

Fraction	Prefix	Symbol	Multiple	Prefix	Symbol
10^{-1}	deci	d	10^1	deka	da
10^{-2}	centi	c	10^2	hecto	h
10.3	milli	m	10^3	kilo	k
10^{-6}	micro	µ	10^6	mega	M
10^{-9}	nano	n	10^9	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f			
10^{-18}	atto	a			

Note: Compound prefixes should not be used, namely, 10^{-9} metre is represented by 1nm, and not 1m µm.

Conversion Factors

The equivalent values or the conversion factors of some important units in common usage are given in the following table. In each table, the SI units are written within thick boxes

Conversion Factors for Units of Force

Units of force	Newton (N)	Kilogram force (kgf) = kp	Dyne
1N	1	0.102	10^5
1 kgf	≈ 9.81	1	9.81×10^5
1 dyne	10^{-5}	$\approx 1.02 \times 10^{-6}$	1

Units of work, energy and torque	Joule	Kilowatt-hour	Newton-metre	Erg	kgf m
1 joule (J) = 1 Watt-second (Ws)	1	0.277778×10^{-6}	1	10^7	0.102
1 kilowatt-hour (kwh)	3.6×10^6	1	3.6×10^6	3.6×10^{13}	$\approx 0.367 \times 10^6$
1 Newton-metre (N m) 1 Volt-ampere-second (VAs)	1	0.277778×10^{-6}	1	10^7	0.102
1 erg	10^{-7}	0.277778×10^{-13}	10^{-7}	1	$\approx 0.102 \times 10^{-7}$
1 kilogram force metre = 1 kgfm	9.81	2.72407×10^{-6}	9.81	$\approx 9.81 \times 10^7$	1

Units of power and energy rate	wan	Kilowatt	Newton-metre per second	kgfm/s	Metric horse-power (PS)
1 watt (W) = 1 joule/second (J/s)	1	10^{-3}	1	0.102	1.36×10^{-3}
1 kilowatt (kW)	10^3	1	10^3	102	1.36
1 newton metre/second (1 Nm/s)	1	10^{-3}	1	0.102	$\approx 1.36 \times 10^{-3}$
1 kilogram-forcem per second (1 kgf m/s)	9.81	$\approx 9.81 \times 10.3$	9.81	1	$\frac{1}{75}$
1 metric horse-power (PS)	735.5	0.7355	735.5	75	1

Conversion Factors for Units of Stress

Units of stress	Pa = N/m2	M Pa = N/mm^2	N/cm^2	kgf/mm^2	kgf/cm^2
1 pascal (Pa) = 1 N/m^2	1	10^{-6}	10.4	$\approx 0.102 \times 10^{-6}$	$\approx 0.102 \times 10^{-4}$
1 megapascal (MPa) = 1 N/mm^2	10^6	1	10^2	0.102	10.2
1 N/cm^2	10^4	10^{-2}	1	$\approx 0.102 \times 10^{-2}$	0.102
1 kgf/mm^2	$\approx 9.81 \times 10^6$	9.81	$\approx 9.81 \times 10^2$	1	10^2
1 kgf/cm^2	$\approx 9.81\ 10^4$	$\approx 9.81 \times 10^{-2}$	9.81	10^{-2}	1

Conversion Factors for Units of Pressure

Units of pressure	Pa = N/m^2	Bar	kgf/cm^2	Atm	Torr
1 pascal (Pa) = 1 N/m^2	1	10^{-5}	$\approx 0.102 \times 10^{-4}$	0.98692×10^{-5}	0.75006×10^{-2}
1 bar = 0.1 MPa = 10^6 dyne/cm^2	10^5	1	1.01972	0.98692	750
1 technical atmosphere = 1 at = 1 kgf/cm^2	$\approx 9.81 \times 10^4$	$\approx 9.81 \times 10^{-1}$	1	9.67841×10^{-1}	736
1 physical atmosphere = 1 atm	$\approx 1.01 \times 10^5$	1.01325	1.03323	1	760
1 torr = 1 mm of mercury = 1 mm of Hg	$\approx 1.33 \times 10^2$	1.333224×10^{-3}	1.35951×10^{-3}	1.31579×10^{-3}	1

The equivalent values of other units in SI units are given in the following table.

Some Other Conversion Factors

Sl. No.	*Quantity*	Unit	*Equivalent*
1	Length	1 angstrom 1 inch 1 foot 1 yard 1 mile 1 nautical mile, international 1 chain 1 engineer's chain	10^{-10} m 0.0254 m 0.3048 m 0.9144 m 1.60934 km 1.85318 km 20.12 m 30.48 m
2	Area	1 square inch 1 square foot 1 square yard 1 square mile 1 acre	645.16 mm^2 0.092903 m^2 0.836127 m^2 2.58999 km^2 4047 m^2
3	Velocity	1 inch per second 1 foot per second 1 foot per minute 1 mile per hour	25.4 mm/s 0.3048 m/s 0.00508 m/s 1.609 km/h
4	Acceleration	1 foot per second squared	0.3048 m/s^2
5	Mass	1 pound 1 slug	0.45359237 kg 14.5939 kg
6	Density	1 pound per cubic inch 1 pound per cubic foot	2.76799×10^4 kg m^{-3} 16.0185 kg m^{-3}

(Contd)

(Conrd)

Sl. No.	*Quantity*	*Unit*	*Equivalent*
7	Force	1 dyne 1 poundal 1 pound-force 1 kilogram-force	10^{-5} N 0.138255 N 4.44822 N 9.80655 N
8	Pressure	1 atmosphere (phys) 1 pound force per square inch 1 torr	101 325 kN m^{-2} 6 89476 k N m^{-2} 133 322 N m^{-2}
9	Energy	1 erg 1 calorie (IT) 1 calorie (15°C) 1 calorie (thermochemical) 1 Btu 1 foot poundal 1 foot pound-force	10^{-7} J 4 1868 J 4 1855 J 4.184 J 1055 06 J 0 042149 J 135582 J
10	Power	1 hp (horse power) 1 metric horse power (PS)	745 7 W 735.5 W
11	Volume	1 litre 1 cubic inch 1 cubmfoot 1 British gallon	10^{-3} m3 163871 × 10^{-5} m^3 0 0283168 m^3 0 004546092 m^3

APPENDIX U

Tables of Preferred Numbers and Sizes

Preferred Numbers

Basic numbers				*Exact numbers*	*Variation of the basic numbers from the exact numbers* %	*Mantissas*
Basic series						
R5	*R10*	*R20*	*R40*			
			1.00	1.0000	0	000
			1.06	1.0593	+0.07	025
		1.00	1.12	1.1220	−0.18	050
	1.00	1.12	1.18	1.1885	−0.71	075
1.00	1.25	1.25	1.25	1.2589	0.71	100
		1.40	1.32	1.3335	−1.01	125
			1.40	1.4125	−0.88	150
			1.50	1.4962	+0.25	175
			1.60	1.5849	+0.95	200
			1.70	1.6788	+1.26	225
		1.60	1.80	1.7783	+1.22	250
	1.60	1.80	1.90	1.8836	+0.87	275
1.60	2.00	2.00	2.00	1.9953	+0.24	300
		2.24	2.12	2.1135	+0.31	325
			2.24	2.2387	+0.06	350
			2.36	2.3714	−0.48	375
			2.50	2.5119	−0.47	400
			2.65	2.6607	−0.40	425
		2.50	2.80	2.8184	−0.65	450
	2.50	2.80	3.00	2.9854	+0.49	475
2.50	3.15	3.15	3.15	3.1623	−0.39	500
		3.55	3.35	3.3497	+0.01	525
			3.55	3.5481	+0.05	550
			3.75	3.7584	−0.22	575
			4.00	3.9811	+0.47	600
			4.25	4.2170	+0.78	625
		4.00	4.50	4.4668	+0.74	650
	4.00	4.50	4.75	4.7315	+0.39	675
4.00	5.00	5.00	5.00	5.0119	−0.24	700
		5.60	5.30	5.3088	−0.17	725
			5.60	5.6234	−0.42	750
			6.00	5.9566	+0.73	775

(Contd)

(Contd)

Basic numbers				Exact numbers	Variation of the basic numbers from the exact numbers	Mantissas
Basic series						
R5	*R10*	*R20*	*R40*		%	
			6.30	6.3096	-0.15	800
			6.70	6.6834	+0.25	825
		6.30	7.10	7.0795	+0.29	850
	6.30	7.10	7.50	7.4989	+0.01	875
6.30	8.00	8.00	8.00	7.9433	+0.71	900
		9.00	8.50	8.4140	+1.02	925
			9.00	8.9125	+0.98	950
			9.50	9.4406	+0.63	975

Preterred Sizes

Preferred sizes

0.1	1	10	100				370
			105				375
	1.1	11	110			38	380
			115				390
0.12	1.2	12	120	0.4	4	40	400
			125				410
		13	130			42	420
			135				430
	1.4	14	140			44	440
			145		4.5	45	450
	1.5	15	150			46	460
			155				470
0.16	1.6	16	160			48	480
			165				490
		17	170	0.5	5	50	500
			175			52	520
	1.8	18	180			53	530
			185		5.5	55	550
		19	190			56	560
			195			58	580
0.2	2	20	200	0.6	6	60	600
		21	210			62	
	2.2	22	220			63	630
		23	230			65	650
		24	240			67	670
0.25	2.5	25	250			68	
		26	260		7	70	700
			270			71	710
	2.8	28	280			72	
			290			75	750
0.3	3	30	300			78	
			310	0.8	8	80	800
			315			82	
	3.2	32	320			85	850
			330			88	
		34	340		9	90	900
	3.5	35	350			92	
			355			95	950
		36	360			98	

APPENDIX V

Surface Quality Symbols and Their Values

It is not possible to achieve an ideal surface of a work-piece in practice. There will always be some deviations from the ideal and the actual values may be quite different. Hence, to limit such deviations of dimensional and surface quality values, elaborate systems of quality control methods have been devised. Since the ideal conditions are ordinarily not attainable in practice, except at exhorbitant costs which do not permit the product to be economically viable and competitive in market, some kind of tolerance and permissible surface conditions of the work-piece have to be conceded for the dimensions of the part as well as for its surface. The dimensional tolerances have been given in Appendix S. In this appendix, the surface quality and the permissible values thereof are dealt with. Besides other relevant data, an engineering drawing must, therefore, also contain information about these allowable values of deviations.

Surface condition is a function of the process of manufacture, machining or finishing procedures adopted and sometimes, of the post heat-treatment imparted to the work-piece. Theoretically, it is possible to prescribe all varieties of surface quality. In practice, however, the surface roughness values are determined in relation to the nominal dimension and the tolerance-grade of the job. There is a direct relationship between the dimensional tolerance on a part and the permissible surface roughness. Assigning of undue accuracy will only enhance the production cost of the piece. Hence, the designer should very carefully select the tolerance, zone and surface quality required, keeping in mind the ultimate use to which the product will be subjected.

Different methods are being followed for indicating surface roughness in an engineering drawing. The undermentioned method is followed as per IS: 696.

The basic grade symbol consists of two legs of unequal lengths representing the surface under consideration as shown in Fig. V.1 (a). If the removal of material by machining is required, then a cross-bar is added to the basic symbol as shown in Fig. V.1 (b). The condition where such removal is not permitted is indicated by adding a circle as shown in Fig. V.1 (c). When special surface characteristics have to be indicated, a horizontal line is added to the longer leg. [Fig. V. 1 (d)]. Positions of the specifications of surface roughness and related data with respect to the symbol have been summarised in Fig. V. 1(e). Criterion for roughness R_t indicates the value of the peak-to-valley height on an (uneven) surface and R_a is the mean roughness index which is the arithmetic mean of the absolute values of the distances between the actual and the mean profiles of the surface under consideration. Previously, root mean square (rms) values were used for R_t. Details about the surface quality, its inspection, control measures, etc. are given in books on machining and quality control and inspection.

The principal criterion of roughness (R_a) may be indicated by the corresponding grade symbol as shown in Table V.1.

Table V.1 Grade symbols for roughness index

Roughness values, R_a μm	*Grade symbol*	*Roughness values, R_a μm*	*Grade symbol*
50	N 12	0.8	N 6
25	N 11	0.4	N 5
12.5	N 10	0.2	N 4
6.3	N 9	0.1	N 3
3.2	N 8	0.05	N 2
1.6	N 7	0.025	N 1

Due to the rapid development of modern techniques and steadily increasing accuracy required for the working surfaces of the products, it has become imperative in certain cases to directly indicate the characteristics of the surfaces by inserting the roughness value, production methods, etc. in the drawing itself as shown in Fig. V. 1(e).

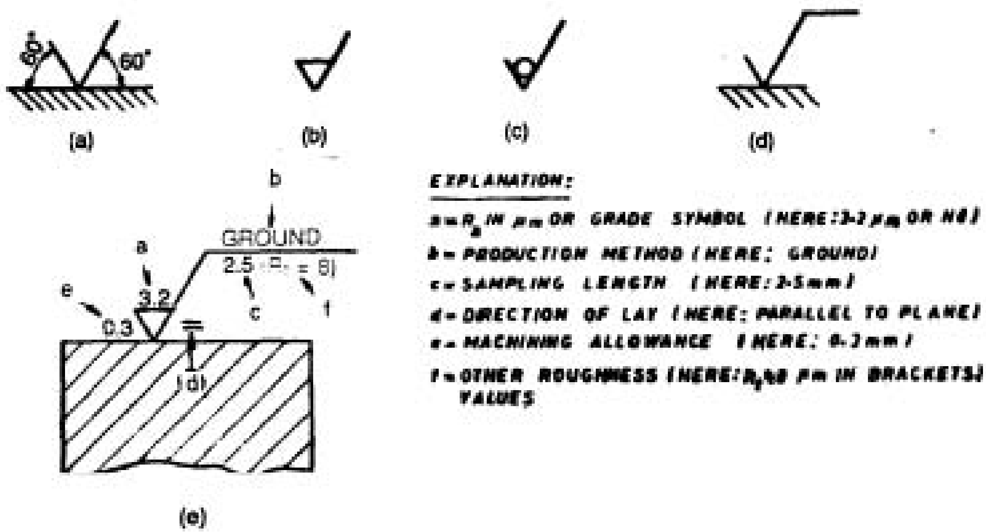
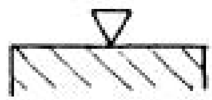
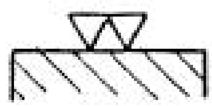
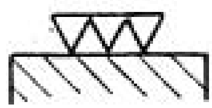
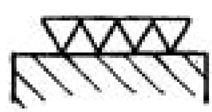


Fig. V.1 Grade symbol

However, indications of the surface quality by means of suitable triangular symbols on the drawings is also followed by many industries and design offices. Such surface quality symbols along with their corresponding values are given in Table V.2. These values are mainly based on the German Industrial Standards, DIN 3141. If the surface quality is indicated in this way, then the relevant "series" used is to be mentioned in the title of the drawing or on the drawing itself.

Table V.2 Surface quality symbols and their values

SURFACE QUALITY SYMBOL	SERIES 1	2	3	4
	MAXIMUM ALLOWABLE SURFACE ROUGHNESS R_t IN μm			
	160	100	63	25
	40	25	16	10
	16	6-3	4	2-5
	-	1	1	0.4

APPENDIX W

Checking of Gears by Means of. Balls or Rollers

In shops and inspection departments, gears are often checked by measuring a pre-specified distance between two identical balls or rollers placed in diametrically opposite tooth gaps of the gear. For this purpose, pins or wires are also employed. This indirect checking of tooth thickness is a very accurate method of gear inspection. The gauge distance is measured by means of Vernier calipers or micrometers or similar measuring devices. Within reasonable limits, the choice of the diameter of the gauging balls or rollers is arbitrary, provided of course that the diameter chosen is accurately known. These balls, etc., used for measurement purposes are accurately ground and polished to the specified dimensions.

In Sec. 2.28, we have discussed the "base tangent length" measurement system for gear checking. Though widely used, this method has its own limitations. For example, it cannot be used for checking internal gears. The method of checking by balls or rollers can be employed for both external as well as internal gears.

If the actual measurement over the balls confirms to the calculated pre-determined value M, then it can be concluded that the gear teeth have been accurately cut as per the specified requirements and size.

Basic Principle of Measurement over Balls

The underlying principles of this type of gear checking are explained below:
Referring to Fig. W.1, the following relations can be established

$$r_b = r\cos\alpha = r_1\cos\alpha_1$$

$$\therefore \quad r_1 = r\,\frac{\cos\alpha}{\cos\alpha_1}$$

whence we get the centre distance between the two opposite balls given by

$$2r_1 = 2r\,\frac{\cos\alpha}{\cos\alpha_1}$$

or

$$d_1 = d\,\frac{\cos\alpha}{\cos\alpha_1} = mz\,\frac{\cos\alpha}{\cos\alpha_1} \qquad \text{(W.1)}$$

The angle α_1 is found from the following relation (see Fig. W.1)

$$\text{inv}\,\alpha_1 = \frac{d_e/2}{r_b} + \text{inv}\,\alpha + \frac{s/2}{r} - \frac{\pi}{z} \qquad \text{(all angles in radians)}$$

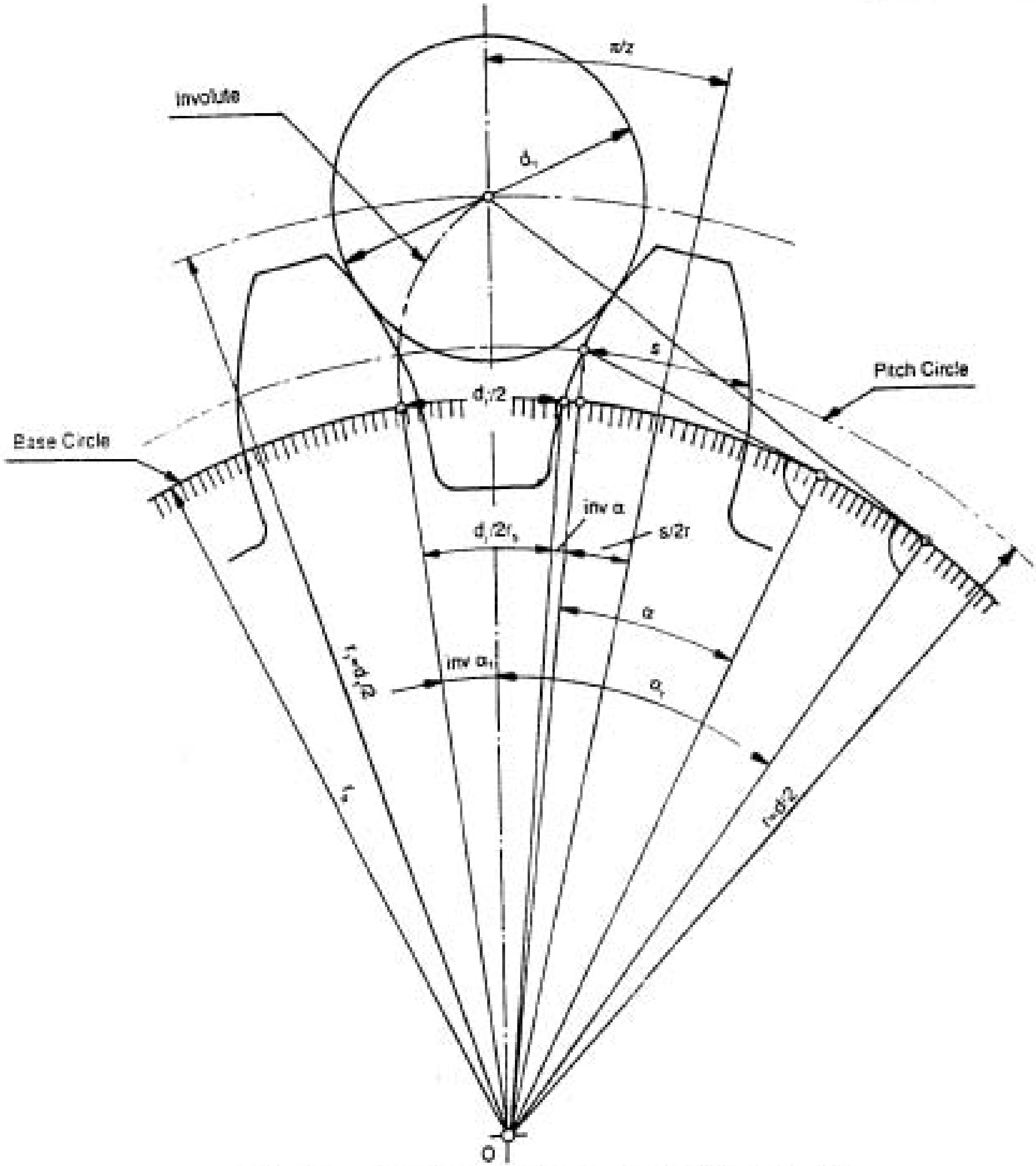


Fig. W.1 Method for measurement over balls or rollers

or

$$\operatorname{inv} \alpha_1 = \frac{d_r}{2r_b} + \operatorname{inv} \alpha + \frac{s}{2r} - \frac{\pi}{z}$$

$$= \frac{d_r}{d \cos \alpha} + \operatorname{inv} \alpha + \frac{s}{d} - \frac{\pi}{z} \qquad \textbf{(W.2)}$$

Here d_r is the diameter of the balls or rollers, the other symbols and subscripts having the usual meanings as before. [Recall the relation: $2r_b = d_b$ (base circle diameter) $= d$ (pitch circle diameter) $\times \cos a$. Also, arc $\frac{d_r}{2}$ = base circle radius, $r_b \times$ subtended angle at the centre, or, angle $= \frac{d_r}{2} / r_b = \frac{d_r}{2r_b}$. Similarly, arc $\frac{s}{2}$ = pitch circle radius, r x angle at the centre].

For uncorrected gears, tooth thickness at the pitch circle

$$s = \frac{\pi m}{2}$$

For corrected gears

$$s = \frac{\pi m}{2} \pm 2xm \tan a. \qquad \text{(As per Eq. 2.28 and 2.29)}$$

Finally, we arrive at the following expression for the value of measurement over two opposite balls in case of an external spur gear having even number of teeth (see Fig. W.2),

$$M_a = d_1 + d_r \qquad \textbf{(W.3)}$$

In case of a gear having odd number of teeth, the balls are to be placed as nearly opposite to each other as possible (see Fig. W.3). In this case, from triangle ABC, we have,

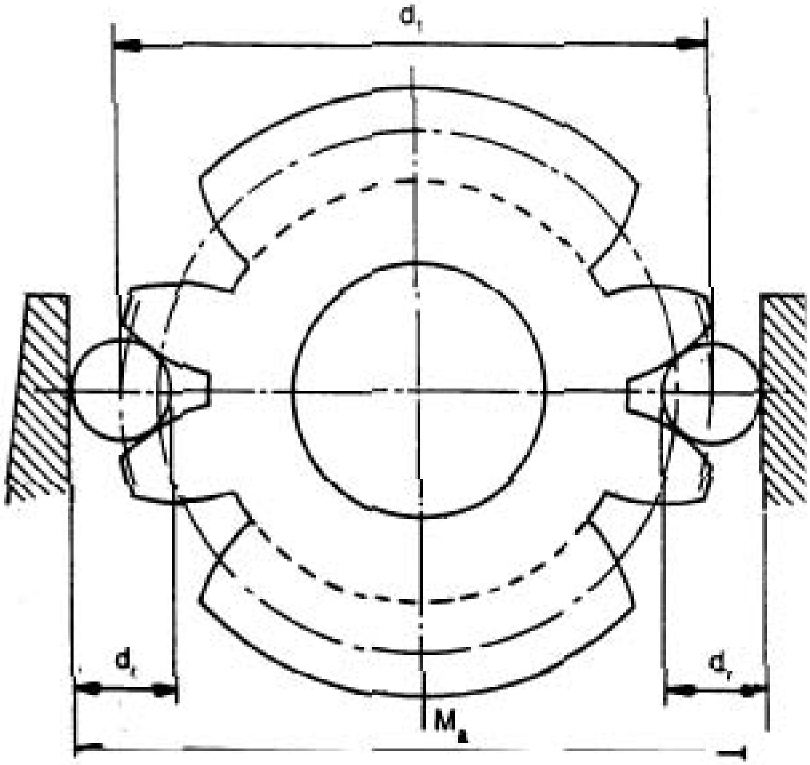


Fig. W.2 Measurement over balls in case of an external gear with an even number of teeth

$$\frac{d_1'}{2} / \frac{d_1}{2} = \frac{d_1'}{d_1} = \cos\frac{90^\circ}{z}$$

whence

$$d_1' = d_1 \cos\frac{90^\circ}{z}$$

and

$$M_a = d_1' + d_r = d_1 \cos\frac{90^\circ}{z} + d_r \quad \text{(W.4)}$$

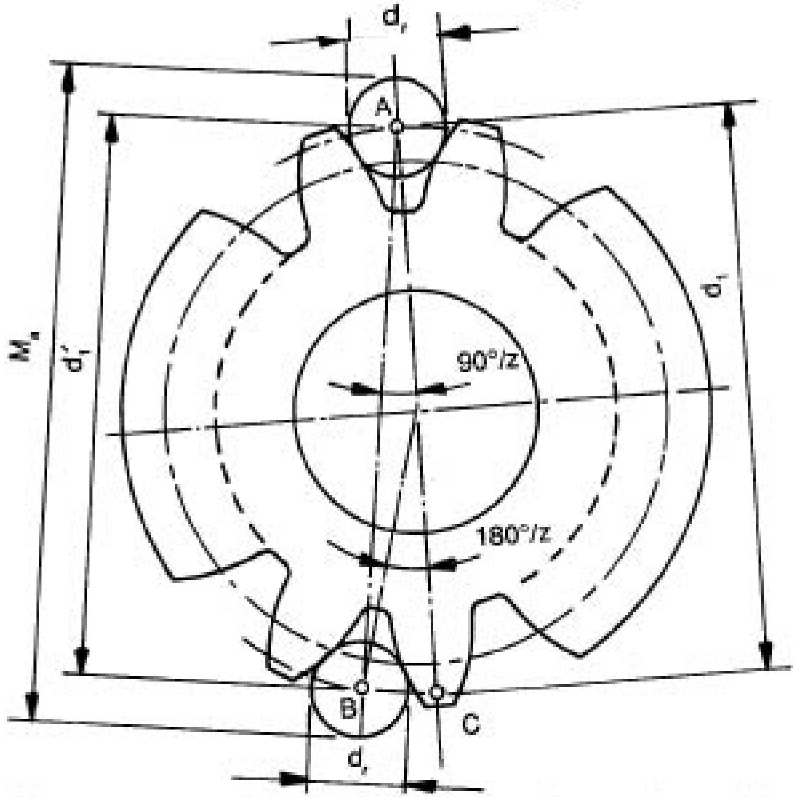


Fig. W.3 Measurement over balls in case **of** an external gear with an **odd** number of teeth

Similarly, the dimensions and parameters for checking teeth of internal spur gears can be derived. The relevant equations are as follows

$$d_1 = d\frac{\cos\alpha}{\cos\alpha_1} \quad \text{(As in Eq. W.1)}$$

$$\text{inv}\,\alpha_1 = -\frac{d_r}{d\cos\alpha} + \text{inv}\,\alpha - \frac{S}{d} + \frac{\pi}{z} \quad \text{(W.5)}$$

In case **of** an internal gear having even number of teeth (see Fig. W.4)

$$M_i = d_1 - d_r \quad \text{(W.6)}$$

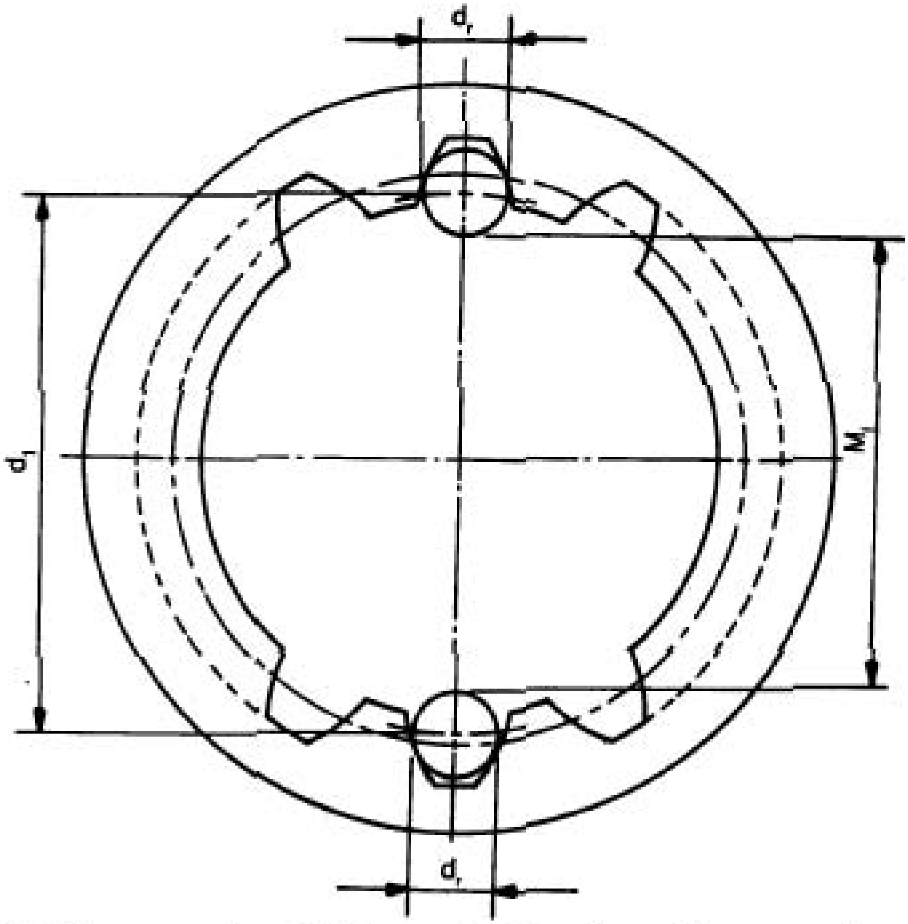


Fig. W.4 Measurement over balls in case of an internal gear with even number of teeth

In case of an internal gear having an odd number of teeth (see **Fig. W.5**),

$$M_i = d_1 \cos\frac{90^\circ}{z} - d_r \tag{W.7}$$

For helical gears, the above expressions are modified in view of the helical orientation of the gear teeth. The relevant equations are summarised below. As before, the subscripts n and t stand for the normal and transverse sections of the helical gear, respectively (see Chap. 3 on Helical Gears).

For external helical gear with even or odd teeth, we have

$$d_1 = \frac{d\cos\alpha_t}{\cos\alpha_{t_1}} = \frac{m_n\cdot z}{\cos\beta}\cdot\frac{\cos\alpha_t}{\cos\alpha_{t_1}} \tag{W.8}$$

$$\text{inv}\,\alpha_{t_1} = \frac{d_r}{m_n\cdot z\cos\alpha_t} + \text{inv}\,\alpha_t + \frac{s_n}{m_n\cdot z} - \frac{\pi}{z} \tag{W.9}$$

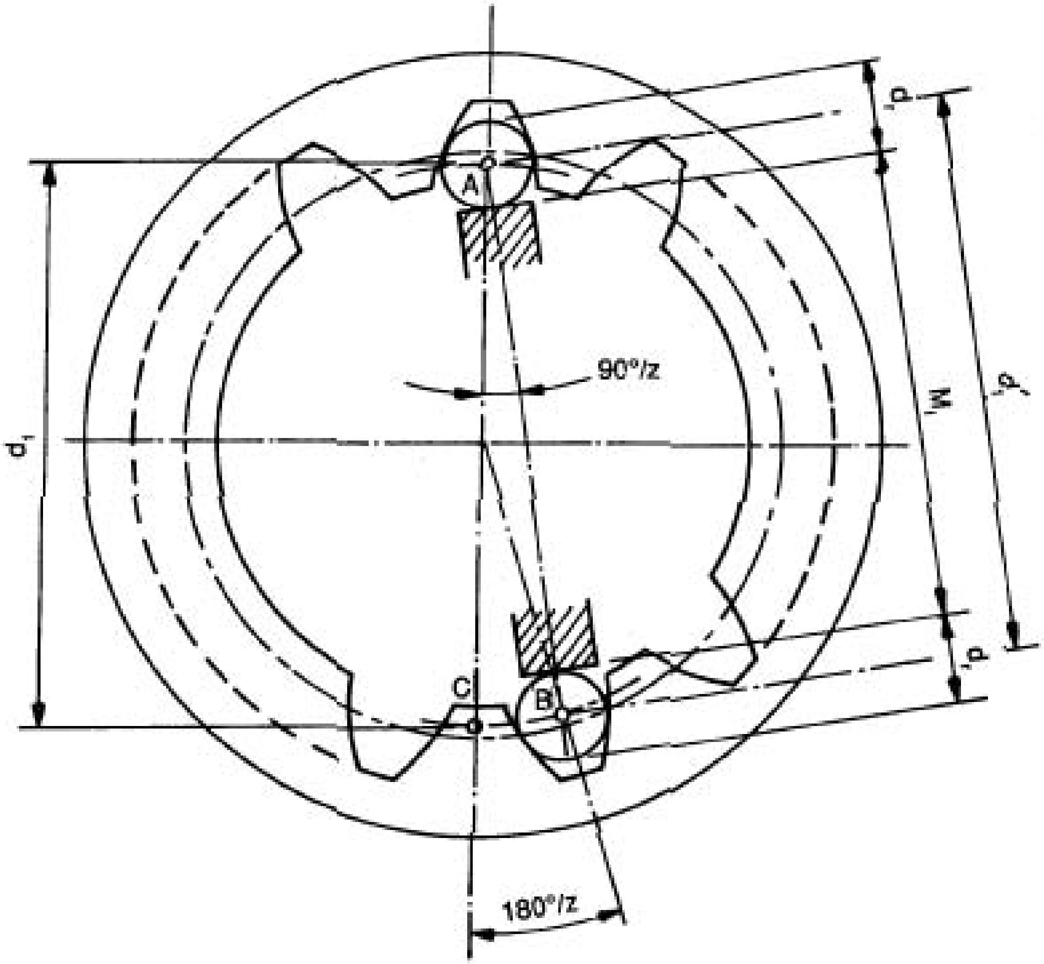


Fig. W.5 Measurement over balls in case of an internal gear with an odd number of teeth

with $s_n = \frac{\pi \cdot m_n}{2}$ or $\frac{\pi \cdot m_n}{2} \pm 2 x m_n \tan \alpha_n$, as the case may be.

For even number of teeth, $M_a = d_1 + d_r$, and for odd number of teeth, $M_a = d_r \cos \frac{90^\circ}{z} + d_r$ as before. In a similar manner, expressions for internal helical gears can be derived by using Eq. W.8 and suitably modifying Eqs W.5 W.6 and W 7 by taking cognizance of the helical aspect as done in Eq W.9.

Determination of ball diameter

As stated before, the diameter of the measuring balls or rollers or wires, etc., is not a critical dimension as can be easily seen by studying Fig. W.1 and the relevant equations thereof. As such, it can be arbitrarily chosen. However, it is desirable that the balls, etc., should touch the teeth surfaces at the pitch circle of the gear or as near as possible, as shown in Fig. W. 6 Under such condition, the diameter of the balls is found as follows:

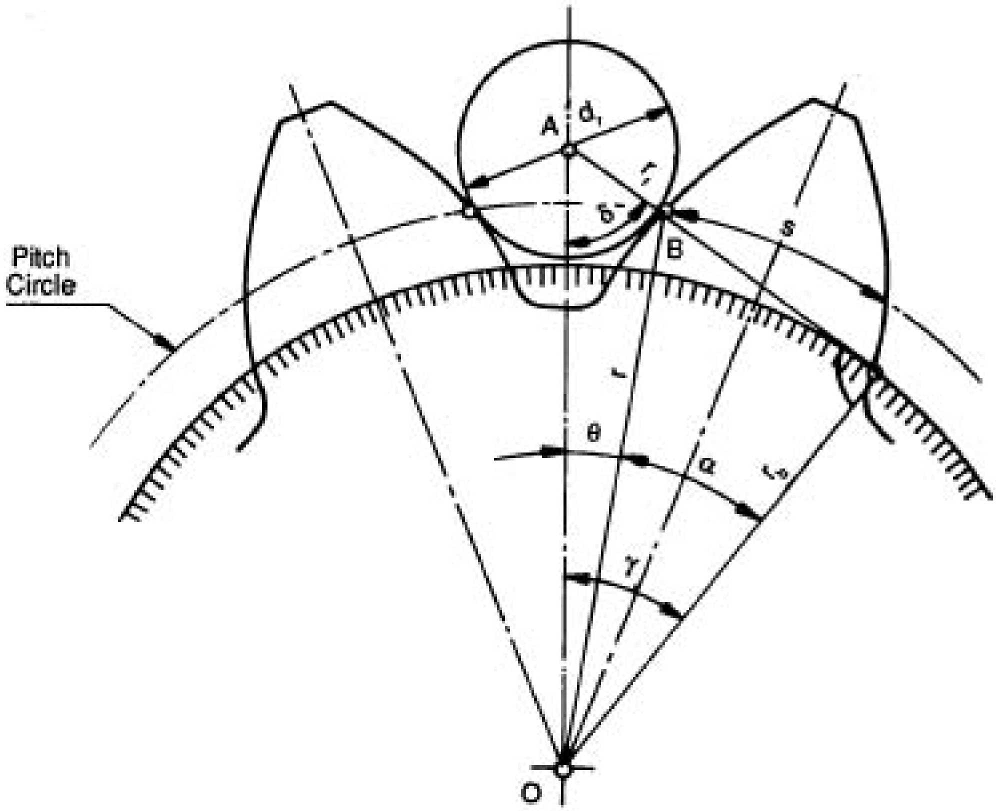


Fig. W.6 Measuring ball which touches the tooth surfaces at the pitch circle

By applying the law of sines in the triangle ABO, we have,

$$\frac{d_r/2}{r} = \frac{r_r}{r} = \frac{\sin\theta}{\sin\delta} \text{ or } \frac{d_r}{d} = \frac{\sin\theta}{\sin\delta},$$

whence

$$d_r = d\frac{\sin\theta}{\sin\delta} = d\,\frac{\sin\theta}{\cos\gamma} = d\,\frac{\sin\theta}{\cos(\alpha\pm\theta)} \tag{W.10}$$

The negative sign is valid for internal toothing.

Again

$$e = \frac{2\pi}{2z} - \frac{\cdots}{r} \text{ in rad.}$$

$$= \frac{360°}{2z} - \frac{s}{2r} \cdot \frac{180°}{\pi} \text{ in deg.}$$

$$= \frac{180°}{z} - \frac{180° \cdot s}{m z \pi}$$

$$= \frac{180°}{z} \times \left(1 - \frac{s}{\pi m}\right) \quad \text{(W.11)}$$

In case of uncorrected gears, the circular tooth thickness, $s = \frac{\pi m}{2}$, so that $\theta = \frac{90°}{z}$ If the available balls or rollers have diameters which are near to the values calculated as per Eq. W. 10, then these can also be used for measurement purposes. The following example illustrate the calculation procedures discussed so far.

Example W.1 The following data are given for an uncorrected internal spur gear:

$$z = 35, m = 4, \alpha = 20°, \text{ quality of tolerance: 7d.}$$

To calculate the toleranced values of measurement over balls.

Solution $d = mz = 4 \times 35 = 140$ mm, $s = \frac{\pi \cdot m}{2} = \frac{\pi \cdot 4}{2} = 6.283$ mm

From Appendix J, corresponding to $d = 140, m = 4$ and tolerance zone $= 7d$, we find the tooth thickness tolerances to be:

$$\text{Upper limit } A_{sU} = -0.056 \text{ mm}$$

$$\text{Lower limit } A_{sL} = -0.084 \text{ mm}$$

where subscripts U and L stand for upper and lower values, respectively. The two corresponding values of tooth thickness are given by

$$s_U = s - A_{sU} = 6.283 - 0.056 = 6.227 \text{ mm}$$

$$s_L = s - A_{sL} = 6.283 - 0.084 = 6.199 \text{ mm}$$

Incidentally, while calculating the above values in case of a helical gear, the tolerances in the normal section, calculated as per Eq. 2.129, are to be inserted in the relevant equations.

To find the ball diameter, we use Eqs. W. 11 and W. 10,

$$\theta = \frac{90°}{z} = \frac{90°}{35} = 2° \, 34' \, 17', 14''$$

$$d_r = d \frac{\sin\theta}{\cos(\alpha - \theta)} - \frac{140 \sin 2° \, 34' \, 17.14''}{\cos(20° - 2° \, 34' \, 17.34'')}$$

$$= 6.58 \text{ mm}$$

We can take balls of 7 mm diameter, which will serve the purpose. Balls and rollers of standard sizes are generally stocked in the shops.

Using **Eq. W.5** and the above two values of tooth thickness, we have the following two values of α_1

$$\text{inv}\,\alpha_{1A} = -\frac{7}{140\cos 20^\circ} + \text{inv}\,20^\circ - \frac{6.227}{140} + \frac{\pi}{35}$$
$$= 0.006977$$

whence α_{1A} = **15° 37' 55"**, by interpolation from Appendix H on Involute functions.

Similarly

$$\text{inv}\,\alpha_{1B} = -\frac{7}{140\cos 20^\circ} + \text{inv}\,20^\circ - \frac{6.199}{140} + \frac{\pi}{35}$$
$$= 0.007176$$

$$\therefore \quad \alpha_{1B} = 15^\circ\, 46'\, 35''$$

From **Eq. W.1** we get

$$d_{1A} = 140\,\frac{\cos 20^\circ}{\cos 15^\circ\, 37'\, 55''} = 136.610 \text{ mm}$$

$$d_{1B} = 140\,\frac{\cos 20^\circ}{\cos 15^\circ\, 46'\, 35''} = 136.707 \text{ mm}$$

Finally, by using **Eq. W.7**, we get the two values of measurement between balls

$$M_{1A} = d_{1A}\cos\frac{90^\circ}{z} - d_r = 136.610\cos\frac{90^\circ}{35} - 7 = 129.472 \text{ mm}$$

$$M_{1B} = d_{1B}\cos\frac{90^\circ}{z} - d_r = 136.707\cos\frac{90^\circ}{35} - 7 = 129.569 \text{ mm}$$

The tolerance between the above two values of M_i amounts to

$$129.569 - 129.472 = 0.097 \text{ mm}$$

Determination of Corrected Value of M

In manufacturing and inspection shops, ready-made tables and charts are usually available from which the M values, calculated on the basis of standard ball diameters, can be directly read off to alleviate the tedium of elaborate calculation procedures. However, it may so happen that the shop has a set of non-standard balls. In such cases, if the M value using a standard ball diameter is known (from available tables, etc.), then the M value using a non-standard ball can be calculated, provided the difference in ball diameters is slight. The method gives sufficiently accurate results, and is explained below. If the deviation in ball diameters is $\pm\Delta d_r$, then

$$M_{corrected} = M_{standard} \pm \Delta M_{standard} \qquad \text{(W.12)}$$

$$\Delta M_{standard} = \Delta d_r \;\text{C} \qquad \text{(W.13)}$$

where $M_{corrected}$ is the new value of M using balls of diameter $d_r \pm \Delta d_r$, $M_{standard}$ is the M

value using standard balls and found from the relevant tables and C is a factor, the value of which is also given normally in tables and charts. The factor C can be calculated by differentiation, using Eqs W.3 and W.4 in case of external gears, and Eqs W.6 and W.7 in case of internal gears. Thus

$$C = \frac{d(M)}{d(d_r)}$$

where M stands for M_a or M_i as the case may be. We have the following relations by differentiation:

For external spur gear with an even number of teeth: $$C = \frac{1}{\sin \alpha_r} + 1 \qquad (W.14)$$

For external spur gear with an odd number of teeth: $$C = \frac{\cos \frac{90°}{z}}{\sin \alpha_r} + 1 \qquad (W.15)$$

For internal spur gear with an even number of teeth: $$C = -\left(\frac{1}{\sin \alpha_t} + 1\right) \qquad (W.16)$$

For internal spur gear with an odd number of teeth: $$C = -\left(\frac{\cos \frac{90°}{z}}{\sin \alpha_r} + 1\right) \qquad (W.17)$$

Example W.2 A factory produces standard corrected gear as per the 05-system of toothing (see Sec. 8.12). The following data of an external spur gear are given: $z = 24$, $m = 3$, $d_r = 6$ mm, $M_a = 84.064$ mm and $C = 3.0047$ (as per chart).
Find the corrected value of M_a if balls having a diameter of 6.1 mm are used.

Solution $\Delta d_r = 6.1 - 6 = +0.1$ mm

$\therefore$ $\Delta M_{standard} = \Delta d_r \cdot C = 0.1 \times 3.0047 = +0.30047$ (as per Eq. W.13)

$\Delta M_{corrected} = M_{standard} + \Delta M_{standard}$ (as per Eq. W.12)

$= 84.064 + 0.30047$

$= 84.364$ mm

Check Using Eq. W.2 and taking $x = +0.5$ and $\alpha = 20°$, we have

$$\text{inv}\,\alpha_r = \frac{d_r}{d \cos \alpha} + \text{inv}\,\alpha + \frac{\pi m/2 + 2xm \tan \alpha}{d} - \frac{\pi}{z}$$

$$= \frac{6}{72 \times 0.93969} + 0.014904 + \frac{\pi\, 3/2 + 2 \times 0.5 \times 3 \times 0.36397}{72} - \frac{\pi}{24}$$

$= 0.05330$, whence $\alpha_r = 29° 55' 22''$

$$d_1 = 72 \frac{\cos 20°}{\cos 29° 55' 22''} = 78.064 \qquad \text{(as per Eq. W.1)}$$

$$M_{standard} = d_1 + d_r = 78.064 + 6 = 84.064 \qquad \text{(as per Eq. W.3)}$$

$$C = \frac{1}{\sin 29° \; 55' \; 22''} + 1 = 3.0047 \qquad \text{(as per Eq. W. 14)}$$

$$\Delta M_{standard} = \Delta d_r \cdot C = 0.1 \times 3.0047 = +0.30047 \text{ mm} \qquad \text{(as per Eq. W.13)}$$

$$\therefore \quad M_{corrected} = M_{standard} + \Delta M_{standard} \qquad \text{(as per Eq. W.12)}$$

$$= 84.064 + 0.30047 = 84.364 \text{ mm}$$

The above value tallies with the value found previously. Using the relevant formulae, viz., Eqs W.1, W.2, and W.3, and taking d_r = 6.1 mm, the reader may recalculate the value of M_a as follows:

$$\text{inv}\, a_r = \frac{6.1}{72 \cos 20°} + \text{inv}\, 20° + \frac{15\pi + 2 \times 0.5 \times 3 \tan 20°}{72} - \frac{\pi}{24}, \; \therefore \; a_r = 30° \; 10' \; 315''$$

$$d_1 = 72 \cos 20°/\cos 30° \; 10' \; 315'' = 78.263 \quad \therefore M_a = 78.263 + 6.1 = 84.363 \text{ mm}$$

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